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ON ALMOST HYPERBOLIC SPACES

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Summary

Dube [1]* has defined an almost hyperpolic Hermitian manifold, almost hyperbolic Kahlerian and nearly hyperbolic Kahlerian space. In this paper we have given a large number of definitions involving Nijenhuis tensor. With the help of these definitions in an almost hyperbolic spaces some results have been obtained.

1. Introduction

Let Mn be a C^∞ real differentiable manifold of dimension n, enclosed with a real vector valued function F such that

(1.1) $$\overline{\overline{X}}=X,$$

where

(1.2) $$\overline{X} \stackrel{\text{def}}{=} F(X).$$

Let us further suppose that in Mn there is given a Riemannian metric g, such that

(1.3) $$g(\overline{X}, \overline{Y}) = -g(X, Y).$$

Then Mn is called an almost hyperbolic Hermitian manifold.

Let $F(X, Y) \stackrel{\text{def}}{=} g(\overline{X}, Y)$,

so that we have

(1.4) $$F(X, \overline{Y}) = g(\overline{\overline{X}}, \overline{Y}) = -g(X\ Y) = -F(\overline{X}, Y),$$

(1.5) $$F(\overline{X}, \overline{Y}) = g(X, \overline{Y}) = -g(\overline{X}\ Y) = -F(X, Y),$$

(1.6) $$F(X, Y) = -F(Y, X).$$

* Numbers in the square brackets indicate the references at the end of this paper.

From (1.5) and (1.6) it can be seen that $F(X, Y)$ is hybrid and skew-symmetric in its covariant slots X and Y.

Let D be a Riemannian connection in Mn, such that [3]

(1.7)a $$D_X Y - D_Y X = [X, Y]$$

and

(1.7)b $$X g(Y, Z) = g(D_X Y, Z) + g(Y, D_X Z).$$

Also for an almost hyperbolic Hermite space, we have [1]

(1.8) $$(D_X F)(Y, \bar{Z}) = (D_X F)(\bar{Y}, Z),$$

(1.9) $$(D_X F)(\bar{Y}, \bar{Z}) = (D_X F)(Y, Z).$$

Let N be (1.2) Nijenhuis tensor defined as follows [2]

(1.10) $$N(X, Y) = [\bar{X}, \bar{Y}] + [X, Y] - \overline{[\bar{X}, Y]} - \overline{[X, \bar{Y}]}.$$

We have also from [3]

(1.11) $$(D_X F)(Y) = D_X FY - F D_X Y.$$

D e f i n i t i o n (1.1) *An almost hyperbolic Hermitian manifold is called almost hyperbolic Kahlerian iff* $dF = 0$,

where $dF \stackrel{\text{def}}{=} (D_X F)(Y, Z) + (D_Y F)(Z, X) + (D_Z F)(X, Y)$.

D e f i n i t i o n (1.2) *An almost hyperbolic manifold is called nearly hyperbolic Kahlerian iff*

$$(D_X F)(Y, Z) + (D_Y F)(X, Z) = 0.$$

2. Some Results

In an almost hyperbolic space Nijenhuis tensor is

(2.1) $$N(X, Y) = [\bar{X}, \bar{Y}] + F^2[X, Y] - F[\bar{X}, Y] - F[X, \bar{Y}].$$

Let us put $M(X, Y) \stackrel{\text{def}}{=} (D_{\bar{X}} F)(Y) - \overline{(D_X F)(Y)}$.
Thus

$$M(X, Y) - M(Y, X) = D_X(FY) - F D_X Y - \overline{D_X(FY)} + F \overline{D_X Y}$$

(2.2) $$- D_{\bar{Y}}(FX) + F D_{\bar{Y}} X + \overline{D_Y(FX)} - F \overline{D_Y X}$$

$$= [\bar{X}, \bar{Y}] + [X, Y] - F[\bar{X}, Y] - F[X, \bar{Y}] = N(X, Y).$$

T h e o r e m (2.1) *We prove that* $N(X, Y)$ *is equal to each of the following expressions:*

(2.3)

