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TRANSMISSION OF WAVES THROUGH A MAGNETIZED LAYER IMBEDDED BETWEEN TWO HALF-SPACES

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Summary: The transmission of waves propagated normally through magnetized elastic layer imbedded between two elastic half-spaces is considered. The transmission ratios are calculated and the effects of magnetic field on this transmission are shown graphically, and it has been observed that the effects are more prominent for the waves of small wave lengths.

1. Introduction

The transmission of normally incident waves through a sandwiched homogeneous elastic layer has been considered by Sinha [2][1]. The author [1 *a*, *b*, *c*] has discussed the transmission of waves through layers having visco-elastic and relaxation properties, and also the transmission in the presence of a second-order fluid layer. In the present paper the effects of magnetic field on the transmission of waves propagated normally through a layer of finite thickness lying between two half-spaces have been studied. All the media are homogeneous, isotropic and elastic. The half-spaces have identical properties which are different from those of the intermediate layer. The imbedded layer is subjected to a constant magnetic field. The transmission ratios are calculated numerically and the effects of magnetic field on this transmission are observed.

2. Solution of the Problem

The problem is solved by taking the cartesian coordinate system with the z-axis vertically downwards. Let the regions $-\infty \leqslant z \leqslant -\frac{H}{2}$ and $H/2 \leqslant z \leqslant \infty$ be occupied by homogeneous isotropic elastic medium with rigidity μ_1 and density ρ_1. Let the homogeneous isotropic elastic layer with rigidity modulus μ_2 and

[1]) Numbers within square brackets refer to References.

mass density ρ_2 occupy the region $-H/2 \leqslant z \leqslant H/2$. We assume that this intermediate layer is subjected to a constant magnetic field $\vec{H}=(H_x, H_y, H_z)$. We concern ourselves only with the case of transverse waves propagated normally to the discontinuity surfaces. Therefore we can assume that the displacement components $(u, v, w)=(0, v, 0)$ and the condition $\partial/\partial x=\partial/\partial y=0$. Let v_1, v_2, v_3 be the components of displacement in the upper medium, layer and lower medium respectively. Assuming that $H_y=0$ and that the magnetic permeability is unity the equation of motion in the layer reduces to

$$\rho_2 \frac{\partial^2 v_2}{\partial t^2}=\left(\mu_2+\frac{H_z^2}{4\pi}\right)\frac{\partial^2 v_2}{\partial z^2} \tag{1}$$

The induced magnetic field $\vec{h}$ and the induced electric field $\vec{E}$ are given by

$$\left.\begin{aligned} \vec{h}&=\left(0,\ H_z\frac{\partial v_2}{\partial z},\ 0\right) \\ \vec{E}&=\frac{1}{c}(-H_z,\ 0,\ H_x)\frac{\partial v_2}{\partial t} \end{aligned}\right\} \tag{2}$$

where c denotes the velocity of light.

The equations of motion for the upper and lower media become

$$\rho_1 \frac{\partial^2 v_1}{\partial t^2}=\mu_1 \frac{\partial^2 v_1}{\partial z^2} \tag{3}$$

$$\rho_1 \frac{\partial^2 v_3}{\partial t^2}=\mu_1 \frac{\partial^2 v_3}{\partial z^2} \tag{4}$$

The solutions of the above wave equations may be taken as

$$v_A=A \exp\{i(pt=fz)\} \tag{5}$$

$$v_{A'}=A' \exp\{i(pt+fz)\} \tag{6}$$

$$v_B=B \exp\{i(pt-gz)\} \tag{7}$$

$$v_{B'}=B' \exp\{i(pt+gz)\} \tag{8}$$

$$v_C=C \exp\{i(pt-fz)\} \tag{9}$$

where

$$p=f\beta_1, \qquad \beta_1=\sqrt{\mu_1/\rho_1},$$

$$f=2\pi/L, \qquad g=\frac{p}{\beta_2}\frac{1}{(1+\eta)^{1/2}},$$

$$\beta_2=\sqrt{\mu_2/\rho_2}, \qquad \eta=H_z^2/4\pi\mu_2$$

and A, A', B, B', C are unkown constants to be deteimined from the boundary conditions and L is the wave length.

The boundary conditions which must be satisfied at interfaces are

(i) the continuity of the tangential component of the electric field,

(ii) the continuity of the magnetic field vector,

(iii) the continuity of the displacements and

(iv) the continuity of the total shear stress.

The boundary conditions relevant to the determination of the transmission for the type of waves considered in this problem are (iii) and (iv).

