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RICCI TYPE IDENTITIES IN A SUBSPACE OF A SPACE OF NON-SYMMETRIC AFFINE CONNEXION

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Summary In [2] for a space of non-symmetric affine connexion we deduced 10 identities which correspond to Ricci identity in the Riemann space. In this work we obtain corresponding identities for a subspace of a space of non-symmetric affine connexion, using two kinds of covariant differentiation. In this way we obtain 4 curvature tensors [the equations (6a), (10), (55a), (55b)] and 15 magnitudes, which we call "curvature pseudotensors".

1. Introduction

Let L_N be a space of non-symmetric affine connexion $\Gamma^{\gamma}_{\alpha\beta}$ (see [1], § 89). If y^{α} $(\alpha = 1, \ldots, N)$ are coordinates in L_N, then the equations

$$(1) \qquad y^{\alpha} = y^{\alpha}(x^1, \ldots, x^M) \quad (M < N)$$

determine a subspace L_M of the space L_N. In this work we signify by comma (,) the partial derivation, for example

$$(2) \qquad \frac{\partial y^{\alpha}}{\partial x^i} = y^{\alpha}_{,i}.$$

Consider a mixed tensor $a^{\alpha\beta\gamma i}_{\delta kl}$ defined in the points of the L_M, in which case the Greek indices relate to L_N, and Latin indices to L_M. Because of non-symmetry of the connexion coefficients, we can define two kinds of covariant derivations:

$$(3) \qquad a^{\alpha\beta\gamma i}_{\delta kl \underset{1}{|} m} = a^{\alpha\beta\gamma i}_{\delta kl, m} + \left(\Gamma^{\alpha}_{\pi\mu} a^{\pi\beta\gamma i}_{\delta kl} + \cdots - \Gamma^{\pi}_{\delta\mu} a^{\alpha\beta\gamma i}_{\pi kl}\right) y^{\mu}_{,m} +$$
$$+ \Gamma^{i}_{pm} a^{\alpha\beta\gamma p}_{\delta kl} - \Gamma^{p}_{km} a^{\alpha\beta\delta i}_{\delta pl} - \Gamma^{p}_{lm} a^{\alpha\beta\gamma i}_{\delta kp},$$

$$(4) \qquad a^{\alpha\beta i}_{\delta kl \underset{2}{|} m} = a^{\alpha\beta\gamma i}_{\delta kl, m} + \left(\Gamma^{\alpha}_{\mu\pi} a^{\pi\beta\gamma i}_{\delta kl} + \cdots - \Gamma^{\pi}_{\mu\delta} a^{\alpha\beta\gamma i}_{\pi kl}\right) y^{\mu}_{,m} +$$
$$+ \Gamma^{i}_{mp} a^{\alpha\beta\gamma p}_{\delta kl} - \Gamma^{p}_{mk} a^{\alpha\beta\gamma i}_{\delta pl} - \Gamma^{p}_{ml} a^{\alpha\beta\gamma i}_{\delta kp}$$

2. Identities

2.1. On the base of (3) we have

$$a^{\alpha\beta\gamma i}_{\delta kl\underset{1}{|}mn}=\left(a^{\alpha\beta\gamma i}_{\delta kl\underset{1}{|}m}\right)_{\underset{1}{|}n}=\left(a^{\alpha\beta\gamma i}_{\delta kl\underset{1}{|}m}\right)_{,n}+\left(\Gamma^{\alpha}_{\sigma\nu}\,a^{\sigma\beta\gamma i}_{\delta kl\underset{1}{|}n}+\cdots\right.$$

$$\left.-\Gamma^{\sigma}_{\delta\nu}\,a^{\alpha\beta\gamma i}_{\sigma kl\underset{1}{|}m}\right)y^{\nu}_{,n}+\Gamma^{i}_{sn}\,a^{\alpha\beta\gamma s}_{\delta kl\underset{1}{|}m}-\Gamma^{s}_{kn}\,a^{\alpha\beta\gamma i}_{\delta sl\underset{1}{|}m}-\Gamma^{s}_{ln}\,a^{\alpha\beta\gamma i}_{\beta ks\underset{1}{|}m}-\Gamma^{s}_{mn}\,a^{\alpha\beta\gamma i}_{\delta kl\underset{1}{|}s}.$$

Using (3), from the preceding equation we obtain

(5) $$a^{\alpha\beta\gamma i}_{\delta kl\underset{1}{|}mn}-a^{\alpha\beta\gamma i}_{\delta kl\underset{1}{|}nm}=\left(\underset{1}{R}{}^{\alpha}_{\pi\mu\nu}\,a^{\pi\beta\gamma i}_{\delta kl}+\cdots-\underset{1}{R}{}^{\pi}_{\delta\mu\nu}\,a^{\alpha\beta\gamma i}_{\pi kl}\right)y^{\mu}_{,m}\,y^{\nu}_{,n}+$$

$$+\underset{1}{R}{}^{i}_{pmn}\,a^{\alpha\beta\gamma p}_{\beta kl}-\underset{1}{R}{}^{p}_{kmn}\,a^{\alpha\beta\gamma i}_{\delta pl}-\underset{1}{R}{}^{p}_{lmn}\,a^{\alpha\beta\gamma i}_{\delta kp}-2\,\Gamma^{s}_{\underset{\smile}{mn}}\,a^{\alpha\beta\gamma i}_{\delta kl\underset{1}{|}s},$$

where the magnitudes

(6a) $$\underset{1}{R}{}^{\alpha}_{\beta\mu\nu}=\Gamma^{\alpha}_{\beta\mu,\nu}-\Gamma^{\alpha}_{\beta\nu,\mu}+\Gamma^{\pi}_{\beta\mu}\,\Gamma^{\alpha}_{\pi\nu}-\Gamma^{\pi}_{\beta\nu}\,\Gamma^{\alpha}_{\pi\mu},$$

(6b) $$\underset{1}{R}{}^{i}_{jmn}=\Gamma^{i}_{jm,n}-\Gamma^{i}_{jn,m}+\Gamma^{p}_{jm}\,\Gamma^{i}_{pn}-\Gamma^{p}_{jn}\,\Gamma^{i}_{pm}$$

are *Riemann-Cristoffel curvature tensors of the* 1^{st} *kind* of the space L_N, respectively L_M, and $\underset{\smile}{mn}$ designates the antisymmetrisation over m, n, i.e.

$$\Gamma^{s}_{\underset{\smile}{mn}}=\frac{1}{2}\left(\Gamma^{s}_{mn}-\Gamma^{s}_{nm}\right).$$

Generally, we have

(7) $$a^{\rho_1\ldots\rho_\theta\, r_1\ldots r_u}_{\tau_1\ldots\tau_\omega\, t_1\ldots t_v\underset{1}{|}mn}-a^{\rho_1\ldots\rho_\theta\, r_1\ldots t_u}_{\tau_1\ldots\tau_\omega\, t_1\ldots r_v\underset{1}{|}nm}=$$

$$=\left[\sum_{\lambda=1}^{\theta}\underset{1}{R}{}^{\rho_\lambda}_{\pi\mu\nu}\binom{\pi}{\rho_\lambda}a^{\cdots}_{\cdots}-\sum_{\varphi=1}^{\omega}\underset{1}{R}{}^{\pi}_{\tau_\varphi\mu\nu}\binom{\tau_\varphi}{\pi}a^{\cdots}_{\cdots}\right]y^{\mu}_{,m}\,y^{\nu}_{,n}+$$

$$+\sum_{l=1}^{u}\underset{1}{R}{}^{r_l}_{pmn}\binom{p}{r_l}a^{\cdots}_{\cdots}-\sum_{f=1}^{v}\underset{1}{R}{}^{p}_{t_f mn}\binom{t_f}{p}a^{\cdots}_{\cdots}-$$

$$-2\,\Gamma^{s}_{\underset{\smile}{mn}}\,a^{\rho_1\ldots r_u}_{\tau_1\ldots t_v\underset{1}{|}s},$$

where, for example, it is

(8) $$\binom{p}{r_l}a^{\cdots}_{\cdots}=a^{\rho_1\ldots\rho_\theta\, r_1\ldots r_{l-1}\,p\,r_{l+1}\ldots r_u}_{\tau_1\ldots\tau_\omega\, t_1\ldots t_v}.$$

The equations (5) and (7) are *the* 1^{st} *Ricci identity* of a subspace of a space of non-symmetric affine connexion.

2.2. Starting from (4), analogically to the preceding case, we obtain

(9) $$a^{\alpha\beta\gamma i}_{\delta kl\underset{2}{|}mn}-a^{\alpha\beta\gamma i}_{\delta kl\underset{2}{|}nm}=\left(\underset{2}{R}{}^{\alpha}_{\mu\pi\nu}\,a^{\pi\beta\gamma i}_{\delta kl}+\cdots-\underset{2}{R}{}^{\pi}_{\mu\delta\nu}\,a^{\alpha\beta\gamma i}_{\pi kl}\right)y^{\mu}_{,m}\,y^{\nu}_{,n}+$$

$$+\underset{2}{R}{}^{i}_{mpn}\,a^{\alpha\beta\gamma p}_{\delta kl}-\underset{2}{R}{}^{p}_{mkn}\,a^{\alpha\beta\gamma i}_{\delta pl}-\underset{2}{R}{}^{p}_{mln}\,a^{\alpha\beta\gamma i}_{\delta kp}+2\,\Gamma^{s}_{\underset{\smile}{mn}}\,a^{\alpha\beta\gamma i}_{\delta kl\underset{2}{|}s},$$

where

$$R_{\underset{2}{}}{}^{\alpha}_{\beta\mu\nu}=\Gamma^{\alpha}_{\beta\mu,\,\nu}-\Gamma^{\alpha}_{\nu\mu,\,\beta}+\Gamma^{\pi}_{\beta\mu}\Gamma^{\alpha}_{\nu\pi}-\Gamma^{\pi}_{\nu\mu}\Gamma^{\alpha}_{\beta\pi} \tag{10}$$

and analogically $\underset{2}{R}{}^{i}_{jmn}$ are *Riemann-Cristoffel curvature tensors of the* 2^{nd} *kind.*