(*a*) $P(X, Y) - \overline{P(X, \bar{\bar{Y}})}$,

(*b*) $P(X, Y) + P(\bar{X}, \bar{Y})$,

(*c*) $Q(X, Y) - \overline{Q(\bar{X}, Y)}$,

(*d*) $T(X, Y) - \overline{T(X, Y)}$,

where

$$\text{(2.4)} \qquad (a) \quad P(X, Y) \overset{\text{def}}{=} [\bar{X}, \bar{Y}] - \overline{[X, \bar{Y}]},$$

$$(b) \quad Q(X, Y) \overset{\text{def}}{=} [\bar{X}, \bar{Y}] - \overline{[\bar{X}, Y]},$$

$$(c) \quad T(X, Y) \overset{\text{def}}{=} [\bar{X}, \bar{Y}] + [X, Y].$$

P r o o f: From (2.4)a, we have

$$\text{(2.5)} \qquad \overline{P(X, Y)} = \overline{[\bar{\bar{X}}, Y]} - [X, Y].$$

Subtracting (2.5) from (2.4)a and using (2.1), we get (2.3)a. For the proof of (2.3)b barıing X and Y both in (2.4)a and adding the resulting equation in (2.4)a, we gєt (2.3)b.

From (2.4)b, we have

$$\text{(2.6)} \qquad \overline{Q(\bar{X}, Y)} = \overline{[X, \bar{\bar{Y}}]} - [X, Y].$$

Subtracting (2.6) from (2.4)b and using (2.1), we get (2.3)c.

From (2.4), we have

$$\text{(2.7)} \qquad \overline{T(\bar{X}, Y)} = \overline{[X, \bar{\bar{Y}}]} + \overline{[\bar{\bar{X}}, Y]}.$$

Subtracting (2.7) from (2.4) *c*, we get required result (2.3)d.

C o r o l l a r y (2.1). From (2.4)a, b, c we see that $P(X, Y)$, $Q(X, Y)$ and $T(X, Y)$ are skew-symmetric in X and Y. It can also be seen that $T(X, Y)$ is hybrid in its covariant slots.

T h e o r e m (2.2) *From the definition of* (24)a, b, c, *we have the following*

$$\text{(i)} \quad \overline{Q(\bar{X}, Y)} = -P(\bar{X}, \bar{Y}),$$

$$\text{(ii)} \quad P(X, Y) + \overline{T(\bar{X}, Y)} - Q(X, Y) = 2\,\overline{[\bar{\bar{X}}, Y]}.$$

P r o o f. (i) $Q(X, Y) = [\bar{X}, \bar{Y}] - \overline{[\bar{X}, Y]}$.

Therefore,

$$\overline{Q(\bar{\bar{X}}, Y)} = \overline{[X, \bar{\bar{Y}}]} - [X, Y] = -P(\bar{\bar{X}}, \bar{Y}).$$

$$\text{(ii)} \quad P(X, Y) + \overline{T(\bar{\bar{X}}, \bar{Y})} - Q(X, Y) = [\bar{X}, \bar{Y}] - \overline{[X, \bar{\bar{Y}}]} + [X, \bar{Y}] +$$

$$+ \overline{[\bar{X}, Y]} - [\bar{X}, \bar{Y}] + [\bar{X}, Y] = 2\,[\bar{X}, Y].$$

3. On Almost Hyperbolic Spaces

Definition (3.1). We will define $H(X, Y, Z)$, $J(X, Y, Z)$ and $M(X, Y, Z)$ in the following manner

$$H(X, Y, Z) \stackrel{\text{def}}{=} (D_X F)(Y, Z) - (D_Y F)(X, Z), \tag{3.1}$$

$$J(X, Y, Z) \stackrel{\text{def}}{=} (D_X F)(Y, Z) + (D_Y F)(Z, X) + (D_Z F)(X, Y), \tag{3.2}$$

$$M(X, Y, Z) \stackrel{\text{def}}{=} (D_{\bar{X}} F)(Y, Z) - (D_X F)(Y, \bar{Z}). \tag{3.3}$$

Theorem (3.1). *A necessary and sufficient condition for an almost hyperbolic Hermitian manifold to be a hyperbolic Kahlerian space is*

$$H(X, Y, Z) + H(Z, X, Y) - H(Y, Z, X) = 0. \tag{3.4}$$

Proof. From (3.1), we have

$$H(X, Y, Z) + H(Z, X, Y) - H(Y, Z, X) = (D_X F)(Y, Z) - (D_Y F)(X, Z) + \\ + (D_Z F)(X, Y) - (D_X F)(Z, Y) - (D_Y F)(Z, X) + \\ + (D_Z F)(Y, X) = 2(D_X F)(Y, Z) = 0,$$

since the space is hyperbolic Kahlerian [4].
The converse is also true.

