PUBLICATIONS DE L'INSTITUT MATHÉMATIQUE
Nouvelle série, tome 16 (30), 1973 pp. 53—60

TOPOLOGICAL STRUCTURES ON CLASSES III

Dedicated to Professor Đuro Kurepa on the occasion of forty years of his fertile scientific work

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(Communicated January 20, 1973)

1. Introduction.

This paper contains a further study of spatial structures in $\mathcal{U}$, i. e. of formations of certain wholes in $\mathcal{U}$ being capable of creative activities in themselves. In the paper [1] we regarded a kind of formations of such wholes. There we regarded a pair $(s_{i+1}, p_{i+1}(t_{i+1}))$ consisting of a class s_{i+1} and a fundamental semigroupoid $p_{i+1}(t_{i+1})$ such that $s_{i+1} \prec p_{i+1}(t_{i+1})$ and a many-valued rule $\tau_{i+1}: s_{i+1} \to p_{i+1}(t_{i+1})$ which assigns to each object s_i of s_{i+1} a filter $\tau_{i+1}(s_i)$ of $p_{i+1}(t_{i+1})$. This rule satisfying the conditons of the Definition 1 of the mentioned paper we called a topology. By means of it we formed a spatial whole in $\mathcal{U}$ called a topological space. Topological organizations are, certainly, not only possible spatial organizations of $\mathcal{U}$. There are also many other ways to form a spatial whole in $\mathcal{U}$. We shall regard here two ways. First we shall regard a weaker form of topology τ_{i+1} on the above mentioned pair, the case when $\tau_{i+1}(s_i)$ is not assumed to be a filter but only an antiresidual subclass of $p_{i+1}(t_{i+1})$. Such a topology on $(s_{i+1}, p_{i+1}(t_{i+1}))$ we shall call a pretopology and denote it by τ_{i+1} too. We shall study this topology through a discussion of the material given in [1] which is concerned with topology τ_{i+1} and emphasize assertions which are valid for it. We shall further weaken pretopology on $(s_{i+1}, p_{i+1}(t_{i+1}))$ by assuming that $\tau_{i+1}(s_i)$ is simply a subclass of $p_{i+1}(t_{i+1})$ without any condition imposed. Such a topology on a pair will be the weakest form of its organization. We shall only announce the way for its involving.

In the second part of the paper we shall be concerned with a particular way of formations of spatial wholes. This way as well as pretopological one does not require previous organizations of subclasses of the fundamental semigroupoid which takes a part in the formation of wholes. It has essentially a constructive character on the semigroupoid. Its main feature is a choice of subclasses of the semigroupoid and a requirement for them to possess constructive ends of themselves. We shall realize this formation by means of a class of many-valued rules having certain special properties. This class we shall call an intuitionistic topology. The world formed by means of this topology will manifest itself by closed constructive wholes on its particular domains and their mutual lawful dependences. These wholes are worlds for themselves all being joined together in a global whole. We shall explain now the main features of intuitionistic topology.

Let us regard two fundamental semigroupoids $q_{i+1}(s_{i+1})$ and $p_{i+1}(t_{i+1})$ and a single-valued funhom F_{i+1} of $q_{i+1}(s_{i+1})$ to $p_{i+1}(t_{i+1})$. As we know, to form a spatial whole from these two semigroupoids by means of a topology we have to require that $q_{i+1}(s_{i+1})$ is discrete, i. e. that $q_{i+1}(s_{i+1})=s_{i+1}$ and that between them there is the strict domination. The requirement of discreteness, however, is not necessary. Namely, we could also regard formations of spatial wholes by means of a topology even in the case that $q_{i+1}(s_{i+1})$ is not discrete but an arbitrary fundamental semigroupoid. We shall not deal with this case here. However we may say that in any case the formation of wholes by topology utilizes the class s_{i+1} or more generally the fundamental semigroupoid $q_{i+1}(s_{i+1})$ for completing filters of $p_{i+1}(t_{i+1})$. Otherwise, we can also utilize this semigroupoid in another sense. Namely, we can utilize it to make a choice of certain subclasses of $p_{i+1}(t_{i+1})$ for which we shall claim then to allow formations of constructive ends in themselves. These ends are the last objects of the subclasses. Regarding the choice of subclasses we assume that besides the objects of $q_{i+1}(s_{i+1})$ and the funhom F_{i+1} it also depends upon objects of $p_{i+1}(t_{i+1})$, i. e. that it is to be done with respect to objects of $p_{i+1}(t_{i+1})$. We decided in the paper on the following choice. If s_i is an object of $q_{i+1}(s_{i+1})$ and t_i an object of $p_{i+1}(t_{i+1})$ then we choose a subclass of $p_{i+1}(t_{i+1})$ in such a manner that its class of objects consists of all those objects t_i' of $p_{i+1}(t_{i+1})$ for which there exist p_{i+1}-rules $p_i: F_{i+1}(s_i)\wedge t_i' \to t_i$, where $F_{i+1}(s_i)\wedge t_i'$ denotes a presequent of these two objects in $p_{i+1}(t_{i+1})$. Certainly, to make this choice possible we must claim that these presequents exist in $p_{i+1}(t_{i+1})$.

