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ON UNSTEADY BOUNDARY LAYERS IN A ROTATING FLUID WITH TIME-DEPENDENT SUCTION

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Summary. A Fourier series analysis of the rotating flow of an incompressible, viscous fluid past an infinite, rotating, porous plate is carried out under the following assumptions: (i) consant plate velocity (ii) variable plate velocity (iii) time-dependent suction (iv) rigid body rotation. Approximate solutions to the mean velocity components u, v and the mean temperature θ are derived and the functions affecting u, v and θ are shown graphically. Also, the numerical values of the functions affecting the mean skin-friction and the mean rate of heat transfer are entered in tables. The effects of E (the Ekman number) and ω (the frequency) on these functions are discussed quantitatively.

1. Introduction

Because of the interesting applications of rotating flows in cosmical and geophysical fluid dynamics and in mechanical and nuclear engineering the study of such flows is receiving the attention of many research workers. Batchelor [1] has discussed the simple flow of rotating fluid on a plate and a number of interesting features have been brought out. It has been observed that near the plate there exists a layer of fluid, called the Ekman layer, where the viscous and Coriolis forces are of the same order of magnitude. Unsteady rotating flow has been considered by Greenspan and Howard [2], Chawla [3], Pop [4], Singh and Sathi [5] in the case of an impermeable plate under the assumption of rigid body rotation. The Stokes and Ekman layers, in the case of the plate performing non-torsional oscillations in its own plane were analysed by Thornley [6]. Recently, Thornley's problem was extended by Đordjević [7] to the case of non-homogeneous fluid whereas Debnath and Mukherjee [8] have considered the case of constant suction or blowing. The effects of variable suction on the unsteady rotating fluid bounded by an infinite oscillating plate, when both the fluid and the plate are in solid body rotation have been analysed by Pop and Soundalgekar [9] for the unsteady velocity field. It was assumed that the oscillatory flow is superimposed on the mean steady flow. The problem has been solved by the method suggested by Lighthill [10] for small-amplitude oscillations. Lighthill's method has also been employed by Stuart [11], Messiha

[12] and Soundalgekar [13]. However, it has been observed in the last three papers that the mean flow is not affected by the frequency of the oscillating flow. Also in the analysis by Pop and Soundalgekar [9] it has been observed that the mean flow has nct been affected by the frequency of the oscillatory plate. In order to understand the effects of the frequency on the mean steady flow in the presence of the time-dependent suction Kelly [14] suggested the method of Fourier series. The purpose of the present paper is to ascertain the effect of oscillating plate frequency on the mean flow of the rotating fluid using the method of Fourier series analysis, already employed, when the plate and the fluid are in solid body rotation.

In section 2 the mathematical analysis presented and the approximate solutions for the mean velocity field, the mean skin-friction and the mean heat transfer are derived for the cases where (i) the plate has a constant velocity (ii) the plate is oscillating. The functions affecting the mean velocity and temperature field are shown graphically followed by a discussion of the predictions. In section 3 the conclusions are set out.

2. Mathematical analysis

a) *Velocity field.* Here a cartesian co-ordinate system rotating uniformly with the fluid with angular velocity Ω' about the z' — axis (taken positive in the vertically upward direction) is considered. The x' and y' axes are assumed to coincide with the porous infinite horizontal plate. Then the unsteady flow is governed by the following equations:

$$\frac{\partial w'}{\partial z'} = 0, \tag{1}$$

$$\frac{\partial u'}{\partial t'} + w' \frac{\partial u'}{\partial z'} - 2\Omega' v' = \nu \frac{\partial^2 u'}{\partial z'^2}, \tag{2}$$

$$\frac{\partial v'}{\partial t'} + w' \frac{\partial v'}{\partial z'} + 2\Omega' u' = \nu \frac{\partial^2 v'}{\partial z'^2}, \tag{3}$$

and the boundary conditions are

$$\begin{aligned} &u' + iv' = U'(t'), \quad w' = w'(t') &&\text{at} \quad z' = 0, \\ &u', v' \to 0, &&\text{as} \quad z' \to \infty. \end{aligned} \tag{4}$$

Here u', v', w' are the velocity components, $U'(t')$ the plate velocity and ν is the kinematic viscosity.

