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ON FIXED POINTS OF GENERALIZED CONTRACTIONS ON PROBABILISTIC METRIC SPACES

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1. Introduction. V. Sehgal and A. Bharucha-Reid [8] introduced a notion of a contraction mapping on a probabilistic metric space and proved fixed-point theorems which are extensions of the classical Banach's fixed-point principle and a fixed-point theorem of M. Edelstein [4].

In the present note we introduce a notion of a generalized contraction map on a probabilistic metric space and prove a fixed point theorem which is an extension of some results of [1] and [8]. Then we consider a sequence of maps on a probabilistic metric space and prove a theorem which extends some results of [3] and [5].

2. Statistical or probabilistic metric spaces were introduced by K. Menger [7]. A probabilistic metric space (briefly a Pm-space) is an orderer pair $(X, \mathcal{F})$, where X is an abstract set of elements and $\mathcal{F}$ is a mapping of $X \times X$ into a collection $\mathcal{L}$ of all distribution functions F (a distribution function F is a nondecreasing and leftcontinuous mapping of reals into $[0, 1]$ with $\inf F(x) = 0$ and $\sup F(x) = 1$). The value of $\mathcal{F}$ at $(u, v) \in X \times X$ will be denoted by $F_{u,v}$. The functions $F_{u,v}$, $u, v \in X$, are assumed to satisfy the following conditions:

(a) $F_{u,v}(x) = 1$ for all $x > 0$, if and only if $u = v$.

(b) $F_{u,v}(o) = o$.

(c) $F_{u,v} = F_{v,u}$

(d) $F_{u,v}(x) = 1$ and $F_{v,w}(y) = 1$ imply $F_{u,w}(x+y) = 1$.

The value $F_{u,v}(x)$ of $F_{u,v}$ at $x \in R$ may be interpreted as the probability that the distance between u and v is less than x.

A mapping $t: [0, 1] \times [0, 1] \to [0, 1]$ is a t-norm if it satisfies

1. $t(a, 1) = 1, \quad t(0, 0) = 0,$
2. $t(a, b) = t(b, a),$
3. $t(c, d) \geqslant t(a, b)$ for $c \geqslant a,\ d \geqslant b,$
4. $t(t(a, b), c) = t(a, t(b, c)).$

A Menger space is a triplet $(X, \mathcal{F}, t)$, where $(X, \mathcal{F})$ is a *Pm*-space and *t*-norm t is such that the Menger's triangle inequality

$$\text{(M)} \qquad f_{u,w}(x+y) \geqslant t\,[F_{u,v}(x),\ F_{v,w}(y)]$$

is satisfied for all $u, v, w \in X$ and for all $x \geqslant 0$, $y \geqslant 0$. A topology in $(X, \mathcal{F}, t)$ is introduced by the family $\{U_v(\varepsilon, \lambda): v \in X,\ \varepsilon > 0,\ \lambda > 0\}$, where the set

$$U_v(\varepsilon, \lambda) = \{u \in X: F_{u,v}(x) > 1-\lambda\},\ \varepsilon > 0,\ \lambda > 0$$

is called an (ε, λ)-neighborhood of $v \in X$.

3. We now introduce a notion of a generalized contraction on a *Pm*-space.

Definition 1 A mapping T on *Pm*-space $(X, \mathcal{F})$ will be called a *generalized contraction* iff there exists a constant q, $0<q<1$, such that for every $u, v \in X$,

$$\text{(1)} \qquad F_{Tu,Tv}(qx) \geqslant \min\{F_{u,v}(x),\ F_{u,Tu}(x),\ F_{v,Tv}(x),\ F_{u,Tv}(2x),\ F_{v,Tu}(2x)\}$$

for all $x > 0$.

Now we shall prove the following result.