We shall see later that the above discussion we can express by certain rules. We shall regard a many-valued funhom $\overset{t_i}{\sigma}_{i+1}$ of $q_{i+1}(s_{i+1})$ to $p_{i+1}(t_{i+1})$ which assigns to each object s_i of $q_{i+1}(s_{i+1})$ a subclass $\overset{t_i}{\sigma}_{i+1}(s_i)$ of $p_{i+1}(t_{i+1})$ the choice of which depends on the image $F_{i+1}(s_i)$ of the object s_i under the funhom F_{i+1} and on a fixed object t_i of $p_{i+1}(t_{i+1})$ as above described. Of course, we shall consider that $p_{i+1}(t_{i+1})$ is such to allow this choice. The next requirement in the sense of the above story is that there exists a single-valued funhom $\overset{t_i}{S}_{i+1}$ and a natural rule $\overset{t_i}{\eta}_{i+1}: \overset{t_i}{\sigma}_{i+1} \to \overset{t_i}{S}_{i+1}$ such that the image of an object s_i under the funhom $\overset{t_i}{\mathfrak{F}}_{i+1}=(\overset{t_i}{\sigma}_{i+1}, \overset{t_i}{\eta}_{i+1}, \overset{t_i}{S}_{i+1})$ is *fc*, see [3], in $\overset{t_i}{\sigma}_{i+1}(s_i)$. Hence, the object $\overset{t_i}{S}_{i+1}(s_i)$ is the last object in the chosen subclass $\overset{t_i}{\sigma}_{i+1}(s_i)$. For the funhom $\overset{t_i}{S}_{i+1}$ we shall say then to be the last selection funhom of $\overset{t_i}{\sigma}_{i+1}$ and for the subclass $\overset{t_i}{\sigma}_{i+1}(s_i)$, $s_i \in q_{i+1}(s_{i+1})$ to have the last object. In such a way we shall have certain constructively closed wholes on $p_{i+1}(t_{i+1})$. It remains still to join all these wholes in a global whole. We shall do this by specifying the funhoms $\overset{t_i}{\sigma}_{i+1}$, $t_i \in p_{i+1}(t_{i+1})$ and connections between them. In that way we shall get intuitionistic topology. Namely, under an intuitionistic topology on a fundamental semigroupoid $p_{i+1}(t_{i+1})$ determined by a fundamental semigroupoid $q_{i+1}(s_{i+1})$ and a funhom $F_{i+1}: q_{i+1}(s_{i+1}) \to p_{i+1}(t_{i+1})$ in the above given sense we shall understand a class of contravariant many-valued funhoms $\overset{t_i}{\sigma}_{i+1}: q_{i+1}(s_{i+1}) \to p_{i+1}(t_{i+1})$, $t_i \in p_{i+1}(t_{i+1})$ such that each of them has a last selection funhom $\overset{t_i}{S}_{i+1}$.

Hence, as a conclusion, we have that the essential feature of intuitionistic topology is a choice of subclasses of the regarded fundamental semigroupoid and creations of constructively closed wholes from them. If we compare a world formed by means of an intuitionistic topology and a topology we shall see that intuitionistic formation is weaker than formation by a topology. It is better to say that it is more free in the choice and allows much more organizational possibilities.