Theorem (3.2). *We have*

$$H(\bar{Z}, \bar{Y}, X) + H(X, \bar{Y}, \bar{Z}) + H(X, \bar{Z}, \bar{Y}) = 2(D_{\bar{Z}} F)(\bar{Y}, X). \tag{3.5}$$

Proof. Barring Y and Z in (3.1) and by virtue of (1.9), we have

$$H(X, \bar{Y}, \bar{Z}) + H(X, \bar{Z}, \bar{Y}) + H(\bar{Z}, \bar{Y}, X) = (D_X F)(Y, Z) - (D_{\bar{Y}} F)(X, \bar{Z}) + \\ + (D_X F)(Z, Y) - (D_{\bar{Z}} F)(X, \bar{Y}) + (D_{\bar{Z}} F)(\bar{Y}, X) \\ - (D_{\bar{Y}} F)(\bar{Z}, X) = 2(D_{\bar{Z}} F)(\bar{Y}, X).$$

Lemma (3.1). *For an almost hyperbolic Kahlerian space, we have*

$$(D_{\bar{X}} F)(\bar{Y}, Z) + (D_{\bar{Y}} F)(\bar{Z}, X) + (D_{\bar{Z}} F)(\bar{X}, Y) = 0. \tag{3.6}$$

Proof. Since for an almost hyperbolic Kahlerian space $dF = 0$, that is

$$(D_X F)(Y, Z) + (D_Y F)(Z, X) + (D_Z F)(X, Y) = 0$$

Therefore,

$$(D_{\bar{X}} F)(\bar{Y}, Z) = -(D_{\bar{Y}} F)(Z, \bar{X}) - (D_Z F)(X, Y). \tag{3.7}$$

Similarly

$$(D_{\bar{Y}} F)(\bar{Z}, X) = -(D_{\bar{Z}} F)(X, \bar{Y}) - (D_X F)(Y, Z) \tag{3.8}$$

and

$$(D_{\bar{Z}} F)(\bar{X}, Y) = -(D_{\bar{X}} F)(Y, \bar{Z}) - (D_Y F)(Z, X). \tag{3.9}$$

Adding (3.7), (3.8), (3.9) and using (1.8), we get

$$2\{(D_{\overline{X}}F)(\overline{Y},Z)+(D_{\overline{Y}}F)(\overline{Z},X)+(D_{\overline{Z}}F)(\overline{X},Y)\}=$$
$$=-\{(D_Z F)(X,Y)+(D_X F)(Y,Z)+(D_Y F)(Z,X)\}.$$

But for an almost hyperbolic Kahlerian space,

$$(D_X F)(Y,Z)+(D_Y F)(Z,X)+(D_Z F)(X,Y)=0.$$

Hence

$$(D_{\overline{X}}F)(\overline{Y},Z)+(D_{\overline{Y}}F)(\overline{Z},X)+(D_{\overline{Z}}F)(\overline{X},Y)=0.$$

Theorem (3.3). *A necessary condition for an almost hyperbolic Hermitian manifold to be an almost hyperbolic Kahlerian space is*

(3.10) $$H(X,\overline{Y},\overline{Z})+H(Y,\overline{Z},\overline{X})+H(Z,\overline{X},\overline{Y})=0.$$

Proof. From the definition of (3.1), (1.7) and (1.8), we get

(3.11) $$H(X,\overline{Y},\overline{Z})=(D_X F)(Y,Z)-(D_{\overline{Y}}F)(X,\overline{Z}).$$

Similarly

(3.12) $$H(Z,\overline{X},\overline{Y})=(D_Z F)(X,Y)-(D_{\overline{X}}F)(Z,\overline{Y})$$

and

(3.13) $$H(Y,\overline{Z},\overline{X})=(D_Y F)(Z,X)-(D_{\overline{Z}}F)(Y,\overline{X}).$$

Adding (3.11), (3.12) and (3.13), we get

$$H(X,\overline{Y},\overline{Z})+H(Y,\overline{Z},\overline{X})+H(Z,\overline{X},\overline{Y})$$
$$=(D_X F)(Y,Z)+(D_Y F)(Z,X)+(D_Z F)(X,Y)$$
$$-\{(D_{\overline{Y}}F)(X,\overline{Z})+(D_{\overline{Z}}F)(Y,\overline{X})+(D_{\overline{X}}F)(Z,\overline{Y})\}=0,$$

where we have made use of (3.6) and the definition of an almost hyperbolic Kahlerian space.