Theorem 1. *Let $(X, \mathcal{F}, t)$ be a Menger space, where t is continuous and satisfies $t(x, x) \geqslant x$ for each $x \in [0, 1]$. If $T: X \to X$ is a generalized contraction on X and X is T-orbitally complete, then T has a unique fixed-point $v \in X$ and* $\lim_n T^n_u = v$ *for every $u \in X$.*

Proof. Let $u \in X$ be arbitrary and consider the sequence:

$$\text{(2)} \qquad u_0 = u,\ u_1 = Tu_0,\ u_2 = Tu_1,\ \ldots,\ u_n = Tu_{n-1},\ \ldots$$

We shall show that the sequence (2) is fundamental in X, i. e., that for each $\varepsilon > 0$, $\lambda > 0$, there is an integer $K(\varepsilon, \lambda)$ such that $m, n \geqslant K(\varepsilon, \lambda)$ imply $F_{u_m, u_n}(\varepsilon) > 1-\lambda$.

First observe that by (*a*)

(3) $u \neq v$ implies $F_{u,v}(qx) < F_{u,v}(x)$ for some $x > 0$ and that (M), 3. and $t(x, x) \geqslant x$ imply

$$\text{(4)} \qquad F_{u,w}(x+y) \geqslant \min\{F_{u,v}(x),\ F_{v,w}(y)\}$$

for all $u, v, w \in X$ and for all $x \geqslant 0$, $y \geqslant 0$.

Suppose that in the sequence (2) $u_{n-1} \neq u_n$ for every integer n, since $u_{n-1} = u_n = Tu_{n-1}$ for some n implies immediately that (2) is fundamental. Then for $u_{n-1}, u_n \in X$ by (1)

$$F_{u_n, u_{n+1}}(qx) = F_{Tu_{n-1}, Tu_n}(qx) \geqslant$$

$$\min\{F_{u_{n-1}, u_n}(x),\ F_{u_{n-1}, u_n}(x),\ F_{u_n, u_{n+1}}(x),\ F_{u_{n-1}, u_{n+1}}(2x),\ F_{u_n, u_n}(2x)\}$$

$$= \min\{F_{u_{n-1}, u_n}(x),\ F_{u_n, u_{n+1}}(x),\ F_{u_{n-1}, u_{n+1}}(2x),\ 1\}.$$

Since by (4)

$$F_{u_{n-1}, u_{n+1}}(2x) \geqslant \min\{F_{u_{n-1}, u_n}(x),\ F_{u_n, u_{n+1}}(x)\},$$

we have

$$F_{u_n, u_{n+1}}(qx) \geqslant \min\{F_{u_{n-1}, u_n}(x),\ F_{u_n, u_{n+1}}(x)\} \text{ for all } x>0.$$

Since we assume that $u_n \neq u_{n+1}$ for each integer n, (3) implies that

$$F_{u_n, u_{n+1}}(qx) \geqslant F_{u_n, u_{n+1}}(x) \text{ for all } x>0$$

is impossible. Then it follows that for each integer n,

$$(5)\qquad F_{u_n, u_{n+1}}(qx) \geqslant F_{u_{n-1}, u_n}(x) \text{ for all } x>0.$$

For an arbitrary integer n we have by (5)

$$F_{u_n, u_{n+1}}(x) \geqslant F_{u_{n-1}, u_n}\left(\frac{x}{q}\right) \geqslant \cdots \geqslant F_{u_0, u_1}\left(\frac{x}{q^n}\right).$$

Let now ε, λ be arbitrary positive reals. Since $F_{u_0, u_1}\left(\frac{x}{q^n}\right) \to 1$ when $n \to \infty$, it follows that there exists an integer $K = K\left(\frac{1-q}{q}\varepsilon, \lambda\right)$ such that

$$(6)\qquad F_{u_{n-1}, u_n}\left(\frac{1-q}{q}\varepsilon\right) > 1-\lambda \text{ for each } n \geqslant K.$$

Then by (5) for $n \geqslant K$ we have

$$(7)\qquad F_{u_{n+p}, u_{n+p+1}}(\varepsilon) \geqslant F_{u_{n+p-1}, u_{n+p}}(\varepsilon) \geqslant \cdots \geqslant F_{u_{n-1}, u_n}(\varepsilon) \geqslant$$
$$\geqslant F_{u_{n-1}, u_n}\left(\frac{1-q}{q}\varepsilon\right) > 1-\lambda$$

for every $p \geqslant 0$, as F is a non-decreasing function $\left(\text{we may suppose that } q \geqslant \frac{1}{2}\right)$.