From equation (1), it is clear that w' is either a function of time only or constant. Hence following Kelly [14] we assume it to be of the form

$$w' = -w_0'[1 + \delta (e^{i\omega' t'} + e^{-i\omega' t'})], \tag{5}$$

where w_0' is the constant suction velocity, ω' is the frequency of the oscillation and δ ($<<1$) is assumed to be a constant amplitude of the suction velocity.

On introducing

(6) $$p' = u' + iv', \quad i = \sqrt{-1},$$

and considering the non-dimensional quantities

(7) $$z = \frac{|w_0'|}{\nu} z', \quad t = \frac{w_0'^2 t'}{4\nu}, \quad E = \frac{\nu}{w_0'^2} \Omega',$$
$$p = p'/U_0', \quad \omega = \frac{4\nu}{w_0'^2} \omega', \quad U = U'/U_0',$$

equations (2), (3) and (4) reduce to

(8) $$\frac{\partial^2 p}{\partial z^2} + [1 + \delta (e^{i\omega t} + e^{-i\omega t})] \frac{\partial p}{\partial z} - 2iEp - \frac{1}{4} \frac{\partial p}{\partial t} = 0,$$

(9) $$p = U(t) \quad \text{at} \quad z = 0; \qquad p \to 0 \quad \text{as} \quad z \to \infty.$$

As we are interested in understanding the nature of the mean flow in the Ekman layer under the influence of the time-dependent suction, we represent the velocity by the following Fourier series

(10) $$p(z, t) = p_0(z) + \sum_{n=1}^{\infty} p_n(z)\, e^{in\omega t} + \sum_{n=1}^{\infty} \overline{p_n(z)}\, e^{-in\omega t},$$

where the bar denotes the complex conjugate. We now consider the following two cases for the plate velocity: (i) constant and (ii) varying.

C a s e (i). Constant plate velocity

(1) $$U(t) = 1.$$

On substituting (10) in (8) and (9), equating the harmonic and the non-harmonic terms, we get the following set of equations

(12) $$\begin{cases} p_0'' + p_0' + \delta (p_1' + \bar{p}_1') - 2iEp_0 = 0, \\ p_0(0) = 1, \quad p_0(\infty) = 0; \end{cases}$$

(13) $$\begin{cases} p_1'' + p_1' + \delta (p_0' + p_2') - i\left(2E + \frac{\omega}{4}\right) p_1 = 0, \\ p_1(0) = 0, \quad p_1(\infty) = 0; \end{cases}$$

(14) $$\begin{cases} p_n'' + p_n' + \delta (p_{n-1}' + p_{n+1}') - i\left(2E + \frac{n\omega}{4}\right) p_n = 0, \\ p_n(0) = 0, \quad p_n(\infty) = 0, \quad n \geqslant 2; \end{cases}$$

and similar equations for $\bar{p}_1$ and $\bar{p}_n$. Here primes denote differentiation with respect to z. In order to solve these coupled equations, we expand p_n's in powers of δ. Hence we assume

(15) $$p_n = \sum_{j=0}^{\infty} p_{nj}(z)\, \delta^j.$$

From (12) to (15) we have

(16)
$$\begin{cases} p_{00}'' + p_{00}' - 2iEp_{00} = 0, \\ p_{00}(0) = 1, \ p_{00}(\infty) = 0; \end{cases}$$

(17)
$$\begin{cases} p_{10}'' + p_{10}' - i\left(2E + \dfrac{\omega}{4}\right)p_{10} = 0, \\ p_{10}(0) = 0, \quad p_{10}(\infty) = 0; \end{cases}$$