Corollary (3.1). From (3.1) we have the following obvious results:

(a) $$H(\overline{X},\overline{Y},\overline{Z})-H(\overline{X},\overline{Z},\overline{Y})=2(D_{\overline{X}}F)(Y,Z)+$$
$$+(D_{\overline{Y}}F)(Z,X)+(D_{\overline{Z}}F)(X,Y),$$

(b) $$H(X,Y,\overline{Z})-H(X,\overline{Z},Y)=2(D_X F)(Y,\overline{Z})+$$
$$+(D_Y F)(\overline{Z},X)+(D_{\overline{Z}}F)(X,Y),$$

(3.14) (c) $$H(X,\overline{Y},Z)-H(X,Z,\overline{Y})=2(D_X F)(\overline{Y},Z)+$$
$$+(D_{\overline{Y}}F)(Z,X)+(D_Z F)(X,\overline{Y}),$$

(d) $$H(\overline{X},\overline{Z},Y)-H(\overline{X},Y,\overline{Z})=2(D_{\overline{X}}F)(\overline{Z},Y)-$$
$$-(D_{\overline{Z}}F)(\overline{X},Y)+(D_Y F)(X,Z).$$

Theorem (3.4). *If the space has any two of the following, it has the third one also,*

(a) *the space is almost hyperbolic Kahlerian,*

(b) *the space is hyperbolic Kahlerian,*

(c) *it is the manifold for which*

$$H(X, Y, Z) = H(X, Z, Y).$$

Proof. From (3.1) and (3.2), we have

$$H(X, Y, Z) - H(X, Z, Y) = J(X, Y, Z) + (D_X F)(Y, Z). \tag{3.15}$$

Theorem (3.5). *A necessary condition for an almost hyperbolic Hermitian manifold to be an almost hyperbolic Kahlerian is expressed as follows:*

$$H(X, \overline{Y}, \overline{Z}) + H(\overline{X}, Y, \overline{Z}) + H(X, Y, Z) = \tag{3.16}$$

$$= 2(D_X F)(Y, Z) + 2(D_Y F)(Z, X) - (D_{\overline{Z}} F)(\overline{X}, Y).$$

Proof. In view of (1.9) and the definition of $H(X, Y, Z)$, we have

$$H(X, \overline{Y}, \overline{Z}) + H(\overline{X}, Y, \overline{Z}) + H(X, Y, Z) = 2(D_X F)(Y, Z) +$$

$$+ 2(D_Y F)(Z, X) + (D_{\overline{X}} F)(\overline{Y}, Z) + (D_{\overline{Y}} F)(Z, \overline{X}),$$

which by virtue of (3.6) yields

$$H(X, \overline{Y}, \overline{Z}) + H(\overline{X}, Y, \overline{Z}) + H(X, Y, Z) = 2(D_X F)(Y, Z) +$$

$$+ 2(D_Y F)(Z, X) - (D_{\overline{Z}} F)(\overline{X}, Y).$$

Theorem (3.6). *From the definition* (3.1) *and* (3.3), *we have*

$$M(X, Y, Z) - M(Y, X, Z) = -H(X, Y, Z) + H(\overline{X}, \overline{Y}, \overline{Z}) \tag{3.17}$$

and consequently for an almost hyperbolic Kahlerian space

$$H(\overline{X}, \overline{Y}, Z) + H(Y, \overline{Z}, \overline{X}) = (D_Y F)(Z, X). \tag{3.18}$$

Proof. We have from (3.3)

$$M(X, Y, Z) = (D_{\overline{X}} F)(Y, Z) - (D_X F)(Y, \overline{Z}) \tag{3.19}$$

and

$$M(Y, X, Z) = (D_{\overline{Y}} F)(X, Z) - (D_Y F)(X, \overline{Z}). \tag{3.20}$$