Now we shall show that for each $n \geqslant K$

$$(8)\qquad F_{u_n, u_{n+p}}(\varepsilon) > 1-\lambda \text{ for } p=0, 1, 2, \ldots$$

Since (8) holds trivialy for $p=0$, we may proceed by induction on p. Assume that (8) is valid for some fixed p. Then by definition of u_n and (1)

$$F_{u_n, u_{n+p+1}}(\varepsilon) = F_{Tu_{n-1}, Tu_{n+p}}\left(q\frac{\varepsilon}{q}\right) \geqslant$$

$$\min\left\{F_{u_{n-1}, u_{n+p}}\left(\frac{\varepsilon}{q}\right), F_{u_{n-1}, u_n}\left(\frac{\varepsilon}{q}\right), F_{u_{n+p}, u_{n+p+1}}\left(\frac{\varepsilon}{q}\right), F_{u_{n-1}, u_{n+p+1}}\left(\frac{2\varepsilon}{q}\right), F_{u_n, u_{n+p}}\left(\frac{2\varepsilon}{q}\right)\right\}.$$

Since by (4)

$$F_{u_{n-1}, u_{n+p}}\left(\frac{\varepsilon}{q}\right) \geqslant \min\left\{F_{u_{n-1}, u_n}\left(\frac{1-q}{q}\varepsilon\right), F_{u_n, u_{n+p}}(\varepsilon)\right\}$$

and

$$F_{u_{n-1},\, u_{n+p+1}}\left(\frac{2\varepsilon}{q}\right) \geqslant \min\left\{F_{u_{n-1},\, u_{n+p}}\left(\frac{\varepsilon}{q}\right),\ F_{u_{n+p},\, u_{n+p+1}}\left(\frac{\varepsilon}{q}\right)\right\},$$

we obtain

$$F_{u_n,\, u_{n+p+1}}(\varepsilon) \geqslant \min\left\{F_{u_{n-1},\, u_n}\left(\frac{1-q}{q}\varepsilon\right),\ F_{u_n,\, u_{n+p}}(\varepsilon),\ F_{u_{n-1},\, u_n}\left(\frac{\varepsilon}{q}\right),\right.$$

$$\left.F_{u_{n+p},\, u_{n+p+1}}\left(\frac{\varepsilon}{q}\right),\ F_{u_n,\, u_{n+p}}\left(\frac{2\varepsilon}{q}\right)\right\}$$

$$\geqslant \min\left\{F_{u_{n-1},\, u_n}\left(\frac{1-q}{q}\varepsilon\right),\ F_{u_n,\, u_{n+p}}(\varepsilon),\ F_{u_{n+p},\, u_{n+p+1}}(\varepsilon)\right\}.$$

Using (6), the inductive assumption and (7) we have

$$F_{u_n,\, u_{n+p+1}}(\varepsilon) > 1-\lambda \text{ for all } n \geqslant K.$$

Therefore, (8) is valid for all $n \geqslant K$ and for every $p = 0, 1, 2, \ldots$ Hence (2) is a fundamental sequence. Since (2) is an orbit of T at $u \in X$ and X is T-orbitally complete, there is a point $v \in X$ such that

$$v = \lim_n u_n = \lim_n T^n u.$$

We now prove that

(9) $$Tv = \lim_n u_{n+1} = v.$$

Let $U_{Tv}(\varepsilon, \lambda)$ be any *nbd* of Tv. Since $\lim_n u_n = v$ there exists an integer K such that