In the paper we shall investigate properties of intuitionistic topology and establish its relationships with regarded kinds of topologies. In the next paper we shall regard all these topologies in the context of a general discussion of various systems of mathematics and formation of a general spatial system. Afterwards we shall deal with a formalization of these investigations.

Finally we mention that not stressed notations, abbreviations and other stipulations in the paper are taken over from earlier author's papers. New ones will be particularly emphasized throughout the paper.

2. Pretopologies.

A pretopology on the pair $(s_{i+1}, p_{i+1}(t_{i+1}))$ is a weaker form of topology on it. The formation of spatial wholes by means of this topology does not require that subclasses of $p_{i+1}(t_{i+1})$ be previously organized in filters. It is enough to claim for them to possess the antiresidual property, see [3]. Subclasses of $p_{i+1}(t_{i+1})$ with this property we shall call *prefilters*. All necessary properties of prefilters one can deduce directly from [3].

First in this section we shall define pretopology and then deal with its reformulation and operators by means of which it can be involved. Thus we shall regard first the material which corresponds to the material contained in the sections 2 and 3 of [1]. Afterwards we shall announce another questions concerning this topology and also its weaker form.

Let us start with the definition. The definition of pretopology is the same as the Definition 1 of [1] only that the word a filter in it is replaced by the word a prefilter. Then $\tau_{i+1}(s_i)$ will mean an η_{i+1}*-neighborhoods prefilter* and the triple $(s_{i+1}, \tau_{i+1}, p_{i+1}(t_{i+1}))$ a *pretopological space* on the $(i+1)$th level in $\mathcal{U}$.

We can easily reformulate the definition of pretopology in the sense of the reformulation concerning topology [1]. Certainly, a reformulated pretopology on $(s_{i+1}, p_{i+1}(t_{i+1}))$ is a subclass $p'_{i+1}(t'_{i+1})$ possessing the following properties:

i) it contains the strict first object o_i^s and the last object 1_i,

ii) it allows *fc* formations on each its subclass.

It also fulfils the connecting property iii) given in [1]. A pretopology is clearly a subclass of $p_{i+1}(t_{i+1})$ consisting of all objects and rules of neighborhoods prefilters $\tau_{i+1}(s_i)$, $s_i \in s_{i+1}$. It obviously does not allow *lcc* formations. The proof of logical equivalence of the original and reformulated definition of pretopology is analogous to that one for the case of topology.

Let us regard now the operators by means of which a pretopology can be involved. The definition of a complementation operator is the same as given in [1]. However, by a closure operator on $p_{i+1}(t_{i+1})$ we shall regard now a covariant funhom of $p_{i+1}(t_{i+1})$ to itself satisfying the conditions $C1$, $C2$ and $C4$ of the Definition 4 of [1]. We shall denote it by $\mathbf{C}'_{i+1}$. By using these two operators we can prove the following

P r o p o s i t i o n 1. *Let* $\langle p_{i+1}(t_{i+1}); \mathscr{C}_{i+1}, \mathbf{C}'_{+1}\rangle$ *be a complemented closure l-semigroupoid. Then the image* $\mathscr{C}_{i+1}\mathbf{C}'_{i+1}(p_{i+1})$ *of* $p_{i+1}(t_{i+1})$ *is a pretopology.*

P r o o f. One can select from the proposition 3 of [1] that part of the proof which corresponds to this case.

Furthermore we have the definition of an interior operator. It is a covariant funhom which satisfies the conditions $O1$, $O2$ and $O4$ of the Definition 6 of [1]. We can define, in an obvious manner, a pretopology by means of this operator. It is enough to consider an interior l-semigroupoid as in [1]. Otherwise, there is a connection between this and the closure operator $\mathbf{C}'_{i+1}$ expressed by means of the operator $\mathscr{C}_{i+1}$. It is the same as for the operator $\mathbf{O}_{i+1}$ and $\mathbf{C}_{i+1}$, see [1].