(18)
$$\begin{cases} p_{11}'' + p_{11}' - i\left(2E + \dfrac{\omega}{4}\right)p_{11} = -p_{00}', \\ p_{11}(0) = 0. \quad p_{11}(\infty) = 0; \end{cases}$$

(19)
$$\begin{cases} p_{02}'' + p_{02}' - 2iEp_{02} = -(p_{11}' + \bar{p}_{11}'), \\ p_{02}(0) = 0, \quad p_{02}(\infty) = 0. \end{cases}$$

The non-trivial solutions of (16) to (19) are

(20)
$$p_{00} = u_{00} + iv_{00} = e^{-hz},$$

(21)
$$p_{11} = u_{11} + iv_{11} = \frac{4ih}{\omega}(e^{-mz} - e^{-hz}),$$

(22)
$$p_{02} = u_{02} + iv_{02} = \frac{4ihm}{\omega(m^2 - m - 2iE)}(e^{-mz} - e^{-hz}) + \frac{4i\bar{h}^2}{\omega(\bar{h} - h - 2iE)}(e^{-\bar{h}z} - e^{-hz}) - \frac{4i\bar{h}\bar{m}}{\omega(\bar{m}^2 - \bar{m} - 2iE)}(e^{-mz} - e^{-hz}),$$

where

$$h = \frac{1}{2}(1 + \sqrt{1 + 8iE}) \quad \text{and} \quad m = \frac{1}{2}\left(1 + \sqrt{1 + 4i\left(2E + \frac{\omega}{4}\right)}\right).$$

Hence, the mean velocity is now given by

$$u_0 = u_{00} + \delta^2 u_{02}, \quad v_0 = v_{00} + \delta^2 v_{02},$$

which is now observed to be affected by the term of $0(\delta^2)$ and hence by ω. The functions u_{00}, v_{00}, u_{02} and v_{02} are shown on figures 1 and 2. We observe from figure 1 that u_{00} and v_{00} decrease with increasing values of Ekman number E. Fig. 2 shows that for constant value of E, an increase in ω leads to a decrease in u_{02} and v_{02}. For possible experimental verification it is seen that for $E = 0.1$, an increase in ω from 5 to 10 leads to 138% decrease in u_{02} and 115% decrease in v_{02} at $z = 0.2$. However, for $\omega = 5$, an increase in E from 0.1 to 0.2 leads to 700% decrease in u_{02} and 253% decrease in v_{02} at $z = 0.2$.

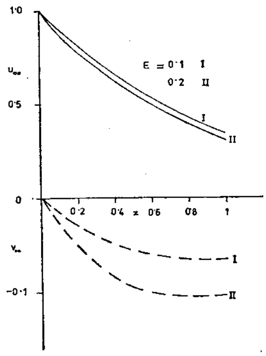

FIG.1: FUNCTION, $u_{..}, v_{..}$

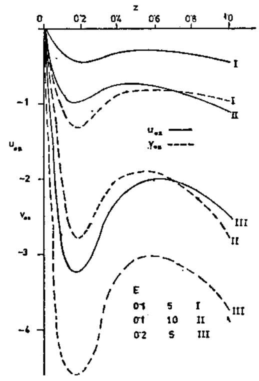

FIG. 2: FUNCTION, $u_{..}, v_{..}$

Knowing the velocity field we can now calculate the skin-friction

$$r = r'/\rho\, u_0' \,|\, w_0' \,| = \left.\frac{dp}{dz}\right|_{z=0} = \left.\frac{dp_{00}}{dz}\right|_{z=0} + \delta^2 \left.\frac{dp_{02}}{dz}\right|_{z=0} . \tag{23}$$