Generally, we have

$$a^{\rho_1\ldots\rho_\theta\, r_1\ldots r_u}_{\tau_1\ldots\tau_\omega\, t_1\ldots t_v\underset{2}{|}mn}-a^{\rho_1\ldots\rho_\theta\, r_1\ldots r_u}_{\tau_1\ldots\tau_\omega\, t_1\ldots t_v\underset{2}{|}nm}= \tag{11}$$

$$=\left[\sum_{\lambda=1}^{\theta}\underset{2}{R}{}^{\rho_\lambda}_{\mu\pi\nu}\left(^{\pi}_{\rho_\lambda}\right)a^{\cdots}_{\cdots}-\sum_{\varphi=1}^{\omega}\underset{2}{R}{}^{\pi}_{\mu\tau_\varphi\nu}\left(^{\tau_\varphi}_{\pi}\right)a^{\cdots}_{\cdots}\right]y^{\mu}_{,m}\,y^{\nu}_{,n}$$

$$+\sum_{l=1}^{u}\underset{2}{R}{}^{r_l}_{mpn}\left(^{p}_{r_l}\right)a^{\cdots}_{\cdots}-\sum_{f=1}^{v}\underset{2}{R}{}^{p}_{mt_f n}\left(^{t_f}_{p}\right)a^{\cdots}_{\cdots}+2\,\Gamma^{s}_{\underset{\vee}{mn}}\,a^{\rho_1\ldots r_u}_{\tau_1\ldots t_v\underset{2}{|}s}\,.$$

The equations (9) and (11) are the *the* 2^{nd} *Ricci identity.*

2.3. Using both kinds of differentiation, on the base of (3) and (4) we obtain

$$a^{\alpha\beta\gamma i}_{\delta kl\underset{1}{|}m\underset{2}{|}n}=\left(a^{\alpha\beta\gamma i}_{\delta kl\underset{1}{|}m}\right)_{\underset{2}{|}n}=\left(a^{\alpha\beta\gamma i}_{\delta kl\underset{1}{|}m}\right)_{,n}+\left(\Gamma^{\alpha}_{\nu\sigma}\,a^{\sigma\beta\gamma i}_{\delta kl\underset{1}{|}m}+\cdots\right.$$

$$\left.-\Gamma^{\sigma}_{\nu\delta}\,a^{\alpha\beta\gamma i}_{\sigma kl\underset{1}{|}m}\right)y^{\nu}_{,n}+\Gamma^{i}_{ns}\,a^{\alpha\beta\gamma s}_{\delta kl\underset{1}{|}m}-\Gamma^{s}_{nk}\,a^{\alpha\beta\gamma i}_{\delta sl\underset{1}{|}m}-\Gamma^{s}_{nl}\,a^{\alpha\beta\gamma i}_{\delta ks\underset{1}{|}m}-\Gamma^{s}_{nm}\,a^{\alpha\beta\gamma i}_{\delta kl\underset{1}{|}s},$$

whence

$$a^{\alpha\beta\gamma i}_{\delta kl\underset{1}{|}m\underset{2}{|}n}-a^{\alpha\beta\gamma i}_{\delta kl\underset{1}{|}n\underset{2}{|}m}=\left(\underset{1}{A}{}^{\alpha}_{\pi\mu\nu}\,a^{\pi\beta\gamma i}_{\delta kl}+\cdots-\underset{2}{A}{}^{\pi}_{\delta\mu\nu}\,a^{\alpha\beta\gamma i}_{\pi kl}\right)y^{\mu}_{,m}\,y^{\nu}_{,n}+ \tag{12}$$

$$+\underset{1}{A}{}^{i}_{pmn}\,a^{\alpha\beta\gamma p}_{\delta kl}-\underset{2}{A}{}^{p}_{kmn}\,a^{\alpha\beta\gamma i}_{\delta pl}-\underset{2}{A}{}^{p}_{lmn}\,a^{\alpha\beta\gamma i}_{\delta kp}+$$

$$+4\,a^{\alpha\beta\gamma i}_{\delta kl\underset{\vee}{<mn>}}+4\,a^{\alpha\beta\gamma i}_{\delta kl\underset{\vee}{\leqslant mn\geqslant}}+2\,\Gamma^{s}_{\underset{\vee}{mn}}\,a^{\alpha\beta\gamma i}_{\delta kl\underset{1}{|}s},$$

where we introduced the denotations

$$\underset{1}{A}{}^{\alpha}_{\beta\mu\nu}=\Gamma^{\alpha}_{\beta\mu,\,\nu}-\Gamma^{\alpha}_{\beta\nu,\,\mu}+\Gamma^{\pi}_{\beta\mu}\Gamma^{\alpha}_{\nu\pi}-\Gamma^{\pi}_{\beta\nu}\Gamma^{\alpha}_{\mu\pi} \tag{13}$$

$$\underset{2}{A}{}^{\alpha}_{\beta\mu\nu}=\Gamma^{\alpha}_{\beta\mu,\,\nu}-\Gamma^{\alpha}_{\beta\nu,\,\mu}+\Gamma^{\pi}_{\mu\beta}\Gamma^{\alpha}_{\pi\nu}-\Gamma^{\pi}_{\nu\beta}\Gamma^{\alpha}_{\pi\mu} \tag{14}$$

and analogically $\underset{1}{A}{}^{i}_{jmn}$ and $\underset{2}{A}{}^{i}_{jmn}$,

$$a^{\alpha\beta\gamma i}_{\delta kl<mn>}=\left(\Gamma^{\alpha}_{\underset{\vee}{\pi\mu}}\,a^{\pi\beta\gamma i}_{\delta kl,\,n}+\cdots-\Gamma^{\pi}_{\underset{\vee}{\delta\mu}}\,a^{\alpha\beta\gamma i}_{\pi kl,\,n}\right)y^{\mu}_{,m}+ \tag{15}$$

$$+\Gamma^{i}_{\underset{\vee}{pm}}\,a^{\alpha\beta\gamma p}_{\delta kl,\,n}-\Gamma^{p}_{\underset{\vee}{km}}\,a^{\alpha\beta\gamma i}_{\delta pl,\,n}-\Gamma^{p}_{\underset{\vee}{lm}}\,a^{\alpha\beta\gamma i}_{\delta kp,\,n}$$

$$a^{\alpha\beta\gamma i}_{\delta kl\leqslant mn\geqslant}=\left(\Gamma^{\alpha}_{[\underset{\smile}{\pi\mu}}\Gamma^{\beta}_{\underset{\smile}{\nu\sigma}]}\,a^{\pi\sigma\gamma i}_{\delta kl}+\Gamma^{\alpha}_{[\underset{\smile}{\pi\mu}}\Gamma^{\gamma}_{\nu\sigma]}\,a^{\pi\beta\sigma i}_{\delta kl}+\right. \tag{16}$$

$$+\Gamma^{\beta}_{[\underset{\smile}{\pi\mu}}\Gamma^{\gamma}_{\underset{\smile}{\nu\sigma}]}\,a^{\alpha\pi\sigma i}_{\delta kl}-\Gamma^{\alpha}_{[\underset{\smile}{\pi\mu}}\Gamma^{\sigma}_{\underset{\smile}{\nu\delta}]}\,a^{\pi\beta\gamma i}_{\sigma kl}-\Gamma^{\beta}_{[\underset{\smile}{\pi\mu}}\Gamma^{\sigma}_{\underset{\smile}{\nu\delta}]}\,a^{\alpha\pi\gamma i}_{\sigma kl}-$$

$$-\Gamma^{\gamma}_{\underset{\smile}{[\pi\mu}}\Gamma^{\sigma}_{\underset{\smile}{\nu\delta]}}a^{\alpha\beta\pi i}_{\sigma kl}\big)y^{\mu}_{,m}y^{\nu}_{,\nu}+\big(\Gamma^{\alpha}_{\underset{\smile}{[\pi\mu}}\Gamma^{i}_{\underset{\smile}{ns]}}a^{\pi\beta\gamma s}_{\delta kl}+\Gamma^{\beta}_{\underset{\smile}{[\pi\mu}}\Gamma^{i}_{\underset{\smile}{ns]}}a^{\alpha\pi\gamma s}_{\delta kl}+$$

$$+\Gamma^{\gamma}_{\underset{\smile}{[\pi\mu}}\Gamma^{i}_{\underset{\smile}{ns]}}a^{\alpha\beta\pi s}_{\delta kl}-\Gamma^{\alpha}_{\underset{\smile}{[\pi\mu}}\Gamma^{s}_{\underset{\smile}{nk]}}a^{\pi\beta\gamma i}_{\delta sl}-\cdots-\Gamma^{\gamma}_{\underset{\smile}{[\pi\mu}}\Gamma^{s}_{\underset{\smile}{nl]}}a^{\alpha\beta\pi i}_{\delta ks}+$$

$$+\Gamma^{\pi}_{\underset{\smile}{[\delta\mu}}\Gamma^{s}_{\underset{\smile}{nk]}}a^{\alpha\beta\gamma i}_{\pi sl}+\Gamma^{\pi}_{\underset{\smile}{[\delta\mu}}\Gamma^{s}_{\underset{\smile}{nl]}}a^{\alpha\beta\gamma i}_{\pi ks}-\Gamma^{\pi}_{\underset{\smile}{[\delta\mu}}\Gamma^{i}_{\underset{\smile}{ns]}}a^{\alpha\beta\gamma s}_{\pi kl}\big)y^{\mu}_{,m}+$$

$$+\Gamma^{i}_{\underset{\smile}{[pm}}\Gamma^{s}_{\underset{\smile}{nk]}}a^{\alpha\beta\gamma p}_{\delta sl}-\Gamma^{i}_{\underset{\smile}{[pm}}\Gamma^{s}_{\underset{\smile}{nl]}}a^{\alpha\beta\gamma p}_{\delta ks}+\Gamma^{p}_{[km}\Gamma^{s}_{\underset{\smile}{nl]}}a^{\alpha\beta\gamma i}_{\delta ps]}\big),$$