(10) $$n \geqslant K \text{ implies } T_{u_n,\, v}\left(\frac{1-q}{2q}\varepsilon\right) > 1-\lambda \text{ and } F_{u_n,\, u_{n+1}}\left(\frac{1-q}{2q}\varepsilon\right) > 1-\lambda.$$

Then by (1)

$$F_{u_{n+1},\, Tv}(\varepsilon) = F_{Tu_n,\, Tv}\left(q \cdot \frac{\varepsilon}{q}\right) \geqslant$$

$$\geqslant \min\left\{F_{u_n,\, v}\left(\frac{\varepsilon}{q}\right),\ F_{u_n,\, u_{n+1}}\left(\frac{\varepsilon}{q}\right),\ F_{v,\, Tv}\left(\frac{\varepsilon}{q}\right),\ F_{u_n,\, Tv}\left(\frac{2\varepsilon}{q}\right),\ F_{u_{n+1},\, v}\left(\frac{2\varepsilon}{q}\right)\right\}.$$

Since by (4)

$$F_{v,\, Tv}\left(\frac{\varepsilon}{q}\right) = F_{v,\, Tv}\left(\frac{1-q}{2q}\varepsilon + \frac{1+q}{2q}\varepsilon\right) \geqslant \min\left\{F_{v,\, u_{n+1}}\left(\frac{1-q}{2q}\varepsilon\right),\ F_{u_{n+1},\, Tv}\left(\frac{1+q}{2q}\varepsilon\right)\right\}$$

and

$$F_{u_n,\, Tv}\left(\frac{2\varepsilon}{q}\right) \geqslant \min\left\{F_{u_n,\, u_{n+1}}\left(\frac{\varepsilon}{q}\right),\ F_{u_{n+1},\, Tv}\left(\frac{\varepsilon}{q}\right)\right\},$$

we obtain, as F is nondecreasing, that

(11) $$F_{u_{n+1},\, Tv}(\varepsilon) \geqslant \min\left\{F_{u_n,\, v}\left(\frac{1-q}{2q}\varepsilon\right),\ F_{u_n,\, u_{n+1}}\left(\frac{1-q}{2q}\varepsilon\right),\ F_{u_{n+1},\, v}\left(\frac{1-q}{2q}\varepsilon\right),\right.$$

$$\left.F_{u_{n+1},\, Tv}\left(\frac{1+q}{2q}\varepsilon\right)\right\}.$$

Hence and by (10)

$$F_{u_{n+1},\,Tv}(\varepsilon) > 1-\lambda \text{ for all } n \geqslant K, \text{ or} \tag{12}$$

$$F_{u_{n+1},\,Tv}(\varepsilon) = F_{u_{n+1},\,Tv}\left(\frac{1+q}{2q}\,\varepsilon_1\right) \text{ for all } n \geqslant K. \tag{13}$$

We proved (9) if (12) is valued. Now if (12) were false, then substituting in (11) ε by $\varepsilon_1 = \frac{1+q}{2q}\varepsilon > \varepsilon$, it would follow

$$F_{u_{n+1},\,Tv}(\varepsilon_1) = F_{u_{n+1},\,Tv}\left(\frac{1+q}{2q}\,\varepsilon_1\right) = F_{u_{n+1},\,Tv}\left[\left(\frac{1+q}{2q}\right)^2\varepsilon\right], \text{ and} \tag{13'}$$

$$F_{u_{n+1},\,Tv}(\varepsilon) = F_{u_{n+1},\,Tv}\left(\frac{1+q}{2q}\,\varepsilon\right) = F_{u_{n+1},\,Tv}\left[\left(\frac{1+q}{2q}\right)^2\varepsilon\right].$$

Proceeding in this direction we would obtain that

$$1-\lambda \geqslant F_{u_{n+1},\,Tv}(\varepsilon) = \cdots = F_{u_{n+1},\,Tv}\left[\left(\frac{1+q}{2q}\right)^k\varepsilon\right] \to 1,\ k \to \infty$$

which is a contradiction. Therefore, the inequality (12) is correct, which implies (9). So we conclude that there exists a fixed point for T.