Substituting from (22) in (23) we get

$$\left.\frac{dp_{02}}{dz}\right|_{z=0} = \frac{4\,ihm\,(h-m)}{\omega\,(m^2-m-2\,iE)} + \frac{4\,i\bar{h}^2(h-\bar{h})}{\omega(\bar{h}^2-\bar{h}-2\,iE)} - \frac{4\,i\bar{h}\bar{m}\,(h-\bar{m})}{\omega\,(\bar{m}^2-m-2\,iE)} .$$

In the case of non-rotation, $E = 0$ and $\left.\frac{dp_{02}}{dz}\right|_{z=0}$ reduces to zero which agrees with Kelly's conclusion. Thus we conclude from (23) that because of the rotation, $u_{02}'(0)$ and $v_{02}'(0)$ are non-zero and hence, the mean skin-friction is affected by term of $0\,(\delta^2)$. The numerical values of $u_{02}'(0)$ and $v_{02}'(0)$ are entered in the table I.

TABLE I

Values of $u_{02}'(0)$

E/ω	5	10	15	20
0.1	1.370	3.224	6.431	11.220
0.2	8.343	22.068	44.047	76.151
		Values of $v_{02}'(0)$		
0.1	— 2.738	— 8.152	—16.869	— 29.730
0.2	—11.603	—32.675	—65.839	—114.130
		Values of $u_{01,1}'(0)$		
0.1	— 1.725	— 1.836	— 1.881	— 1.906
0.2	— 1.527	— 1.687	— 1.762	— 1.802

E/ω	5	10	15	20
		Values of $v'_{01,1}(0)$		
0.1	0.177	0.143	0.123	0.109
0.2	0.273	0.237	0.204	0.184
		Values of $u'_{01,2}(0)$		
0.1	0.058	0.040	0.032	0.027
0.2	0.145	0.107	0.087	0.075
		Values of $v'_{01.2}(0)$		
0.1	0.324	0.222	0.176	0.149
0.2	0.480	0.354	0.290	0.250
		Values of $-\theta'_{02}(0)$		
E	P/ω	5	10	15
0.1	0.71 7	5.0133 65	9.2768 121	13.394 173
0.2	0.71 7	5.3352 70	9.8726 129	14.256 184

We conclude from this table that an increase in E or ω leads to an increase in $u'_{02}(0)$ and a decrease in $v'_{02}(0)$. For possible experimental verification we observe that when E is increased from 0.1 to 0.2, there is a 538% increase in $u'_{02}(0)$ at $\omega = 5$ and a 633% increase in $u'_{02}(0)$ at $\omega = 10$. Hence we conclude that the rate of increase in the value of $u'_{02}(0)$ due to an increase in E, increases owing to an increase in ω. However, an increase in E or ω leads to a decrease in $v'_{02}(0)$.

Case (ii) Variable plate velocity. We now study the case of velocity of the plate oscillating with the same frequency as that of the suction velocity given in (5), but with an arbitrary phase angle α. Thus let

(24) $$U(t) = 1 + 2\varepsilon \cos(\omega t + \alpha) = 1 + \varepsilon_1 e^{i\omega t} + \bar{\varepsilon}_1 e^{-i\omega t},$$

where

$$\varepsilon_1 = \varepsilon e^{i\alpha}.$$

We still assume the velocity field to be given by (10). But in the present case, the equations governing p_{10} are modified and are given by

(25) $$\begin{cases} p''_{10} + p'_{10} - i\left(2E + \dfrac{\omega}{4}\right) p_{10} = 0, \\ p_{10}(0) = \varepsilon_1, \quad p_{10}(\infty) = 0; \end{cases}$$

whose solution is

(26) $$p_{10} = u_{10} + iv_{10} = \varepsilon_1 e^{-mz}.$$

The non-zero velue of $p_{10}(z)$ suggests that $p_{01}(z)$ is also non-zero and is derived from (12) as

$$\text{(27)}\qquad \begin{cases} p_{01}'' + p_{01}' - 2iEp_{01} = -(p_{10}' + \bar{p}_{10}'), \\ p_{01}(0) = 0, \quad p_{01}(\infty) = 0. \end{cases}$$