Subtracting (3.20) from (3.19), we get

$$M(X, Y, Z) - M(Y, X, Z) = (D_{\overline{X}} F)(Y, Z) - (D_X F)(Y, \overline{Z}) -$$

$$- (D_{\overline{Y}} F)(X, Z) + (D_Y F)(X, \overline{Z}),$$

which with the help of (3.1) becomes

$$M(X, Y, Z) - M(Y, X, Z) = -H(X, Y, Z) + H(\overline{X}, \overline{Y}, \overline{Z})$$

and consequently,

$$
\begin{aligned}
H(\overline{X}, \overline{Y}, Z) + H(Y, \overline{Z}, \overline{X}) &= (D_{\overline{X}} F)(\overline{Y}, Z) - (D_{\overline{Y}} F)(\overline{X}, Z) + \\
&+ (D_Y F)(\overline{Z}, \overline{X}) - (D_{\overline{Z}} F)(Y, \overline{X}) = (D_{\overline{X}} F)(\overline{Y}, Z) + \\
&+ (D_{\overline{Y}} F)(Z, \overline{X}) + (D_{\overline{Z}} F)(\overline{X}, Y) + (D_Y F)(\overline{Z}, \overline{X}) \\
&= (D_Y F)(Z, X),
\end{aligned}
$$

by virtue of (1.8), (1.9) and (3.6).

Theorem (3.7). *In an almost hyperbolic Hermitian manifold, we have*

(3.21)a $$M(\overline{X}, \overline{Y}, Z) = -M(X, \overline{Y}, \overline{Z}) = -M(X, Y, Z)$$

and

(3.21)b
$$M(\overline{X}, \overline{Y}, \overline{Z}) = -M(\overline{X}, Y, \overline{Z}),$$
$$M(\overline{X}, Y, Z) = -M(X, \overline{Y}, Z) = -M(X, Y, \overline{Z}).$$

Proof. From (3.3) and (1.8), we have

(3.22) $$M(\overline{X}, \overline{Y}, Z) = (D_X F)(\overline{Y}, Z) - (D_{\overline{X}} F)(Y, Z)$$

and

(3.23) $$M(X, \overline{Y}, \overline{Z}) = (D_{\overline{X}} F)(Y, Z) - (D_X F)(\overline{Y}, Z).$$

Thus from (3.3), (3.22) and (3.23), we get (3.21)a. To prove (3.21)b, we have

$$M(\overline{X}, Y, Z) = (D_X F)(Y, Z) - (D_{\overline{X}} F)(Y, \overline{Z}),$$

$$M(X, \overline{Y}, Z) = (D_{\overline{X}} F)(\overline{Y}, Z) - (D_X F)(Y, Z),$$

$$M(X, Y, \overline{Z}) = (D_{\overline{X}} F)(\overline{Y}, Z) - (D_X F)(Y, Z).$$

Theorem (3.8). *In nearly hyperbolic Kahlerian space,*

(3.24)a $$[\overline{X}, \overline{Y}] - [X, Y] = \overline{D_{\overline{X}} Y} + \overline{D_Y \overline{X}} - D_{\overline{Y}} \overline{X} - D_X Y,$$

(3.24)b $$[X, \overline{Y}] - [\overline{X}, Y] = \overline{D_X Y} + \overline{D_Y X} - D_{\overline{Y}} X - D_{\overline{X}} Y.$$

Proof. We have

$$D_X \overline{Y} + D_Y \overline{X} = (D_X F)(Y) + (D_Y F)(X) + \overline{D_X Y} + \overline{D_Y X}$$

$$D_X \overline{Y} + D_Y \overline{X} = \overline{D_X Y} + \overline{D_Y X},$$

since the space is nearly hyperbolic Kahlerian.

$$D_{\overline{X}} \overline{Y} + D_Y X = \overline{D_{\overline{X}} Y} + \overline{D_Y \overline{X}}$$

$$\text{or } [\overline{X}, \overline{Y}] - [X, Y] = \overline{D_{\overline{X}} Y} + \overline{D_Y \overline{X}} - D_{\overline{Y}} \overline{X} - D_X Y.$$

To prove (3.24)b, we bar X in (3.24)a and apply (1.1).