To prove the uniqueness of the fixed point v in (9), suppose that $w \neq v$ and $Tw = w$. Then by (1)

$$F_{v,w}(qx) = F_{Tv,Tw}(qx) \geqslant \min\{F_{v,w}(x),\ F_{v,Tv}(x),\ F_{w,Tw}(x),\ F_{v,Tw}(2x), F_{w,Tv}(2x)\}$$
$$= \min\{F_{v,w}(x),\ 1,\ 1,\ F_{v,w}(2x),\ F_{w,v}(2x)\} = F_{v,w}(x)$$

for all $x > 0$, which is a contradiction with (3). Therefore, the fixed point is unique.

Corollary 1.1. *Let (M, d) be a metric space and let $T\colon M \to M$ be a mapping. If*

$$d(Tu, Tv) \leqslant q.\max\left\{d(u, v),\ d(u, Tu),\ d(v, Tv),\ \frac{1}{2}d(u, Tv),\ \frac{1}{2}d(v, Tu)\right\} \tag{14}$$

for some $q < 1$ and for all $u, v \in M$ and if M is T-orbitally complete, then T has a unique fixed point $p \in M$ and $\lim_n T^n u = p$ for every $u \in M$.

Proof. The metric **d** induces a mapping $\mathcal{F}\colon M \times M \to \mathcal{L}$, where $\mathcal{F}(u, v) = F_{u,v}\,(u, v \in M)$ is defined by $F_{u,v}(x) = 0$ if $x \leqslant d(u, v)$ and $F_{u,v}(x) = 1$ if $x > d(u, v)$. Further, if $t\colon [0, 1] \times [0, 1] \to [0, 1]$ is defined by $t(a, b) = \min\{a, b\}$, then $(M, \mathcal{F}, t)$ is a T-orbitally complete Menger space, what is easy to prove.

Now we shall show that (14) implies (1). Put $d(a, b) = \max\{d(u, v), d(u, Tu), d(v, Tv)\}$ and $\frac{1}{2}d(c, e) = \max\left\{\frac{1}{2}d(u, Tv),\ \frac{1}{2}d(v, Tu)\right\}$. Suppose first that $d(Tu, Tv) \leqslant qd(a, b)$. Then for $x \leqslant d(a, b)$ one has $F_{a,b}(x) = 0$ and hence $F_{Tu,Tv}(qx) \geqslant F_{a,b}(x)$; and for $x > d(a, b)$ it follows $qx > qd(a, b) \geqslant d(Tu, Tv)$ which implies $F_{Tu,Tv}(qx) = 1$ and hence $F_{Tu,Tv}(qx) \geqslant F_{a,b}(x)$. Therefore, $F_{Tu,Tv}(qx) \geqslant$

$\geqslant F_{a,b}(x)$ for all $x>0$ when $d(Tu, Tv)\leqslant qd(a, b)$. Suppose now that $d(Tu, Tv)\leqslant \leqslant q\frac{1}{2}d(c, e)$. Then $x\leqslant\frac{1}{2}d(c, e)$ implies $F_{c,e}(2x)=0$; and $x>\frac{1}{2}d(c, e)$ implies $qx>d(Tu, Tv)$ and hence $F_{Tu, Tv}(qx)=1$. Thus, $F_{Tu, Tv}(qx)\geqslant F_{c,e}(2x)$ for all $x>0$ when $d(Tu, Tv)\leqslant q\frac{1}{2}d(c, e)$. Therefore, we showed that if T satisfies the condition (14) on (M, d) then T satisfies the condition (1) on $(M, \mathcal{F}, t)$, as $d(f, g)=d(y, z)$ implies $F_{f,g}(x)=F_{y,z}(x)$ for $x>0$. The result now follows by our Theorem.