The total shear stresses in the upper medium, layer and the lower medium respectively are

(10) $$p_{yz} = \mu_1 \frac{\partial v_1}{\partial z}$$

(11) $$p_{yz} = \mu_2 \frac{\partial v_2}{\partial z} + \frac{1}{4\pi} H_z h_y$$

(12) $$p_{yz} = \mu_1 \frac{\partial v_3}{\partial z}$$

where

(13) $$v_1 = v_A + v_{A'}$$

(14) $$v_2 = v_B + v_{B'}$$

(15) $$v_3 = v_C$$

Hence the boundary conditions are

$$\left.\begin{aligned} &\text{I.} \quad v_1 = v_2 \\ &\text{II.} \quad \mu_1 \frac{\partial v_1}{\partial z} = \mu_2 \frac{\partial v_2}{\partial z} + \frac{1}{4\pi} H_z h_y \end{aligned}\right\} \text{ at } z = -H/2,$$

$$\left.\begin{aligned} &\text{III.} \quad v_2 = v_3 \\ &\text{IV.} \quad \mu_2 \frac{\partial v_2}{\partial z} + \frac{1}{4\pi} H_z h_y = \mu_1 \frac{\partial v_3}{\partial z} \end{aligned}\right\} \text{ at } z = H/2.$$

The factor v_C/v_A gives the expression for transmission ratio R which is related with C/A as

(16) $$C/A = \mathrm{R}e^{i\Phi}$$

where Φ is the change in phase.

The above four boundary conditions along with Eqs. (10) to (15) give four equations in five unkowns A, A', B, B' and C, but these equations can be solved for C/A to obtain the expression for the transmission ratio R with the help of Eq (16) in the form

(17) $$R^2 = \frac{4\sigma\xi(1+\eta)}{\left[4\sigma\xi(1+\eta)\cos^2\left\{\frac{2\pi}{k}\sqrt{\frac{\sigma}{\xi(1+\eta)}}\right\} + \{1+\sigma\xi(1+\eta)\}^2 \sin^2\left\{\frac{2\pi}{k}\sqrt{\frac{\sigma}{\xi(1+\eta)}}\right\}\right]}$$

$$(18) \qquad \tan\Phi = \frac{\dfrac{2\sqrt{\sigma\xi(1+\eta)}}{1+\sigma\xi(1+\eta)}\tan\dfrac{2\pi}{k} - \tan\left\{\dfrac{2\pi}{k}\sqrt{\dfrac{\sigma}{\xi(1+\eta)}}\right\}}{\dfrac{2\sqrt{\sigma\xi(1+\eta)}}{1+\sigma\xi(1+\eta)} + \tan\dfrac{2\pi}{k}\tan\left\{\dfrac{2\pi}{k}\sqrt{\dfrac{\sigma}{\xi(1+\eta)}}\right\}},$$

where the dimensionless σ, ξ, η, k are defined as

$$\sigma = \rho_2/\rho_1, \quad \xi = \mu_2/\mu_1,$$
$$k = L/H, \quad \eta = H_z^2/4\pi\mu_2.$$

Numerical results are obtained by taking $\sigma=1$, $\xi=2,5$ and $L/H=1, 2, 5$. Figures 1 and 2 relating $\eta\,(0 \leqslant \eta \leqslant 10)$ and R^2 show the effects of magnetic field on the transmission of normally incident transverse waves. It can be observed from the graphs in the figures that the effects are more prominent for the waves of small wave lengths and also for the small values of η.

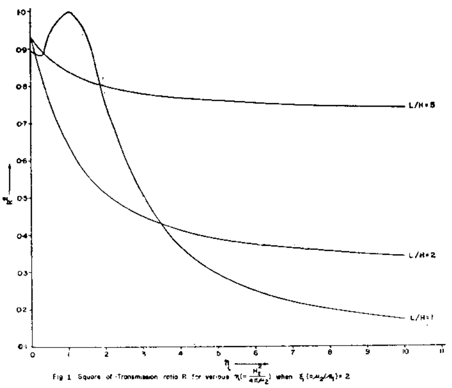

Fig. 1.

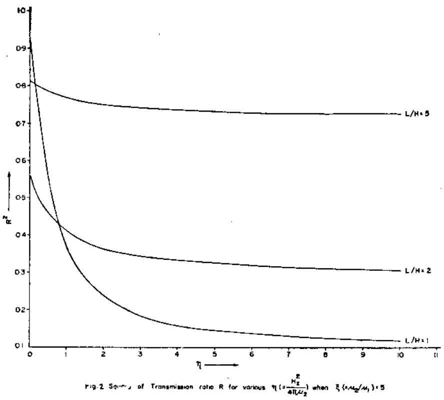

Fig. 2.

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