Theorem (3.9). *An almost hyperbolic Hermitian manifold is hyperbolic Kahlerian if it has the following properties*

(a) *it is an almost hyperbolic Kahlerian,*

(b) *it is nearly hyperbolic Kahlerian,*

(c) $J(X, Y, Z)+H(X, Y, Z)=0$.

Proof.

$$J(X, Y, Z)=(D_X F)(Y, Z)+(D_Y F)(Z, X)+(D_Z F)(X, Y),$$

$$H(X, Y, Z)=(D_X F)(Y, Z)+(D_Y F)(X, Z)-2(D_Y F)(X, Z).$$

Therefore,

$$J(X, Y, Z)+H(X, Y, Z)=(D_X F)(Y, Z)+(D_Y F)(Z, X)+(D_Z F)(X, Y)+$$
$$+(D_X F)(Y, Z)+(D_Y F)(X, Z)-2(D_Y F)(X, Z).$$

For an almost hyperbolic Kahlerian space, nearly hyperbolic Kahlerian space and hyperbolic Kahlerian space, we have respectively:

$$(D_X F)(Y, Z)+(D_Y F)(Z, X)+(D_Z F)(X, Y)=0,$$

$$(D_X F)(Y, Z)+(D_Y F)(X, Z)=0$$

and

$$(D_Y F)(X, Z)=0.$$

Hence the result follows.

Theorem (3.10). *From* (3.2) *and* (3.3), *we have*

(3.25)a $$J(\overline{X}, Y, Z)-J(X, Y, \overline{Z})=M(X, Y, Z)-M(Z, X, Y),$$

and consequently for an almost hyperbolic Kahlerian space

(3.25)b $$M(\overline{X}, Y, Z)+M(\overline{Y}, Z, X)+M(\overline{Z}, X, Y)=J(X, Y, Z).$$

Proof. From (3.2), we have

$$J(\overline{X}, Y, Z)=(D_{\overline{X}} F)(Y, Z)+(D_Y F)(Z, \overline{X})+(D_Z F)(\overline{X}, Y),$$

$$J(X, Y, \overline{Z})=(D_X F)(Y, \overline{Z})+(D_Y F)(\overline{Z}, X)+(D_{\overline{Z}} F)(X, Y).$$

Therefore

$$J(\overline{X}, Y, Z)-J(X, Y, \overline{Z})=(D_{\overline{X}} F)(Y, Z)+(D_Y F)(Z, \overline{X})+(D_Z F)(\overline{X}, Y)$$
$$-(D_X F)(Y, \overline{Z})-(D_Y F)(\overline{Z}, X)-(D_{\overline{Z}} F)(X, Y)$$
$$=(D_{\overline{X}} F)(Y, Z)-(D_X F)(Y, \overline{Z})+(D_Y F)(Z, \overline{X})$$
$$-(D_Y F)(\overline{Z}, X)-(D_{\overline{Z}} F)(X, Y)+(D_Z F)(\overline{X}, Y)$$
$$=M(X, Y, Z)-M(Z, X, Y),$$

which yields (3.25)a.

From (3.3) using (1.1), we get

$$M(\bar{X}, Y, Z) = (D_X F)(Y, Z) - (D_{\bar{X}} F)(Y, \bar{Z}),$$

$$M(\bar{Y}, Z, X) = (D_Y F)(Z, X) - (D_Y F)(Z, X),$$

$$M(\bar{Z}, X, Y) = (D_Z F)(X, Y) - (D_{\bar{Z}} F)(X, \bar{Y}).$$

Hence

$$M(\bar{X}, Y, Z) + M(\bar{Y}, Z, X) - M(\bar{Z}, X, Y) = (D_X F)(Y, Z) + (D_Y F)(Z, X) + \\ + (D_Z F)(X, Y) - \{(D_{\bar{X}} F)(Y, \bar{Z}) + (D_{\bar{Y}} F)(Z, \bar{X}) + \\ + (D_{\bar{Z}} F)(X, \bar{Y})\} = J(X, Y, Z),$$

which follows from the definition (3.2) and the definition of an almost hyperbolic Kahlerian space.

Theorem (3.11). *In an almost hyperbolic Hermitian manifold*

$$\overline{M(\bar{X}, Y)} = -M(X, Y).$$

The proof follows from the definition of $M(X, Y)$.

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