Corollary 1.2. ([8], Th. 3). *Let $(E, \mathcal{F}, t)$ be a complete Menger space, where t is continuous function satisfying $t(x, x)\geqslant x$ for each $x\in[0, 1]$. If T is any contraction mapping of E into itself, i.e. if for each $u, v\in E$*

$$F_{Tu, Tv}(qx)\geqslant F_{u,v}(x) \quad \textit{for all} \quad x>0,$$

then there is a unique $p\in E$ such that $Tp=p$. Moreover, $T^n q\to p$ for each $q\in E$.

4. In this section we shall consider a sequence of maps on a *Pm*-space. We need the following definition (see [5]).

Definition 2. A sequence of maps $T_i: X\to X$ on a *Pm*-space X converges uniformly to a map $T: X\to X$ iff for every $\varepsilon>0$ and $\lambda>0$ there exists a positive integer $K=K(\varepsilon, \lambda)$ such that

$$F_{Tu, T_i u}(\varepsilon)>1-\lambda$$

for every $u\in X$ and all $i\geqslant K$.

Theorem 2. *Let $\{T_i\}_{i\in N}$ be a sequence of maps on a Menger space $(X, \mathcal{F}, t)$, where t is continuous and satisfies $t(x, x)\geqslant x$, $x\in[0, 1]$ and let $T: X\to X$ be a generalized contraction on X and X T-orbitally complete. If each T_i $(i=1, 2, \ldots)$ has at least one fixed point v_i and if the sequence $\{T_i\}_{i\in N}$ on the subset $I=\{u\in X$: there is some T_i such that $u=T_i u\}$ converges uniformly to T, then the sequence $\{u_i\}_{i\in N}$ converges to a unique fixed point v of T.*

Proof. By Theorem 1 the mapping T has a unique fixed point v. To show that $v=\lim_i v_i$, let $U_v(\varepsilon, \lambda)$ be an arbitrary *nbd* of v. We must show that

$$F_{v_i, v}(\varepsilon)>1-\lambda$$

for almost all $i\in N$. Since $\{T_i\}$ converges uniformly to T, there exists $K\in N$ such that

$$F_{T_i u, Tu}\left(\frac{1-q}{2}\varepsilon\right)>1-\lambda \quad \text{for} \quad i\geqslant K \tag{15}$$

for every $u\in X$. For arbitrary $v_i\in X$ for which $T_i v_i=v_i$ we have by (4)

$$F_{v_i, v}(\varepsilon)=F_{v_i, v}\left(\frac{1-q}{2}\varepsilon+\frac{1+q}{2}\varepsilon\right)\geqslant\min\left\{F_{T_i v_i, Tv_i}\left(\frac{1-q}{2}\varepsilon\right),\ F_{Tv_i, v}\left(\frac{1+q}{2}\varepsilon\right)\right\}. \tag{16}$$

Since T is a generalized contraction, $v=Tv$, $v_i=T_i v_i$ and F is nondecreasing, we obtain

$$F_{Tv_i,v}\left(\frac{1+q}{2}\varepsilon\right)=F_{Tv_i,Tv}\left(q\frac{1+q}{2q}\varepsilon\right)\geqslant$$

$$\min\left\{F_{v_i,v}\left(\frac{1+q}{2q}\varepsilon\right),\ F_{v_i,Tv_i}\left(\frac{1+q}{2q}\varepsilon\right),\ F_{v,Tv}\left(\frac{1+q}{2q}\varepsilon\right),\ F_{v_i,Tv}\left(\frac{1+q}{q}\varepsilon\right),\ T_{v,Tv_i}\left(\frac{1+q}{q}\varepsilon\right)\right\}$$

$$\geqslant\min\left\{F_{v_i,v}\left(\frac{1+q}{2q}\varepsilon\right),\ F_{T_iv_i,Tv_i}\left(\frac{1+q}{2q}\varepsilon\right),\ F_{v,Tv_i}\left(\frac{1+q}{q}\varepsilon\right)\right\}.$$