If we assume

$$\text{(28)}\qquad \varepsilon_1 = \varepsilon_{1r} + i\varepsilon_{1i}, \quad p_{01} = \varepsilon_{1r} p_{01,1} + \varepsilon_{1i} p_{01,2},$$

then in view of (26), the equations for $p_{01,1}$ and $p_{01,2}$ are given by

$$\text{(29)}\qquad \begin{cases} p_{01,1}'' + p_{01,1}' - 2iEp_{01,1} = -2\,[m_r \cos(m_i z) - m_i \sin(m_i z)] \cdot e^{-m_r z}, \\ p_{01,1}(0) = 0, \quad p_{01,1}(\infty) = 0; \end{cases}$$

$$\text{(30)}\qquad \begin{cases} p_{01,2}'' + p_{01,2}' - 2iEp_{01,2} = 2\,[m_i \cos(m_i z) + m_r \sin(m_i z)] \cdot e^{-m_r z}, \\ p_{01,2}(0) = 0, \quad p_{01,2}(\infty) = 0. \end{cases}$$

The solutions of these equations are

$$\text{(31)}\qquad \begin{aligned} p_{01,1} = u_{01,1} + iv_{01,1} = {} & \frac{2(m_i n_2 - m_r n_1)}{n_1^2 + n_2^2}\,[e^{-hz} - e^{-m_r z}\cos(m_i z)] + \\ & + \frac{2(m_r n_2 - m_i n_1)}{n_1^2 + n_2^2}\, e^{-m_r z} \sin(m_i z), \end{aligned}$$

$$\text{(32)}\qquad \begin{aligned} p_{01,2} = u_{01,2} + iv_{01,2} = {} & \frac{2(m_i n_1 + m_r n_2)}{n_1^2 + n_2^2}\,[e^{-hz} - e^{-m_r z}\cos(m_i z)] + \\ & + \frac{2(m_r n_1 - m_i n_2)}{n_1^2 + n_2^2}\, e^{-m_r z} \sin(m_i z), \end{aligned}$$

where

$$n_1 = m_r^2 - m_i^2 - m_r, \quad n_2 = m_i - 2m_r m_i - 2E.$$

The functions $u_{01,j}$ and $v_{01,j}$ are plotted on figures 3 and 4 respectively. Figure 3 indicates that away from the plate $u_{01,1}$ increases with increasing E or ω. An increase in ω leads to a decrease in $u_{01,2}$ while an increase in E leads to an increase in $u_{01,2}$. But $v_{01,j}$ decreases with increasing ω and increases with E. The numerical values of the functions affecting the mean skin-friction viz. $u_{01,j}'(0)$ and $v_{0,1j}'(0)$ are entered in table I.

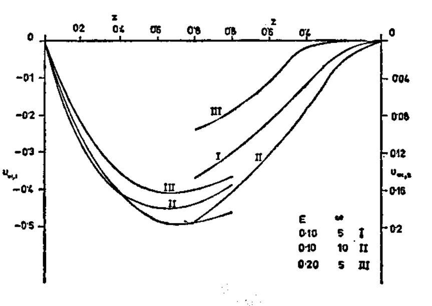

FIG. 3: FUNCTION, $u_{01,1}, u_{01,2}$

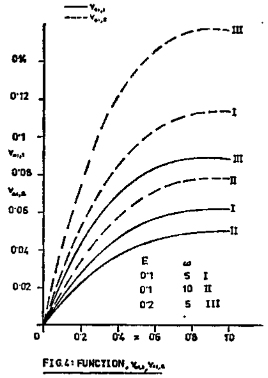

FIG. 4: FUNCTION, $v_{01,1}, v_{01,2}$

b) *Temperature field.* Knowing the velocity field it is of interest to study the temperature field. The energy equation, on taking into account the viscous dissipation, is given by

$$\rho c_p \left(\frac{\partial T'}{\partial t'} + w' \frac{\partial T'}{\partial z'} \right) = k \frac{\partial^2 T'}{\partial z'^2} + \mu \left[\left(\frac{\partial u'}{\partial z'} \right)^2 + \left(\frac{\partial v'}{\partial z'} \right)^2 \right], \tag{33}$$

and the boundary conditions are

$$T' = T'_w \quad \text{at} \quad z' = 0; \qquad T' \to T'_\infty \quad \text{as} \quad z' \to \infty. \tag{34}$$