(17)
$$\Gamma^{\alpha}_{\underset{\smile}{[\pi\mu}}\Gamma^{\beta}_{\underset{\smile}{\nu\sigma]}}=\frac{1}{2}\big(\Gamma^{\alpha}_{\pi\mu}\Gamma^{\beta}_{\nu\sigma}-\Gamma^{\alpha}_{\mu\pi}\Gamma^{\beta}_{\sigma\nu}\big)$$

(and analogically $\Gamma^{\alpha}_{\underset{\smile}{[\pi\mu}}\Gamma^{i}_{\underset{\smile}{ns]}}$ etc.). The magnitudes $\underset{1}{A}{}^{\alpha}_{\beta\mu\nu}$, $\underset{2}{A}{}^{\alpha}_{\beta\mu\nu}$ $(\underset{1}{A}{}^{i}_{jmn},\ \underset{2}{A}{}^{i}_{jmn})$ given by (13) and (14) are not tensors and we call them *curvature pseudotensors of the* 1*st respectively* 2*nd kind.*

In general the following is valid

(18)
$$a^{\rho_1\ldots\rho_\theta r_1\ldots r_u}_{\tau_1\ldots\tau_\omega t_1\ldots t_v\underset{1}{|}m\underset{2}{|}n}-a^{\rho_1\ldots\rho_\theta r\ldots r_u}_{\tau_1\ldots\tau_\omega t_1\ldots t_v\underset{1}{|}n\underset{2}{|}m}=$$
$$=\Big[\sum_{\lambda=1}^{\theta}\underset{1}{A}{}^{\rho_\lambda}{}_{\pi\mu\nu}\binom{\pi}{\rho_\lambda}a^{\cdots}_{\cdots}-\sum_{\varphi=1}^{\omega}\underset{2}{A}{}^{\pi}{}_{\tau_\varphi\mu\nu}\binom{\tau_\varphi}{\pi}a^{\cdots}_{\cdots}\Big]y,^{\mu}_{m}y,^{\nu}_{n}+$$
$$\sum_{l=1}^{u}\underset{1}{A}{}^{r_l}{}_{pmn}\binom{p}{r_l}a^{\cdots}_{\cdots}-\sum_{f=1}^{v}\underset{2}{A}{}^{p}_{t_f mn}\binom{t_f}{p}a^{\cdots}_{\cdots}+$$
$$+4\,a^{\rho_1\ldots r_u}_{\tau_1\ldots t_v<\underset{\vee}{mn}>}+4\,a^{\rho_1\ldots r_u}_{\tau_1\ldots t_v\leqslant\underset{\vee}{mn}\geqslant}+2\,\Gamma^{s}_{\underset{\vee}{mn}}a^{\rho_1\ldots r_u}_{\tau_1\ldots t_v\underset{1}{|}s},$$

where we introduce the denotations

(19)
$$a^{\rho_1\ldots r_u}_{\tau_1\ldots t_v<mn>}=$$
$$=\Big[\sum_{\lambda=1}^{\theta}\Gamma^{\rho_\lambda}_{\underset{\vee}{\pi\mu}}\binom{\pi}{\rho_\lambda}a^{\cdots}_{\cdots,n}-\sum_{\varphi=1}^{\omega}\Gamma^{\pi}_{\underset{\vee}{\tau_\varphi\mu}}\binom{\tau_\varphi}{\pi}a^{\cdots}_{\cdots,n}\Big]y,^{\mu}_{m}+$$
$$+\sum_{l=1}^{r}\Gamma^{r_l}_{\underset{\vee}{pm}}\binom{p}{r_l}a^{\cdots}_{\cdots,n}-\sum_{f=1}^{v}\Gamma^{p}_{\underset{\vee}{t_f m}}\binom{t_f}{p}a^{\cdots}_{\cdots,n},$$

(20)
$$a^{\rho_1\ldots r_u}_{\tau_1\ldots t_v\leqslant mn\geqslant}=$$
$$=\Big[\sum_{\substack{\lambda=1\\(\lambda<\varphi)}}^{\theta-1}\sum_{\varphi=2}^{0}\Gamma^{\rho_\lambda}_{\underset{\smile}{[\pi\mu}}\Gamma^{\rho_\varphi}_{\underset{\smile}{\nu\sigma]}}\binom{\pi}{\rho_\lambda}\binom{\sigma}{\rho_\varphi}a^{\cdots}_{\cdots}-\sum_{\lambda=1}^{\theta}\sum_{\varphi=1}^{\omega}\Gamma^{\rho_\lambda}_{\underset{\smile}{[\pi\mu}}\Gamma^{\sigma}_{\underset{\smile}{\nu\tau_\varphi]}}\binom{\pi}{\rho_\lambda}\binom{\tau_\varphi}{\sigma}a^{\cdots}_{\cdots}+$$
$$+\sum_{\substack{\lambda=1\\(\lambda<\varphi)}}^{\omega-1}\sum_{\varphi=2}^{\omega}\Gamma^{\pi}_{\underset{\smile}{[\tau_\lambda\mu}}\Gamma^{\sigma}_{\underset{\smile}{\nu\tau_\varphi]}}\binom{\tau_\lambda}{\pi}\binom{\tau_\varphi}{\sigma}a^{\cdots}_{\cdots}\Big]y,^{\mu}_{m}y,^{\nu}_{n}+$$
$$+\Big[\sum_{\lambda=1}^{\theta}\sum_{l=1}^{u}\Gamma^{\rho_\lambda}_{\underset{\smile}{[\pi\mu}}\Gamma^{r_l}_{\underset{\smile}{ns]}}\binom{\pi}{\rho_\lambda}\binom{s}{r_l}a^{\cdots}_{\cdots}-\sum_{\lambda=1}^{\theta}\sum_{f=1}^{v}\Gamma^{\rho_\lambda}_{\underset{\smile}{[\pi\mu}}\Gamma^{s}_{\underset{\smile}{nt_f]}}\binom{\pi}{\rho_\lambda}\binom{t_f}{s}a^{\cdots}_{\cdots}-$$

$$-\sum_{\varphi=1}^{\omega}\sum_{l=1}^{u}\Gamma^{\pi}_{[\underset{\smile}{\tau_\varphi}\mu}\Gamma^{r_l}_{\underset{\smile}{ns}]}\binom{\tau_\varphi}{\pi}\binom{s}{r_l}a^{\cdots}_{\cdots}+\sum_{\varphi=1}^{\omega}\sum_{f=1}^{v}\Gamma^{\pi}_{[\underset{\smile}{\tau_\varphi}\mu}\Gamma^{s}_{\underset{\smile}{nt_f}]}\binom{\tau_\varphi}{\pi}\binom{t_f}{s}a^{\cdots}_{\cdots}\Bigg]y^{\mu}_{,m}+$$

$$+\sum_{\substack{l=1\\(l<f)}}^{u-1}\sum_{f=2}^{u}\Gamma^{r_l}_{[\underset{\smile}{pm}}\Gamma^{r_f}_{\underset{\smile}{ns}]}\binom{p}{r_l}\binom{s}{r_f}a^{\cdots}_{\cdots}-\sum_{l=1}^{u}\sum_{f=1}^{v}\Gamma^{r_l}_{[\underset{\smile}{pm}}\Gamma^{s}_{\underset{\smile}{nt_f}]}\binom{p}{r_l}\binom{t_f}{s}a^{\cdots}_{\cdots}+$$

$$+\sum_{\substack{l=1\\(l<f)}}^{v-1}\sum_{f=2}^{v}\Gamma^{p}_{[\underset{\smile}{t_l m}}\Gamma^{s}_{\underset{\smile}{nt_f}]}\binom{t_l}{p}\binom{t_f}{s}a^{\cdots}_{\cdots}.$$

The equations (12) and (18) we call the 3[rd] *Ricci identity* of a subspace of a space of non-symmetric affine connexion.

2.4. Applying two kinds of covariant differentiation in inversed order than in preceding case, we obtain

$$(21)\qquad a^{\alpha\beta\gamma i}_{\delta kl\underset{2}{|}m\underset{1}{|}n}-a^{\alpha\beta\gamma i}_{\delta kl\underset{2}{|}n\underset{1}{|}m}=\big(\underset{3}{A}{}^{\alpha}_{\mu\pi\nu}a^{\pi\beta\gamma i}_{\delta kl}+\cdots-\underset{4}{A}{}^{\pi}_{\mu\delta\nu}a^{\alpha\beta\gamma i}_{\pi kl}\big)y^{\mu}_{,m}y^{\nu}_{,n}+$$

$$+\underset{3}{A}{}^{i}_{mpn}a^{\alpha\beta\gamma p}_{\delta kl}-\underset{4}{A}{}^{p}_{mkn}a^{\alpha\beta\gamma i}_{\delta pl}-\underset{4}{A}{}^{p}_{mln}a^{\alpha\beta\gamma i}_{\delta kp}+$$

$$-4\,a^{\alpha\beta\gamma i}_{\delta kl<\underset{\smile}{mn}>}-4\,a^{\alpha\beta\gamma i}_{\delta kl\leqslant\underset{\smile}{mn}\geqslant}-2\,\Gamma^{s}_{\underset{\smile}{mn}}a^{\alpha\beta\gamma i}_{\delta kl\underset{2}{|}s},$$

where the magnitudes

$$(22)\qquad \underset{3}{A}{}^{\alpha}_{\beta\mu\nu}=\Gamma^{\alpha}_{\beta\mu,\nu}-\Gamma^{\alpha}_{\nu\mu,\beta}+\Gamma^{\pi}_{\beta\mu}\Gamma^{\alpha}_{\pi\nu}-\Gamma^{\pi}_{\nu\mu}\Gamma^{\alpha}_{\pi\beta},$$

$$(23)\qquad \underset{4}{A}{}^{\alpha}_{\beta\mu\nu}=\Gamma^{\alpha}_{\beta\mu,\nu}-\Gamma^{\alpha}_{\nu\mu,\beta}+\Gamma^{\pi}_{\mu\beta}\Gamma^{\alpha}_{\nu\pi}-\Gamma^{\pi}_{\mu\nu}\Gamma^{\alpha}_{\beta\pi},$$

we call the *curvature pseudotensors of the* 3[rd] *respectively* 4[th] *kind* and analogically $\underset{3}{A}{}^{i}_{jmn}$, $\underset{4}{A}{}^{i}_{jmn}$.

