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ON THE CONVERGENCE IN THE SPACE OF MIKUSIŃSKI'S OPERATORS

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Class C of continuous complex-valued functions of a non-negative real variable forms a commutative algebra without zero divisors where the product is defined as the finite convolution and the sums and scalar products are defined in the usual way. The quotient field $\mathcal{M}$ of this algebra is the operator field of Mikusiński. In this operator field we have two types of convergence:

Definition 1. A sequence $a_n \in \mathcal{M}$ is said to be G_1 convergent to $a \in \mathcal{M}$, denoted by $a_n \overset{G_1}{\rightarrow} a$, if and only if there exists $q \in C$, $q \neq \{o\}$ such that $qa_n, qa \in C$ and $qa_n \overset{C}{\rightarrow} qa$, where the C convergence means the almost uniform convergence.

Definition 2. A sequence $a_n \in \mathcal{M}$ is said to be G_2 convergent to $a \in \mathcal{M}$, denoted by $a_n \overset{G_2}{\rightarrow} a$, if and only if $a_n = f_n/g_n$, $a = f/g$ where $f_n, g_n, f, g \in C$; $g_n, g \neq \{o\}$ and $f_n \overset{C}{\rightarrow} f$, $g_n \overset{C}{\rightarrow} g$.

We will repeat some well-known facts (see [1]).

Proposition 1. $G_1 \subset G_2$ in the sense that every G_1 convergent sequence is also G_2 convergent.

Proposition 2. G_1 is not topological in the sense that there is not a topology for $\mathcal{M}$ such that G_1 is exactly the convergence class of that topology.

In the work [2] there is the theorem which states:

Theorem 2. G_2 convergence is not topological

The proof is not correct because the sequence $nI/(s-nI)$ was used as a C convergent sequence but it is only G_2 convergent.

Let LTG_1 be the smallest topological convergence greater than G_1. It consists of G_1 convergent sequences and of such sequences which every subsequence possesses a convergent subsequence. There is an interesting question raised by J. Mikusiński, R. Bednarek [3]. What is the relationship between G_2 and LTG_1?

The next theorem in the same work [2] states:

Theorem 3. $G_2 \subset LTG_1$

It is also not correct because:

Theorem. *Some G_2 convergent sequences are not LTG_1 convergent.*

Proof: Let $x_n = f_n/g_n$ G_2 converge to $x = f/g$ and let $s(f_n) = s(f) = s(g_n) = o$ while $s(g) = s_o > 0$. Here $s(h)$ denotes the support number of the function $h \in C$, defined by D. Norris [4]. Suppose that every subsequence of x_n, denoted by $x_{n,k}$, has a subsequence $x_{n,k,m}$ such that $x_{n,k,m} \xrightarrow{G_1} x$. Then there exists $q \in C$ such that $qx_{n,k,m}$, $qx \in C$ and $qx_{n,k,m} \xrightarrow{C} qx$.

From $qx \in C$, according to Titchmarsh's theorem, it follows that $q = 0$ on $[0, \gamma]$ where $\gamma \geqslant s_0$. Then $qx_{n,k,m} = 0$ on $[0, \gamma]$ and $qx = 0$ on $[0, \gamma - s_0]$. But if a sequence h_n converges in C to h then lim sups $(h_n) \leqslant s(h)$, as was shown by T. Boehme [1], and we get the counterdiction.

An interesting question is where the mistake in the proof of the theorem 3. is. It lies in the wrong application of T. Boehme's theorem from [1], i.e. in the neglecting of the case when it cannot be used.

LITERATURE

[1] T. Boehme, *On Mikusiński operators*, Studia Math. 33, 1969 (127—140).

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[4] D. Norris, *A topology for Mikusiński operators*, Studia Math. 24 1964 (245—255).