In view of (6) it is easy to show that

$$\left(\frac{\partial p'}{\partial z'} \right) \overline{\left(\frac{\partial p'}{\partial z'} \right)} = \left(\frac{\partial u'}{\partial z'} \right)^2 + \left(\frac{\partial v'}{\partial z'} \right)^2.$$

Hence in combination with (7), equations (33) and (34) reduce to following non-dimensional form

$$\begin{cases} \dfrac{\partial^2 \theta}{\partial z^2} + P[1 + \delta(e^{i\omega t} + e^{-i\omega t})] \dfrac{\partial \theta}{\partial z} - \dfrac{P}{4} \dfrac{\partial \theta}{\partial t} = P E_c \left(\dfrac{\partial p}{\partial z} \right) \overline{\left(\dfrac{\partial p}{\partial z} \right)}, \\ \theta(0) = 1, \quad \theta(\infty) = 0; \end{cases} \tag{35}$$

where $P = \mu c_p / k$, Prandtl number, $E_c = U_0'^2 / c_p (T'_w - T'_\infty)$, Eckert number,

$$\theta = (T' - T'_w)/(T'_w - T'_\infty).$$

If we now assume that in the boundary layer, the mean temperature has superimposed on it the unsteady temperature, then θ can be represented by the following Fourier series:

$$\theta = \theta_0(z) + \sum_{n=1}^{\infty} \theta_n(z) e^{in\omega t} + \sum_{n=1}^{\infty} \overline{\theta}_n(z) e^{-in\omega t}. \tag{36}$$

Substituting (36) in (35), equating the harmonic and the non-harmonic terms, we have

$$\begin{cases} \theta_0'' + P\theta_0' + P\delta(\theta_1' + \overline{\theta}_1') + PE_c(\bar{p}_0' \overline{p}_0' + 2 p_1' \overline{p}_1') = 0, \\ \theta_0(0) = 1, \quad \theta_0(\infty) = 0; \end{cases} \tag{37}$$

$$\begin{cases} \theta_1'' + P\theta_1' - \dfrac{i\omega P}{4} \theta_1 + P\delta(\theta_0' + \theta_2') + PE_c(p_0' p_1' + \bar{p}_0' p_1' + 2 p_1' p_2') = 0, \\ \theta_1(0) = \theta_1(\infty) = 0; \end{cases} \tag{38}$$

$$\begin{cases} \theta_2'' + P\theta_2' - \dfrac{i\omega P}{4} \theta_2 + P\delta(\theta_1' + \overline{\theta_1'} \theta_3') + PE_c(p_0' p_2' + \overline{p_0'} p_2') = 0, \\ \theta_2(0) = \theta_2(\infty) = 0. \end{cases} \tag{39}$$

To solve the set of coupled equations (37) to (39), we assume for the θ_n's

$$\theta_n = \sum_{j=0}^{\infty} \theta_{nj}(z)\,\delta^j. \tag{40}$$

Substituting (40) and (15) in (37) to (39) and equating the coefficients of different powers of δ, we obtain

$$\begin{cases} \theta''_{00} + P\theta'_{00} = -PE_c p'_{00}\bar{p}'_{00}, \\ \theta_{00}(0) = 1, \quad \theta_{00}(\infty) = 0; \end{cases} \tag{41}$$