Generally

$$(24)\qquad a^{\rho_1\ldots r_u}_{\tau_1\ldots t_v\underset{2}{|}m\underset{1}{|}n}-a^{\rho_1\ldots r_u}_{\tau_1\ldots t_v\underset{2}{|}n\underset{1}{|}m}=$$

$$=\Bigg[\sum_{\lambda=1}^{\theta}\underset{3}{A}{}^{\rho_\lambda}_{\mu\pi\nu}\binom{\pi}{\rho_\lambda}a^{\cdots}_{\cdots}-\sum_{\varphi=1}^{\omega}\underset{4}{A}{}^{\pi}_{\mu\tau_\varphi\nu}\binom{\tau_\varphi}{\pi}a^{\cdots}_{\cdots}\Bigg]y^{\mu}_{,m}y^{\nu}_{,n}+$$

$$+\sum_{l=1}^{u}\underset{3}{A}{}^{r_l}_{mpn}\binom{p}{r_l}a^{\cdots}_{\cdots}-\sum_{f=1}^{v}\underset{4}{A}{}^{p}_{mt_f n}\binom{t_f}{p}a^{\cdots}_{\cdots}-$$

$$-4\,a^{\rho_1\ldots r_u}_{\tau_1\ldots t_v<\underset{\smile}{mn}>}-4\,a^{\rho_1\ldots r_u}_{\tau_1\ldots t_v\leqslant\underset{\smile}{mn}\geqslant}-2\,\Gamma^{s}_{\underset{\smile}{mn}}a^{\rho_1\ldots r_u}_{\tau_1\ldots t_v\underset{2}{|}s}.$$

The equations (21) and (24) we call *the* 4[th] *Ricci identity.*

2.5. Using the derivations from the sections 2.1 and 2.2., we get

$$(25)\qquad a^{\alpha\beta\gamma i}_{\delta kl\underset{1}{|}mn}-a^{\alpha\beta\gamma i}_{\delta kl\underset{2}{|}nm}=$$

$$=\big(\underset{5}{A}{}^{\alpha}_{\pi\mu\nu}y^{\mu}_{,m}y^{\nu}_{,n}+2\,\Gamma^{\alpha}_{\underset{\smile}{\pi\mu}}y^{\mu}_{,mn}\big)a^{\pi\beta\gamma i}_{\delta kl}+\cdots-\big(\underset{6}{A}{}^{\pi}_{\delta\mu\nu}y^{\mu}_{,m}y^{\nu}_{,n}+2\,\Gamma^{\pi}_{\underset{\smile}{\delta\mu}}y^{\mu}_{,mn}\big)a^{\alpha\beta\gamma i}_{\pi kl}+$$

$$+\underset{5}{A}{}^{i}_{pmn}a^{\alpha\beta\gamma p}_{\delta kl}-\underset{6}{A}{}^{p}_{kmn}a^{\alpha\beta\gamma i}_{\delta pl}-\underset{6}{A}{}^{p}_{lmn}a^{\alpha\beta\gamma i}_{\delta kp}+$$

$$+4\,a^{\alpha\beta\gamma i}_{\delta kl<\underline{mn}>}+4\,a^{\alpha\beta\gamma i}_{\delta kl\lessdot\underline{mn}\gtrdot}-\Gamma^{s}_{mn}\left(a^{\alpha\beta\beta i}_{\delta kl\,\dot{1}\,s}-a^{\alpha\beta\gamma i}_{\delta kl\,\dot{2}\,s}\right),$$

where the magnitudes

$$\underset{5}{A}{}^{\alpha}_{\beta\mu\nu}=\Gamma^{\alpha}_{\beta\mu,\nu}-\Gamma^{\alpha}_{\nu\beta,\mu}+\Gamma^{\pi}_{\beta\mu}\Gamma^{\alpha}_{\pi\nu}-\Gamma^{\pi}_{\nu\beta}\Gamma^{\alpha}_{\mu\pi},\tag{26}$$

$$\underset{6}{A}{}^{\alpha}_{\beta\mu\nu}=\Gamma^{\alpha}_{\beta\mu,\nu}-\Gamma^{\alpha}_{\nu\beta,\mu}+\Gamma^{\pi}_{\mu\beta}\Gamma^{\alpha}_{\nu\pi}-\Gamma^{\pi}_{\beta\nu}\Gamma^{\alpha}_{\pi\mu}\tag{27}$$

we call *curvature pseudotensors of the* 5[th] *and* 6[th] *kind and* $a^{\alpha\beta\gamma i}_{\delta kl\lessdot mn\gtrdot}$ is given analogically to (16), i e.

$$a^{\alpha\beta\gamma i}_{\delta kl\lessdot mn\gtrdot}=\left(\Gamma^{\alpha}_{[\underset{\smile}{\pi\mu}}\Gamma^{\beta}_{\underset{\smile}{\sigma\nu}]}a^{\pi\sigma\gamma i}_{\delta kl}+\cdots+\Gamma^{\gamma}_{[\underset{\smile}{\pi\mu}}\Gamma^{\sigma}_{\underset{\smile}{\delta\nu}]}a^{\alpha\beta\pi i}_{\sigma kl}\right)y^{\mu}_{,m}y^{\nu}_{,n}+\tag{28}$$

$$+\left(\Gamma^{\alpha}_{[\underset{\smile}{\pi\mu}}\Gamma^{i}_{\underset{\smile}{sn}]}a^{\pi\beta\gamma s}_{\delta kl}+\cdots+\Gamma^{\pi}_{[\underset{\smile}{\delta\mu}}\Gamma^{i}_{\underset{\smile}{sn}]}a^{\alpha\beta\gamma s}_{\pi kl}\right)y^{\mu}_{,m}+$$

$$+\left(-\Gamma^{i}_{[\underset{\smile}{pm}}\Gamma^{s}_{\underset{\smile}{kn}]}a^{\alpha\beta\gamma p}_{\delta sl}+\cdots+\Gamma^{p}_{[\underset{\smile}{km}}\Gamma^{s}_{\underset{\smile}{ln}]}a^{\alpha\beta\gamma i}_{\delta ps}\right).$$

The designation $\underline{mn}$ in (25) (and analogically in other cases) means the symmetrisation over m and n, i.e. for example

$$a^{\alpha\beta\gamma i}_{\delta kl<\underline{mn}>}=\frac{1}{2}\left(a^{\alpha\beta\gamma i}_{\delta kl<mn>}+a^{\alpha\beta\gamma i}_{\delta kl<nm>}\right).$$

Generally

$$a^{\rho_1\ldots r_u}_{\tau_1\ldots t_v\dot{1}\,mn}-a^{\rho_1\ldots r_u}_{\tau_1\ldots t_v\dot{2}\,nm}=\tag{29}$$

$$=\sum_{\lambda=1}^{\theta}\left(\underset{5}{A}{}^{\rho_\lambda}_{\pi\mu\nu}y^{\mu}_{,m}y^{\nu}_{,n}+2\,\Gamma^{\rho_\lambda}_{\underset{\vee}{\pi\mu}}y^{\mu}_{,mn}\right)\binom{\pi}{\rho_\lambda}a^{\cdots}_{\cdots}-$$

$$-\sum_{\varphi=1}^{\omega}\left(\underset{6}{A}{}^{\pi}_{\tau_\varphi\mu\nu}y^{\mu}_{,m}y^{\nu}_{,n}+2\,\Gamma^{\pi}_{\underset{\vee}{\tau_\varphi\mu}}y^{\mu}_{,mn}\right)\binom{\tau_\varphi}{\pi}a^{\cdots}_{\cdots}+$$

$$+\sum_{l=1}^{u}\underset{5}{A}{}^{r_l}_{pmn}\binom{p}{r_l}a^{\cdots}_{\cdots}-\sum_{f=1}^{v}\underset{6}{A}{}^{p}_{t_f mn}\binom{t_f}{p}a^{\cdots}_{\cdots}+$$

$$+4\,a^{\rho_1\ldots r_u}_{\tau_1\ldots t_v<\underline{mn}>}+4\,a^{\rho_1\ldots r_u}_{\tau_1\ldots t_v\lessdot\underline{mn}\gtrdot}-\Gamma^{s}_{mn}\left(a^{\rho_1\ldots r_u}_{\tau_1\ldots t_v\,\dot{1}\,s}-a^{\rho_1\ldots r_u}_{\tau_1\ldots t_v\,\dot{2}\,s}\right),$$

where, analogically to (20), it is

$$a^{\rho_1\ldots r_u}_{\tau_1\ldots t_v\lessdot mn\gtrdot}=\tag{30}$$

$$=\left[\sum_{\substack{\lambda=1\\(\lambda<\varphi)}}^{\theta-1}\sum_{\varphi=2}^{\theta}\Gamma^{\rho_\lambda}_{[\underset{\smile}{\pi\mu}}\Gamma^{\rho_\varphi}_{\underset{\smile}{\sigma\nu}]}\binom{\pi}{\rho_\lambda}\binom{\sigma}{\rho_\varphi}a^{\cdots}_{\cdots}-\cdots\right]y^{\mu}_{,m}y^{\nu}_{,n}+$$

$$+\left[\sum_{\lambda=1}^{\theta}\sum_{l=1}^{u}\Gamma^{\rho_\lambda}_{[\underset{\smile}{\pi\mu}}\Gamma^{r_l}_{\underset{\smile}{sn}]}\binom{\pi}{\rho_\lambda}\binom{s}{r_l}a^{\cdots}_{\cdots}-\cdots\right]y^{\mu}_{,m}+$$

$$+\sum_{\substack{l=1\\(l<f)}}^{u-1}\sum_{f=2}^{u}\Gamma^{r_l}_{[\underset{\smile}{pm}}\Gamma^{r_f}_{\underset{\smile}{sn}]}\binom{p}{r_l}\binom{s}{r_f}a^{\cdots}_{\cdots}-\cdots.$$