Using that

$$F_{v,Tv_i}\left(\frac{1+q}{q}\varepsilon\right)\geqslant\min\left\{F_{vv_i}\left(\frac{1+q}{2q}\varepsilon\right),\ F_{T_iv_i,Tv_i}\left(\frac{1+q}{2q}\varepsilon\right)\right\}$$

we get

$$F_{Tv_i,v}\left(\frac{1+q}{2}\varepsilon\right)\geqslant\min\left\{F_{v_i,v}\left(\frac{1+q}{2q}\varepsilon\right),\ F_{T_iv_i,Tv_i}\left(\frac{1+q}{2q}\varepsilon\right)\right\}\geqslant$$

$$\geqslant\min\left\{F_{v_i,v}\left(\frac{1+q}{2q}\varepsilon\right),\ F_{T_iv_i,Tv_i}\left(\frac{1-q}{2}\varepsilon\right)\right\}.$$

Then (16) results in

$$F_{v_i,v}(\varepsilon)\geqslant\min\left\{F_{T_iv_i,Tv_i}\left(\frac{1-q}{2}\varepsilon\right),\ F_{v_i,v}\left(\frac{1+q}{2q}\varepsilon\right)\right\}.$$

Hence and by (15)

(17) $$F_{v_i,v}(\varepsilon)>1-\lambda \quad \text{for all} \quad i\geqslant K, \text{ or}$$

(18) $$F_{v_i,v}(\varepsilon)=F_{v_i,v}\left(\frac{1+q}{2q}\varepsilon\right) \quad \text{for all} \quad i\geqslant K.$$

The assertion of Theorem follows if (17) is valid. Since $F_{v_i,v}(\varepsilon)\leqslant 1-\lambda$ implies (as in the proof of Theorem 1.)

$$F_{v_i,v}(\varepsilon)=F_{v_i,v}\left(\left(\frac{1+q}{2q}\right)^n\varepsilon\right)\to 1,\ n\to\infty,$$

which is a contradiction, we see that (17) is correct. The theorem is proved.

Corollary 2.1. ([5], Th. 21.1). *Let $\{T_i\}_{i\in N}$ be a sequence of maps on a Menger space $(S, \mathcal{F}, t)$ such that $T_i s_i=s_i$ for some $s_i\in S$ and let T_0 be a contraction mapping on S with a fixed point $s_0\in S$. If $\{T_i\}$ converges uniformly to T_0, then the sequence $\{s_i\}$ converges to s_0.*

REFERENCES

[1] Lj. Ćirić, *Generalized contractions and fixed-point theorems*, Publ. Inst. Math., tome 12 (26), 1971, 19—26.

[2] Lj. Ćirić, *A generalization of Banach's contraction principle*, Proc. Amer. Math. Soc., 45 (1974), 267—273.

[3] Lj. Ćirić, *On families of contractive maps and fixed points*, Publ. Inst. Math., tome 17 (31), 1974, 45—51.

[4] M. Edelstein, *An extension of Banach's contraction principle*, Proc. Amer, Math. Soc., 12 (1961), 7—10.

[5] A. and V. Istratescu, *On the theory of fixed points for some classes of mappings*. VI, Rev. Roum. Math. pures et appl., tome XVII (1973), 1639—1642.

[6] Dj. Kurepa, *Some fixed point theorems*, Math. Balkanica, 2 (1972), 102—108.

[7] K. Menger, *Statistical metrics*, Proc. Nat. Acad. Sci. U.S.A., 28 (1942), 535—537.

[8] V. Sehgal and A. Bharucha—Reid, *Fixed points of contraction mappings on probabilistic metric spaces*, Math. systems theory, 6 (1972), 97—102.

[9] B. Schweizer and A. Sklar, *Statistical metric spaces*, Pacific J. Math., 10 (1960), 313—334.

[10] B. Schweizer, A. Sklar and E. Thorp, *The metrization of statistical metric spaces*, Pacific J. Math., 10 (1960), 673—675.