$$\begin{cases} \theta''_{10} + P\theta'_{10} - \dfrac{i\omega P}{4}\theta_{10} = 0, \\ \theta_{10}(0) = \theta_{10}(\infty) = 0; \end{cases} \tag{42}$$

$$\begin{cases} \theta''_{01} + P\theta'_{01} = 0, \\ \theta_{01}(0) = \theta_{01}(\infty) = 0; \end{cases} \tag{43}$$

$$\begin{cases} \theta''_{20} + P\theta'_{20} - \dfrac{i\omega P}{4}\theta_{20} = 0, \\ \theta_{20}(0) = \theta_{20}(\infty) = 0: \end{cases} \tag{44}$$

$$\begin{cases} \theta''_{11} + P\theta'_{11} - \dfrac{i\omega P}{4}\theta_{11} = -P(\theta'_{00} + \theta'_{20}) - \\ \qquad -PE_c(p'_{00}p'_{11} + p'_{10}p'_{01} + \bar{p}'_{00}p'_{11} + \\ \qquad\qquad + \bar{p}'_{01}p'_{10} + 2\bar{p}'_{10}p_{21} + 2\bar{p}'_{11}p'_{20}), \\ \theta_{11}(0) = \theta_{11}(\infty) = 0; \end{cases} \tag{45}$$

$$\begin{cases} \theta''_{02} + P'_{02} = -P(\theta'_{21} + \bar{\theta}'_{11}) - 2PE_c p'_{11}\bar{p}'_{11}, \\ \theta_{02}(0) = \theta_{02}(\infty) = 0. \end{cases} \tag{46}$$

In view of (20) to (22), the non-trivial solutions of (41) to (46), are

$$\theta_{00} = e^{-Pz} + \frac{PE_c(h_r^2 + h_i^2)}{2h_r(2h_r - P)}(e^{-Pz} - e^{-2h_r z}), \tag{47}$$

$$\begin{aligned} \theta_{11} &= A_1(e^{-Pz} - e^{-lz}) + A_2(e^{-2h_r z} - e^{-lz}) + \\ &+ A_3(e^{-2h_r z} - e^{-lz}) + A_4[e^{-lz} - e^{-(m+h)z}] + \\ &+ A_5[e^{-lz} - e^{-(m+\bar{h})z}] + A_6(e^{-2hz} - e^{-lz}), \end{aligned} \tag{48}$$

$$\theta_{02}=\frac{1}{2h_r(2h_r-P)}\left[\frac{32\,(h_r^2-h_i^2)^2}{\omega^2}-2Ph_r(A_3+\bar{A}_3)\right]e^{-Pz}-e^{-2h_rz})-$$

$$-\frac{32\,\bar{h}\bar{m}h^2/\omega^2-PA_5(h+\bar{m})}{(\bar{h}+m)^2-P\,(\bar{h}+m)}[e^{-Pz}-e^{-(\bar{h}+m)z}]+$$

$$+\frac{12\,h\bar{h}\,m\bar{m}}{\omega^2 m_r(2m_r-P)}(e^{-Pz}-e^{-2m_rz})+$$

$$+\frac{P(A_1+A_2+A_3-A_4-A_5+A_6)}{(l-P)}(e^{-Pz}-e^{-lz})-$$

(49)

$$-\frac{P\,(A_3+\bar{A}_3)}{3h_r-P}(e^{-Pz}-e^{-2h_rz})+\frac{P\bar{A}_4}{(\overline{h+m})-P}[e^{-Pz}-e^{-(\overline{h+m})z}]+$$

$$+\frac{PA_4}{h+m-P}[e^{-Pz}-e^{-(h+m)z}]-\frac{PA_6}{2h-P}(e^{-Pz}-e^{-2hz})+$$

$$+\frac{P(\bar{A}_1+\bar{A}_2+\bar{A}_3-\bar{A}_4-\bar{A}_5+\bar{A}_6)}{\bar{l}-P}(e^{-Pz}-e^{-\bar{l}z})-$$