The equations (29) and (25) are *the 5*[th] *Ricci identity.*

2.6. Using the covariant derivations from the sections 2.1. and 2.3., we have

$$(31)\qquad a^{\alpha\beta\gamma i}_{\delta kl\underset{1}{|}m\underset{2}{|}n}-a^{\alpha\beta\gamma i}_{\delta kl\underset{1}{|}n\underset{2}{|}m}=\Big(\underset{7}{A}{}^{\alpha}_{\pi\mu\nu}\,a^{\pi\beta\gamma i}_{\delta kl}+\cdots-\underset{8}{A}{}^{\pi}_{\delta\mu\nu}\,a^{\alpha\beta\gamma i}_{\pi kl}\Big)\,y^{\mu}_{,m}\,y^{\nu}_{,n}+$$

$$+\underset{7}{A}{}^{i}_{pmn}\,a^{\alpha\beta\gamma p}_{\delta kl}-\underset{8}{A}{}^{p}_{kmn}\,a^{\alpha\beta\gamma i}_{\delta pl}-\underset{8}{A}{}^{p}_{lmn}\,a^{\alpha\beta\gamma i}_{\delta kp}+2\,a^{\alpha\beta\gamma i}_{\delta kl<mn>}+2\,a^{\alpha\beta\gamma i}_{\delta kl\leqslant mn\geqslant},$$

where

$$(32)\qquad \underset{7}{A}{}^{\alpha}_{\beta\mu\nu}=\Gamma^{\alpha}_{\beta\mu,\nu}-\Gamma^{\alpha}_{\beta\nu,\mu}+\Gamma^{\pi}_{\beta\mu}\,\Gamma^{\alpha}_{\pi\nu}-\Gamma^{\pi}_{\beta\nu}\,\Gamma^{\alpha}_{\mu\pi},$$

$$(33)\qquad \underset{8}{A}{}^{\alpha}_{\beta\mu\nu}=\Gamma^{\alpha}_{\beta\mu,\nu}-\Gamma^{\alpha}_{\beta\nu,\mu}+\Gamma^{\pi}_{\mu\beta}\,\Gamma^{\alpha}_{\pi\nu}-\Gamma^{\pi}_{\beta\nu}\,\Gamma^{\alpha}_{\pi\mu}$$

are *curvature pseudotensors of the* 7^{th} *and* 8^{th} *kind*, and

$$(34)\qquad a^{\alpha\beta\gamma i}_{\delta kl\leqslant mn\geqslant}=\Big[\Big(\Gamma^{\alpha}_{\underset{\vee}{\pi\mu}}\,\Gamma^{\beta}_{\sigma\nu}+\Gamma^{\alpha}_{\pi\nu}\,\Gamma^{\beta}_{\underset{\vee}{\sigma\mu}}\Big)\,a^{\pi\sigma\gamma i}_{\delta kl}+$$

$$+\Big(\Gamma^{\alpha}_{\underset{\vee}{\pi\mu}}\,\Gamma^{\gamma}_{\sigma\nu}+\Gamma^{\alpha}_{\pi\nu}\,\Gamma^{\gamma}_{\underset{\vee}{\sigma\mu}}\Big)\,a^{\pi\beta\sigma i}_{\delta kl}+\Big(\Gamma^{\beta}_{\underset{\vee}{\pi\mu}}\,\Gamma^{\gamma}_{\sigma\nu}+\Gamma^{\beta}_{\pi\nu}\,\Gamma^{\gamma}_{\underset{\vee}{\sigma\mu}}\Big)\,a^{\alpha\pi\sigma i}_{\delta kl}-$$

$$-\Big(\Gamma^{\alpha}_{\underset{\vee}{\pi\mu}}\,\Gamma^{\sigma}_{\delta\nu}+\Gamma^{\alpha}_{\pi\nu}\,\Gamma^{\sigma}_{\underset{\vee}{\delta\mu}}\Big)\,a^{\pi\beta\gamma i}_{\sigma kl}-\cdots\Big]\,y^{\mu}_{,m}\,y^{\nu}_{,n}+$$

$$+\Big[\Big(\Gamma^{\alpha}_{\underset{\vee}{\pi\mu}}\,\Gamma^{i}_{sn}\,y^{\mu}_{,m}+\Gamma^{\alpha}_{\pi\nu}\,\Gamma^{i}_{\underset{\vee}{sm}}\,y^{\nu}_{,n}\Big)\,a^{\pi\beta\gamma s}_{\delta kl}+\cdots-\Big(\Gamma^{\alpha}_{\underset{\vee}{\pi\mu}}\,\Gamma^{s}_{kn}\,y^{\mu}_{,m}+\Gamma^{\alpha}_{\pi\nu}\,\Gamma^{s}_{\underset{\vee}{km}}\,y^{\nu}_{,n}\Big)\,a^{\pi\beta\gamma i}_{\delta sl}-$$

$$-\Big(\Gamma^{\alpha}_{\underset{\vee}{\pi\mu}}\,\Gamma^{s}_{ln}\,y^{\mu}_{,m}+\Gamma^{\alpha}_{\pi\nu}\,\Gamma^{s}_{\underset{\vee}{lm}}\,y^{\nu}_{,n}\Big)\,a^{\pi\beta\gamma i}_{\delta ks}-\cdots$$

$$+\Big(\Gamma^{\pi}_{\underset{\vee}{\delta\mu}}\,\Gamma^{s}_{kn}\,y^{\mu}_{,m}+\Gamma^{\pi}_{\delta\nu}\,\Gamma^{s}_{\underset{\vee}{km}}\,y^{\nu}_{,n}\Big)\,a^{\alpha\beta\gamma i}_{\pi sl}+\Big(\Gamma^{\pi}_{\underset{\vee}{\delta\mu}}\,\Gamma^{s}_{ln}\,y^{\mu}_{,m}+\Gamma^{\pi}_{\delta\nu}\,\Gamma^{s}_{\underset{\vee}{lm}}\,y^{\nu}_{,n}\Big)\,a^{\alpha\beta\gamma i}_{\pi ks}-$$

$$-\Big(\Gamma^{\pi}_{\underset{\vee}{\delta\mu}}\,\Gamma^{i}_{sn}\,y^{\mu}_{,m}+\Gamma^{\pi}_{\delta\nu}\,\Gamma^{i}_{\underset{\vee}{sm}}\,y^{\nu}_{,n}\Big)\,a^{\alpha\beta\gamma s}_{\pi kl}\Big]+\Big[-\Big(\Gamma^{i}_{\underset{\vee}{pm}}\,\Gamma^{s}_{kn}+\Gamma^{i}_{pn}\,\Gamma^{s}_{\underset{\vee}{km}}\Big)\,a^{\alpha\beta\gamma p}_{\delta sl}-$$

$$-\Big(\Gamma^{i}_{\underset{\vee}{pm}}\,\Gamma^{s}_{ln}+\Gamma^{i}_{pn}\,\Gamma^{s}_{\underset{\vee}{lm}}\Big)\,a^{\alpha\beta\gamma p}_{\delta ks}+\Big(\Gamma^{p}_{\underset{\vee}{km}}\,\Gamma^{s}_{ln}+\Gamma^{p}_{kn}\,\Gamma^{s}_{\underset{\vee}{lm}}\Big)\,a^{\alpha\beta\gamma i}_{\delta ps}\Big].$$

In general

$$(35)\qquad a^{\rho_1\ldots r_u}_{\tau_1\ldots t_v\underset{1}{|}mn}-a^{\rho_1\ldots r_u}_{\tau_1\ldots t_v\underset{1}{|}n\underset{2}{|}m}=$$

$$=\Big[\sum_{\lambda=1}^{\theta}\underset{7}{A}{}^{\rho_\lambda}_{\pi\mu\nu}\binom{\pi}{\rho_\lambda}a^{\cdots}_{\cdots}-\sum_{\varphi=1}^{\omega}\underset{8}{A}{}^{\pi}_{\tau_\varphi\mu\nu}\binom{\tau_\varphi}{\pi}a^{\cdots}_{\cdots}\Big]\,y^{\mu}_{,m}\,y^{\nu}_{,n}+$$

$$+\sum_{l=1}^{u}\underset{17}{A}{}^{r_l}_{pmn}\binom{p}{r_l}\,a^{\cdots}_{\cdots}-\sum_{f=1}^{v}\underset{8}{A}{}^{p}_{t_f mn}\binom{t_f}{p}\,a^{\cdots}_{\cdots}+$$

$$+2\,a^{\rho_1\ldots r_u}_{\tau_1\ldots t_v<mn>}+2\,a^{\rho_1\ldots r_u}_{\tau_1\ldots t_v\leqslant mn\geqslant},$$

where we denote

$$(36)\qquad a^{\rho_1\ldots r_u}_{\tau_1\ldots t_v\leqslant mn\geqslant}=\Big[\sum_{\lambda=1}^{\theta-1}\sum_{\substack{\varphi=2\\(\lambda<\varphi)}}^{\theta}\Big(\Gamma^{\rho_\lambda}_{\underset{\vee}{\pi\mu}}\,\Gamma^{\rho_\varphi}_{\sigma\nu}+\Gamma^{\rho_\lambda}_{\pi\nu}\,\Gamma^{\rho_\varphi}_{\underset{\vee}{\sigma\mu}}\Big)\binom{\pi}{\rho_\lambda}\binom{\sigma}{\rho_\varphi}\,a^{\cdots}_{\cdots}-$$

$$-\sum_{\lambda=1}^{\theta}\sum_{\varphi=1}^{\omega}\Big(\Gamma^{\rho_\lambda}_{\underset{\vee}{\pi\mu}}\,\Gamma^{\sigma}_{\tau_\varphi\nu}+\Gamma^{\rho_\lambda}_{\pi\nu}\,\Gamma^{\sigma}_{\underset{\vee}{\tau_\varphi\mu}}\Big)\binom{\pi}{\rho_\lambda}\binom{\tau_\varphi}{\sigma}\,a^{\cdots}_{\cdots}+$$

$$+\sum_{\substack{\lambda=1\\(\lambda<\varphi)}}^{\omega-1}\sum_{\varphi=2}^{\omega}\Big(\Gamma^{\pi}_{\underset{\vee}{\tau_\lambda\mu}}\Gamma^{\sigma}_{\tau_\varphi\nu}+\Gamma^{\pi}_{\tau_\lambda\nu}\Gamma^{\sigma}_{\underset{\vee}{\tau_\varphi\mu}}\Big)\binom{\tau_\lambda}{\pi}\binom{\tau_\varphi}{\sigma}a^{\cdots}_{\cdots}\Big]y^{\mu}_{,m}y^{\nu}_{,n}+$$

$$+\sum_{\lambda=1}^{\theta}\sum_{l=1}^{u}\Big(\Gamma^{\rho_\lambda}_{\underset{\vee}{\pi\mu}}\Gamma^{r_l}_{sn}y^{\mu}_{,m}+\Gamma^{\rho_\lambda}_{\pi\nu}\Gamma^{r_l}_{\underset{\vee}{sm}}y^{\nu}_{,n}\Big)\binom{\pi}{\rho_\lambda}\binom{s}{r_l}a^{\cdots}_{\cdots}-$$

$$-\sum_{\lambda=1}^{\theta}\sum_{f=1}^{v}\Big(\Gamma^{\rho_\lambda}_{\underset{\vee}{\pi\mu}}\Gamma^{s}_{t_f n}y^{\mu}_{,m}+\Gamma^{\rho_\lambda}_{\pi\nu}\Gamma^{s}_{\underset{\vee}{t_f m}}y^{\nu}_{,n}\Big)\binom{\pi}{\rho_\lambda}\binom{t_f}{s}a^{\cdots}_{\cdots}-$$

$$-\sum_{\varphi=1}^{\omega}\sum_{l=1}^{u}\Big(\Gamma^{\pi}_{\underset{\vee}{\tau_\varphi\mu}}\Gamma^{r_l}_{sn}y^{\mu}_{,m}+\Gamma^{\pi}_{\tau_\varphi\nu}\Gamma^{r_l}_{\underset{\vee}{sm}}y^{\nu}_{,n}\Big)\binom{\tau_\varphi}{\pi}\binom{s}{r_l}a^{\cdots}_{\cdots}+$$

$$+\sum_{\varphi=1}^{\omega}\sum_{f=1}^{v}\Big(\Gamma^{\pi}_{\underset{\vee}{\tau_\varphi\mu}}\Gamma^{s}_{t_f n}y^{\mu}_{,m}+\Gamma^{\pi}_{\tau_\varphi\nu}\Gamma^{s}_{\underset{\vee}{t_f m}}y^{\nu}_{,n}\Big)\binom{\tau_\varphi}{\pi}\binom{t_f}{s}a^{\cdots}_{\cdots}+$$

$$+\sum_{\substack{l=1\\(l<f)}}^{u-1}\sum_{f=2}^{u}\Big(\Gamma^{r_l}_{\underset{\vee}{pm}}\Gamma^{r_f}_{sn}+\Gamma^{r_l}_{pn}\Gamma^{r_f}_{\underset{\vee}{sm}}\Big)\binom{p}{r_l}\binom{s}{r_f}a^{\cdots}_{\cdots}-$$

$$-\sum_{l=1}^{u}\sum_{f=1}^{v}\Big(\Gamma^{r_l}_{\underset{\vee}{pm}}\Gamma^{s}_{t_f n}+\Gamma^{r_l}_{pn}\Gamma^{s}_{\underset{\vee}{t_f m}}\Big)\binom{p}{r_l}\binom{t_f}{s}a^{\cdots}_{\cdots}+$$

$$+\sum_{\substack{l=1\\(l<f)}}^{v-1}\sum_{f=2}^{v}\Big(\Gamma^{p}_{\underset{\vee}{t_l m}}\Gamma^{s}_{t_f n}+\Gamma^{p}_{t_l n}\Gamma^{s}_{\underset{\vee}{t_f m}}\Big)\binom{t_l}{p}\binom{t_f}{s}a^{\cdots}_{\cdots}.$$

The equations (31) and (36) are the 6^{th} *Ricci identity* of a subspace of a space of non-symmetric affine connexion.

2.7. On the base of the covariant derivations from the sections 2.1. and 2.4., we obtain

$$a^{\alpha\beta\gamma i}_{\delta kl\,\underset{1}{|}\,m\underset{1}{|}\,n}-a^{\alpha\beta\gamma i}_{\delta kl\,\underset{2}{|}\,n\,\underset{1}{|}\,m}=\tag{37}$$

$$=\big(\underset{9}{A}{}^{\alpha}_{\pi\mu\nu}\,y^{\mu}_{,m}\,y^{\nu}_{,n}+2\,\Gamma^{\alpha}_{\underset{\vee}{\pi\mu}}\,y^{\mu}_{,mn}\big)\,a^{\pi\beta\gamma i}_{\delta kl}+\cdots-\big(\underset{10}{A}{}^{\pi}_{\delta\mu\nu}\,y^{\mu}_{,m}\,y^{\nu}_{,n}+2\,\Gamma^{\pi}_{\underset{\vee}{\delta\mu}}\,y^{\mu}_{,mn}\big)\,a^{\alpha\beta\gamma i}_{\pi kl}+$$

$$+\underset{9}{A}{}^{i}_{pmn}\,a^{\alpha\beta\gamma p}_{\delta kl}-\underset{10}{A}{}^{p}_{kmn}\,a^{\alpha\beta\gamma i}_{\delta pl}-\underset{10}{A}{}^{p}_{lmn}\,a^{\alpha\beta\gamma i}_{\delta kp}+$$

$$+2\,a^{\alpha\beta\gamma i}_{\delta kl<nm>}+2\,a^{\alpha\beta\gamma i}_{\delta kl\leqslant nm\geqslant}-\big(\Gamma^{s}_{mn}\,a^{\alpha\beta\gamma i}_{\delta kl\,\underset{1}{|}\,s}-\Gamma^{s}_{nm}\,a^{\alpha\beta\gamma i}_{\delta kl\,\underset{2}{|}\,s}\big),$$

where we call the magnitudes

$$\underset{9}{A}{}^{\alpha}_{\beta\mu\nu}=\Gamma^{\alpha}_{\beta\mu,\nu}-\Gamma^{\alpha}_{\nu\beta,\mu}+\Gamma^{\pi}_{\beta\mu}\Gamma^{\alpha}_{\pi\nu}-\Gamma^{\pi}_{\nu\beta}\Gamma^{\alpha}_{\pi\mu}\tag{38}$$

$$\underset{10}{A}{}^{\alpha}_{\beta\mu\nu}=\Gamma^{\alpha}_{\beta\mu,\nu}-\Gamma^{\alpha}_{\nu\beta,\mu}+\Gamma^{\pi}_{\beta\mu}\Gamma^{\alpha}_{\nu\pi}-\Gamma^{\pi}_{\beta\nu}\Gamma^{\alpha}_{\pi\mu}\tag{39}$$

curvature pseudotensors of the 9^{th} *respectively* 10^{th} *kind.*

Generally in force is

$$(40)\qquad a^{\rho_1 \dots r_u}_{\tau_1 \dots t_v \dot{1} mn} - a^{\rho_1 \dots r_u}_{\tau_1 \dots t_v \dot{2}\; n \dot{1} m} =$$

$$= \sum_{\lambda=1}^{\theta} \Big(\underset{9}{A}{}^{\rho_\lambda}_{\pi\mu\nu}\, y^{\mu}_{,m} y^{\nu}_{,n} + 2\,\Gamma^{\rho_\lambda}_{\underset{\vee}{\pi\mu}}\, y^{\mu}_{,mn}\Big) \binom{\pi}{\rho_\lambda} a^{\cdots}_{\cdots} -$$

$$- \sum_{\varphi=1}^{\omega} \Big(\underset{10}{A}{}^{\pi}_{\tau_\varphi \mu\nu}\, y^{\mu}_{,m} y^{\nu}_{,n} + 2\,\Gamma^{\pi}_{\underset{\vee}{\tau_\varphi \mu}}\, y^{\mu}_{,mn}\Big) \binom{\tau_\varphi}{\pi} a^{\cdots}_{\cdots} +$$

$$+ \sum_{l=1}^{u} \underset{9}{A}{}^{r_l}_{pmn} \binom{p}{r_l} a^{\cdots}_{\cdots} - \sum_{f=1}^{v} \underset{10}{A}{}^{p}_{t_f mn} \binom{t_f}{p} a^{\cdots}_{\cdots} +$$

$$+ 2\, a^{\rho_1 \dots r_u}_{\tau_1 \dots t_v < nm >} + 2\, a^{\rho_1 \dots r_u}_{\tau_1 \dots t_v \leqslant nm \geqslant} - \Big(\Gamma^{s}_{mn}\, a^{\rho_1 \dots r_u}_{\tau_1 \dots t_v \dot{1} s} - \Gamma^{s}_{nm}\, a^{\rho_1 \dots r_u}_{\tau_1 \dots t_v \dot{2} s}\Big).$$