$$-\frac{P\bar{A}_6}{2\bar{h}-P}(e^{-Pz}-e^{-2\bar{h}z}),$$

where

$$l=\frac{1}{2}\,(P+\sqrt{P^2+i\omega P}),\quad A_1=\frac{4P}{i\omega}\left[1+\frac{PE_c(h_r^2+h_i^2)}{2h_r(2h_r-P)}\right],$$

$$A_2=-\frac{P^2E_c(h_1^2+h_i^2)}{\left(4h_r^2-2Ph_r-\frac{i\omega P}{4}\right)(2h_r-P)},$$

$$A_3=\frac{4\,iPE_ch^2\bar{h}}{\omega\left(4h_r^2-2Ph_r-\frac{i\omega P}{4}\right)},$$

$$A_4=\frac{4\,iPE_ch^2m}{\omega\left[(m+h)^2-P\,(m+h)-\frac{i\omega P}{4}\right]},$$

$$A_5=\frac{4\,iPE_ch\bar{h}m}{\omega\left[(m+\bar{h})^2-P\,(m+\bar{h})-\frac{i\omega P}{4}\right]},$$

$$A_6=\frac{4\,iPE_ch^3}{\omega\left(4h^2-2Ph-\frac{i\omega P}{4}\right)}.$$

The functions θ_{00} and θ_{02} are shown on figures 5 and 6 respectively for the case of air and water. θ_{00} increases with increasing E. An increase in ω leads to an increase in θ_{02} but an increase in E leads to a decrease in θ_{02} for air and a rise in θ_{02} for water.

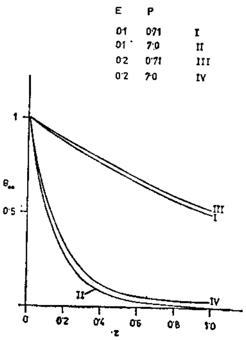

FIG.5: FUNCTION, θ_{00}, E_c=0.02

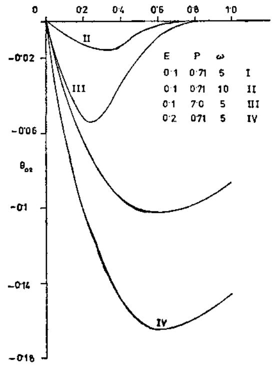

FIG.6 : FUNCTION, θ_{02}, E_c=0.02.

Knowing the temperature field, it is interesting to know the rate of heat transfer, given by

$$(50)\qquad q=-\frac{q'\nu}{k\,|w_0'|\,(T_w'-T_\infty')}=\frac{d\theta}{dz}\bigg|_{z=0}=\frac{d\theta_{00}}{dz}\bigg|_{z=0}+\delta^2\frac{d\theta_{02}}{dz}\bigg|_{z=0}.$$

From (49) we get $\theta_{02}'(0)$, whose numerical values are entered in table I. We observe from this table that an increase in ω or E leads to an increase in $-\theta_{02}(0)$. This is more in the case of water than for air. Quantitatively, we observe that for $\omega=5$, $P=0.71$, there is a 6% rise in $-\theta_{02}(0)$ when E is increased from 0.1 to 0.2. But for $P=0.71$ an increase in ω from 5 to 10 leads to 85% increase in $-\theta_{02}(0)$.

3. Conclusions

1) u_{00}, v_{00} decrease with increasing E.

2) an increase in ω or E leads to a decrease in u_{02}, v_{02}.

3) an increase in E or ω leads to an increase in $u_{02}'(0)$ and decrease in $v_{02}'(0)$.

4) $u_{01,1}$ increases with increasing E or $\omega\cdot u_{01,2}$, $v_{01,j}(j=1,2)$ decreases with increasing ω and increases with increasing E.

5) an increase in ω leads to an increase in θ_{02} and an increase in E leads to a decrease in θ_{02} for air and a rise in θ_{02} for water.

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