The equations (37) and (40) are 7^{th} *Ricci identity.*

2.8. Using the covariant derivations of the second degree from the sections 2.2. and 2.3., we have

$$(41)\qquad a^{\alpha\beta\gamma i}_{\delta kl \dot{2} mn} - a^{\alpha\beta\gamma i}_{\delta kl \dot{1}\; n \dot{2} m} =$$

$$= \Big(\underset{11}{A}{}^{\alpha}_{\mu\pi\nu}\, y^{\mu}_{,m} y^{\nu}_{,n} + 2\,\Gamma^{\alpha}_{\underset{\vee}{\mu\pi}}\, y^{\mu}_{,mn}\Big) a^{\pi\beta\gamma i}_{\delta kl} + \dots - \Big(\underset{12}{A}{}^{\pi}_{\mu\delta\nu}\, y^{\mu}_{,m} y^{\nu}_{,n} + 2\,\Gamma^{\pi}_{\underset{\vee}{\mu\delta}}\, y^{\mu}_{,mn}\Big) a^{\alpha\beta\gamma i}_{\pi kl} +$$

$$+ \underset{11}{A}{}^{i}_{mpn}\, a^{\alpha\beta\gamma p}_{\delta kl} - \underset{12}{A}{}^{p}_{mkn}\, a^{\alpha\beta\gamma i}_{\delta pl} - \underset{12}{A}{}^{p}_{mln}\, a^{\alpha\beta\gamma i}_{\delta kp} -$$

$$- 2\, a^{\alpha\beta\gamma i}_{\delta kl < nm >} + 2\, a^{\alpha\beta\gamma i}_{\delta kl \leqslant mn \geqslant} + \Gamma^{s}_{mn}\, a^{\alpha\beta\gamma i}_{\delta kl \dot{1} s} - \Gamma^{s}_{nm}\, a^{\alpha\beta\gamma i}_{\delta kl \dot{2} s},$$

where

$$(42)\qquad \underset{11}{A}{}^{\alpha}_{\beta\mu\nu} = \Gamma^{\alpha}_{\beta\mu,\nu} - \Gamma^{\alpha}_{\mu\nu,\beta} + \Gamma^{\pi}_{\beta\mu}\Gamma^{\alpha}_{\nu\pi} - \Gamma^{\pi}_{\mu\nu}\Gamma^{\alpha}_{\beta\pi},$$

$$(43)\qquad \underset{12}{A}{}^{\alpha}_{\beta\mu\nu} = \Gamma^{\alpha}_{\beta\mu,\nu} - \Gamma^{\alpha}_{\mu\nu,\beta} + \Gamma^{\pi}_{\beta\mu}\Gamma^{\alpha}_{\pi\nu} - \Gamma^{\pi}_{\nu\mu}\Gamma^{\alpha}_{\beta\pi}$$

and $\underset{11}{A}{}^{i}_{jmn}$, $\underset{12}{A}{}^{i}_{jmn}$ are *curvature pseudotensors of the* 11^{th} *and* 12^{th} *kind,* and analogically to (34), we introduce a denotation

$$(44)\qquad a^{\alpha\beta\gamma i}_{\delta kl \leqslant mn \geqslant} = \Big[\Big(\Gamma^{\alpha}_{\underset{\vee}{\mu\pi}}\Gamma^{\beta}_{\underset{\vee}{\nu\sigma}} + \Gamma^{\alpha}_{\underset{\vee}{\nu\pi}}\Gamma^{\beta}_{\mu\sigma}\Big) a^{\pi\sigma\beta\gamma i}_{\delta kl} + \dots$$

$$- \Big(\Gamma^{\alpha}_{\underset{\vee}{\mu\pi}}\Gamma^{\sigma}_{\nu\delta} + \Gamma^{\sigma}_{\underset{\vee}{\nu\pi}}\Gamma^{\sigma}_{\mu\sigma}\Big) a^{\pi\beta\gamma i}_{\sigma kl} - \dots\Big] y^{\mu}_{,m} y^{\nu}_{,n} +$$

$$+ \Big[\Big(\Gamma^{\alpha}_{\underset{\vee}{\mu\pi}}\Gamma^{i}_{ns}\, y^{\mu}_{,m} + \Gamma^{\alpha}_{\underset{\vee}{\nu\pi}}\Gamma^{i}_{ms}\, y^{\nu}_{,n}\Big) a^{\pi\beta\gamma s}_{\delta kl} + \dots -$$

$$- \Big(\Gamma^{\alpha}_{\underset{\vee}{\mu\pi}}\Gamma^{s}_{nk}\, y^{\mu}_{,m} + \Gamma^{\alpha}_{\underset{\vee}{\nu\pi}}\Gamma^{s}_{mk}\, y^{\nu}_{,n}\Big) a^{\pi\beta\gamma i}_{\delta sl} - \dots$$

$$+ \Big(\Gamma^{\pi}_{\underset{\vee}{\mu\delta}}\Gamma^{s}_{nk}\, y^{\mu}_{,m} + \Gamma^{\pi}_{\underset{\vee}{\nu\delta}}\Gamma^{s}_{mk}\, y^{\nu}_{,n}\Big) a^{\alpha\beta\gamma i}_{\pi sl} + \dots\Big] +$$

$$+ \Big[-\Big(\Gamma^{i}_{\underset{\vee}{mp}}\Gamma^{s}_{nk} + \Gamma^{i}_{\underset{\vee}{np}}\Gamma^{s}_{mk}\Big) a^{\alpha\beta\gamma p}_{\delta sl} - \dots + \Big(\Gamma^{p}_{\underset{\vee}{mk}}\Gamma^{s}_{nl} + \Gamma^{p}_{\underset{\vee}{nk}}\Gamma^{s}_{ml}\Big)\Big].$$

Generally

(45)
$$a^{\rho_1 \dots r_u}_{\tau_1 \dots t_v \underset{2}{|} mn} - a^{\rho_1 \dots r_u}_{\tau_1 \dots t_v \underset{1}{|} n \underset{2}{|} m} =$$

$$= \sum_{\lambda=1}^{\theta} \Big(\underset{11}{A}{}^{\rho_\lambda}{}_{\mu\pi\nu}\, y^{\mu}_{,m}\, y^{\nu}_{,n} + 2\, \underset{\vee}{\Gamma^{\rho_\lambda}_{\mu\pi}}\, y^{\mu}_{,mn} \Big) \binom{\pi}{\rho_\lambda} a^{\dots}_{\dots} -$$

$$- \sum_{\varphi=1}^{\omega} \Big(\underset{12}{A}{}^{\pi}{}_{\mu\tau_\varphi\nu}\, y^{\mu}_{,m}\, y^{\nu}_{,n} + 2\, \underset{\vee}{\Gamma^{\pi}_{\mu\tau_\varphi}}\, y^{\mu}_{,mn} \Big) \binom{\tau_\varphi}{\pi} a^{\dots}_{\dots} +$$

$$+ \sum_{l=1}^{u} \underset{11}{A}{}^{r_l}{}_{mpn} \binom{p}{r_l} a^{\dots}_{\dots} - \sum_{f=1}^{v} \underset{12}{A}{}^{p}{}_{mt_f n} \binom{t_f}{p} a^{\dots}_{\dots} -$$

$$-2\, a^{\rho_1 \dots r_u}_{\tau_1 \dots t_v <nm>} + 2\, a^{\rho_1 \dots r_u}_{\tau_1 \dots t_v \leqslant mn \geqslant} + \Gamma^{s}_{mn}\, a^{\rho \dots r_u}_{\tau \dots t_v \underset{1}{|} s} - \Gamma^{s}_{nm}\, a^{\rho_1 \dots r_u}_{\tau_1 \dots t_v \underset{2}{|} s},$$

where, analogically to (36), we have put

(46)
$$a^{\rho_1 \dots r_u}_{\tau_1 \dots t_v \leqslant mn \geqslant} =$$

$$= \Bigg[\sum_{\substack{\lambda=1 \\ (\lambda<\varphi)}}^{\theta-1} \sum_{\varphi=2}^{\theta} \Big(\Gamma^{\rho_\lambda}_{\mu\pi}\, \underset{\vee}{\Gamma^{\rho_\varphi}_{\nu\delta}} + \underset{\vee}{\Gamma^{\rho_\lambda}_{\nu\pi}}\, \Gamma^{\rho_\varphi}_{\mu\sigma} \Big) \binom{\pi}{\rho_\lambda} \binom{\sigma}{\rho_\varphi} a^{\dots}_{\dots} - \cdots \Bigg] y^{\mu}_{,m}\, y^{\nu}_{,n} +$$

$$+ \sum_{\lambda=1}^{\theta} \sum_{l=1}^{u} \Big(\Gamma^{\rho_\lambda}_{\mu\pi}\, \underset{\vee}{\Gamma^{r_l}_{ns}}\, \Gamma\, y^{\mu}_{,m} + \underset{\vee}{\Gamma^{\rho_\lambda}_{\nu\pi}}\, \Gamma^{r_l}_{ms}\, y^{\nu}_{,n} \Big) \binom{\pi}{\rho_\lambda} \binom{s}{r_l} a^{\dots}_{\dots} - \cdots$$

$$+ \sum_{\substack{l=1 \\ (l<f)}}^{u-1} \sum_{f=2}^{u} \Big(\Gamma^{r_l}_{mp}\, \underset{\vee}{\Gamma^{r_f}_{ns}} + \underset{\vee}{\Gamma^{r_l}_{np}}\, \Gamma^{r_f}_{ms} \Big) \binom{p}{r_l} \binom{s}{r_f} a^{\dots}_{\dots} - \cdots.$$

The equation (45), together with (41), represents *the* 8^{th} *Ricci identity.*

2.9. Using the derivations from the sections 2.2. and 2.4., we get

(47)
$$a^{\alpha\beta\gamma i}_{\delta kl \underset{2}{|} mn} - a^{\alpha\beta\gamma i}_{\delta kl \underset{2}{|} n \underset{1}{|} m} =$$

$$= \Big(\underset{13}{A}{}^{\alpha}{}_{\mu\pi\nu}\, a^{\pi\beta\gamma i}_{\delta kl} + \cdots - \underset{14}{A}{}^{\pi}{}_{\mu\delta\nu}\, a^{\alpha\beta\gamma i}_{\pi kl} \Big)\, y^{\mu}_{,m}\, y^{\nu}_{,n} +$$

$$+ \underset{13}{A}{}^{\iota}{}_{mpn}\, a^{\alpha\beta\gamma p}_{\delta kl} - \underset{14}{A}{}^{p}{}_{mkn}\, a^{\alpha\beta\gamma i}_{\delta pl} - \underset{14}{A}{}^{p}{}_{mln}\, a^{\alpha\beta\gamma i}_{\delta kp} -$$

$$-2\, a^{\alpha\beta\gamma i}_{\delta kl <mn>} + 2\, a^{\alpha\beta\gamma i}_{\delta kl \leqslant nm \geqslant},$$

where we call the magnitudes

(48)
$$\underset{13}{A}{}^{\alpha}{}_{\beta\mu\nu} = \Gamma^{\alpha}_{\beta\mu,\nu} - \Gamma^{\alpha}_{\nu\mu,\beta} + \Gamma^{\pi}_{\beta\mu}\, \Gamma^{\alpha}_{\nu\pi} - \Gamma^{\pi}_{\nu\mu}\, \Gamma^{\alpha}_{\pi\beta}$$

(49)
$$\underset{14}{A}{}^{\alpha}{}_{\beta\mu\nu} = \Gamma^{\alpha}_{\beta\mu,\nu} - \Gamma^{\alpha}_{\nu\mu,\beta} + \Gamma^{\pi}_{\mu\beta}\, \Gamma^{\alpha}_{\nu\pi} - \Gamma^{\pi}_{\nu\mu}\, \Gamma^{\alpha}_{\beta\pi}$$

curvature pseudotensors of the 13^{th} *and* 14^{th} *kind.*

In general

(50)
$$a^{\rho_1 \dots r_u}_{\tau_1 \dots t_v \underset{2}{|} mn} - a^{\rho_1 \dots r_u}_{\tau_1 \dots t_v \underset{2}{|} n \underset{1}{|} m} =$$
$$= \left[\sum_{\lambda=1}^{\theta} \underset{13}{A}{}^{\rho_\lambda}_{\mu\pi\nu} \binom{\pi}{\rho_\lambda} a^{\cdots}_{\cdots} - \sum_{\varphi=1}^{\omega} \underset{14}{A}{}^{\pi}_{\mu\tau_\varphi\nu} \binom{\tau_\varphi}{\pi} a^{\cdots}_{\cdots} \right] y^{\mu}_{,m} y^{\nu}_{,n} +$$
$$= \sum_{l=1}^{u} \underset{13}{A}{}^{r_l}_{mpn} \binom{p}{r_l} a^{\cdots}_{\cdots} - \sum_{f=1}^{v} \underset{14}{A}{}^{p}_{mt_f n} \binom{t_f}{p} a^{\cdots}_{\cdots} -$$
$$- 2\, a^{\rho_1 \dots r_u}_{\tau_1 \dots t_v <mn>} + 2\, a^{\rho_1 \dots r_u}_{\tau_1 \dots t_v \ll nm \gg},$$

We call the equations (47), (50) *the* 9[th] *Ricci identity.*

2.10. On the base of derivations from the sections 2.3., 2.4. it follows

(51)
$$a^{\alpha\beta\gamma i}_{\delta kl \underset{1}{|} m \underset{2}{|} n} - a^{\alpha\beta\gamma i}_{\delta kl \underset{2}{|} n \underset{1}{|} m} =$$
$$= \big(\underset{15}{A}{}^{\alpha}_{\pi\mu\nu} y^{\mu}_{,m} y^{\nu}_{,n} + 2\, \Gamma^{\alpha}_{\underset{\vee}{\pi\mu}} y^{\mu}_{,mn} \big)\, a^{\pi\beta\gamma i}_{\delta kl} + \cdots$$
$$- \big(\underset{15}{A}{}^{\pi}_{\delta\mu\nu} y^{\mu}_{,m} y^{\nu}_{,n} + 2\, \Gamma^{\pi}_{\underset{\vee}{\delta\mu}} y^{\mu}_{,mn} \big)\, a^{\alpha\beta\gamma i}_{\pi kl} +$$
$$+ \underset{15}{A}{}^{i}_{pmn} a^{\alpha\beta\gamma p}_{\delta kl} - \underset{15}{A}{}^{p}_{kmn} a^{\alpha\beta\gamma i}_{\delta pl} - \underset{15}{A}{}^{p}_{lmn} a^{\alpha\beta\gamma i}_{\delta kp} -$$
$$- \Gamma^{s}_{nm} \left(a^{\alpha\beta\gamma i}_{\delta kl \underset{1}{|} s} - a^{\alpha\beta\gamma i}_{\delta kl \underset{2}{|} s} \right),$$

where

(52)
$$\underset{15}{A}{}^{\alpha}_{\beta\mu\nu} = \Gamma^{\alpha}_{\beta,\mu\nu} - \Gamma^{\alpha}_{\nu\beta,\mu} + \Gamma^{\pi}_{\beta\mu} \Gamma^{\alpha}_{\nu\pi} - \Gamma^{\pi}_{\nu\beta} \Gamma^{\alpha}_{\pi\mu}$$

is *curvature pseudotensor of the* 15[th] *kind* (and $\underset{15}{A}{}^{i}_{jmn}$ too).

Generally

(53)
$$a^{\rho_1 \dots r_u}_{\tau_1 \dots t_v \underset{1}{|} m \underset{2}{|} n} - a^{\rho_1 \dots r_u}_{\tau_1 \dots t_v \underset{2}{|} n \underset{1}{|} m} =$$
$$= \sum_{\lambda=1}^{\theta} \left(\underset{15}{A}{}^{\rho_\lambda}_{\pi\mu\nu} y^{\mu}_{,m} y^{\nu}_{,n} + 2\, \Gamma^{\rho_\lambda}_{\underset{\vee}{\pi\mu}} y^{\mu}_{,mn} \right) \binom{\pi}{\rho_\lambda} a^{\cdots}_{\cdots} -$$
$$- \sum_{\varphi=1}^{\omega} \left(\underset{15}{A}{}^{\pi}_{\tau_\varphi\mu\nu} y^{\mu}_{,m} y^{\nu}_{,n} + 2\, \Gamma^{\pi}_{\underset{\vee}{\tau_\varphi\mu}} y^{\mu}_{,mn} \right) \binom{\tau_\varphi}{\pi} a^{\cdots}_{\cdots} +$$
$$+ \sum_{l=1}^{u} \underset{5}{A}{}^{r_l}_{pmn} \binom{p}{r_l} a^{\cdots}_{\cdots} - \sum_{f=1}^{v} \underset{15}{A}{}^{p}_{t_f mn} \binom{t_f}{p} a^{\cdots}_{\cdots} -$$
$$- \Gamma^{s}_{nm} \left(a^{\rho_1 \dots r_u}_{\tau_1 \dots t_v \underset{1}{|} s} - a^{\rho_1 \dots r_u}_{\tau_1 \dots t_v \underset{2}{|} s} \right),$$

which, together with (51), represents *the* 10[th] *Ricci identity.*

The 10^{th} Ricci identity can be transformed into another form. In fact, if we count the term in the brackets in (51) according to (3) and (4), the equation (51) becomes:

(54)
$$a^{\alpha\beta\gamma i}_{\delta kl \underset{1}{|} m \underset{2}{|} n} - a^{\alpha\beta\gamma i}_{\delta kl \underset{2}{|} n \underset{1}{|} m} =$$
$$= \underset{3}{R}{}^{\alpha}_{\pi mn} a^{\pi\beta\gamma i}_{\delta kl} + \cdots - \underset{3}{R}{}^{\pi}_{\delta, mn} a^{\alpha\beta\gamma i}_{\pi kl} +$$
$$+ \underset{3}{R}{}^{i}_{pmn} a^{\alpha\beta\gamma p}_{\delta kl} - \underset{3}{R}{}^{p}_{kmn} a^{\alpha\beta\gamma i}_{\delta pl} - \underset{3}{R}{}^{p}_{lmn} a^{\alpha\beta\gamma i}_{\delta kp},$$

where we call the magnitude

(55a)
$$\underset{3}{R}{}^{\alpha}_{\beta mn} = \underset{15}{A}{}^{\alpha}_{\beta\mu\nu} y^{\mu}_{,m} y^{\nu}_{,n} + 2 \Gamma^{\alpha}_{\underset{\vee}{\beta\mu}} (y^{\mu}_{,mn} - \Gamma^{s}_{nm} y^{\mu}_{,s})$$

a curvature tensor of the 3^{rd} *kind of the space* L_N *with respect to the subspace* L_M, *and*

(55b)
$$\underset{3}{R}{}^{i}_{jmn} = \underset{15}{A}{}^{i}_{jmn} - 2 \Gamma^{i}_{\underset{\vee}{js}} \Gamma^{s}_{nm}$$

is a curvature tensor of the 3^{rd} *kind of the subspace* L_M.

Generally, it is

(56)
$$a^{\rho_1 \ldots r_u}_{\tau_1 \ldots t_v \underset{1}{|} m \underset{2}{|} n} - a^{\rho_1 \ldots r_u}_{\tau_1 \ldots t_v \underset{2}{|} n \underset{1}{|} m} =$$
$$= \sum_{\lambda=1}^{\theta} \underset{3}{R}{}^{\rho_\lambda}_{\pi mn} \binom{\pi}{\rho_\lambda} a^{\cdots}_{\cdots} - \sum_{\varphi=1}^{\omega} \underset{3}{R}{}^{\pi}_{\tau_\varphi mn} \binom{\tau_\varphi}{\pi} a^{\cdots}_{\cdots} +$$
$$+ \sum_{l=1}^{u} \underset{3}{R}{}^{r_l}_{pmn} \binom{p}{r_l} a^{\cdots}_{\cdots} - \sum_{f=1}^{v} \underset{3}{R}{}^{p}_{t_f mn} \binom{t_f}{p} a^{\cdots}_{\cdots}.$$

The equations (54) and (56) are *another form of the* 10^{th} *Ricci identity.*

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