We should now regard another questions concerning pretopologies. However, since this is not our main purpose, then we omit to do it. We shall only announce these questions. So, by following the papers [1] and [2] we could easily define the concepts — open and closed objects, a base for pretopology, then regard conditions which govern formations of pretopological spaces and the rules for relating these spaces.

We could now define a wider class of pretopological spaces by assuming that the pretopology τ_{i+1} is such that $\tau_{i+1}(s_i)$ is an arbitrary subclass of $p_{i+1}(t_{i+1})$ without imposed conditions upon it. Such a pretopology is to be involved by means of a complementation operator $\mathscr{C}_{i+1}$ as defined in [1] and a closure operator $\mathbf{C}''_{i+1}$ being a covariant funhom which satisfies the conditions $C1$ and $C4$, respectively by means of an interior operator $\mathbf{O}''_{i+1}$ satisfying the conditions $O1$ and $O4$ of the corresponding definititions of [1]. We shall not study this kind of pretopology although one should do it.

In the next section we shall see that pretopology and also topology are only certain stronger forms of a spatial organization called intuitionistic.

3. Intuitionistic topologies.

In this section we shall deal with the second type of announced topologies which do not require previous organizations of fundamental semigroupoids which take parts in formations of spatial wholes in $\mathscr{U}$, although we shall require for them to have certain necessary properties. The starting elements for introducing this topology, that we call intuitionistic topology, are fundamental semigroupoids $q_{i+1}(s_{i+1})$ and $p_{i+1}(t_{i+1})$ and a funhom F_{i+1} of $q_{i+1}(s_{i+1})$ to $p_{i+1}(t_{i+1})$. Our purpose is to form a spatial whole from these concepts which will manifest itself by a constructive completeness on its parts. By this we mean that certain subclasses of the whole possess constructive ends. These ends are last objects of these subclasses. The question arises, how to choose the subclasses. We decided on the following choice which is, of course, stipulated by the operations we are acquainted with. We assume that the choice depends on the images of objects of $q_{i+1}(s_{i+1})$ under F_{i+1} and that it is relative to objects of $p_{i+1}(t_{i+1})$. Let us specify in which way. We fix an object t_i of $p_{i+1}(t_{i+1})$ and regard an object s_i of $q_{i+1}(s_{i+1})$. Then the class we are interested in is a subclass $\sigma^{t_i}_{i+1}(s_i)$ of $p_{i+1}(t_{i+1})$ whose class of objects consists of all those objects t'_i of $p_{i+1}(t_{i+1})$ for which there exist p_{i+1}-rules $p_i: F_{i+1}(s_i) \wedge t_i \to t_i'$. Here, $F_{i+1}(s_i)$ denotes the image of s_i under F_{i+1} and $F_{i+1}(s_i) \wedge t_i'$ the presequent of these two objects. Certainly, in order to make the desired choice of subclasses we have to claim that $p_{i+1}(t_{i+1})$ is such to allow presequent formations on any pair of its objects.

We shall now rewrite this story and define intuitionistic topology by means of certain funhoms. Let $q_{i+1}(s_{i+1})$ be an arbitrary fundamental semigroupoid and $p_{i+1}(t_{i+1})$ a fundamental semigroupoid having certain special properties those which will allow us to make desired choices of its subclasses. According to our present decision we claim that it contains presequents of any two its objects. Further, let σ_{i+1} be a many-valued funhom of $q_{i+1}(s_{i+1})$ to $p_{i+1}(t_{i+1})$ which assigns to each object s_i of $q_{i+1}(s_{i+1})$ a subclass $\sigma_{i+1}(s_i)$ of $p_{i+1}(t_{i+1})$ and to each rule $q_i \in q_{i+1}$ a rule $\sigma_{i+1}(q_i)$ between subclasses of $p_{i+1}(t_{i+1})$. Certainly we can assume the rules $\sigma_{i+1}(q_i)$, $q_i \in q_{i+1}$ to be inclusions. We shall adopt here this stipulation. For the funhom σ_{i+1} we shall say to be relative to an object t_i of $p_{i+1}(t_{i+1})$ if the image of an object s_i of $q_{i+1}(s_{i+1})$ under it depends upon this object. The dependence on which we have decided is as follows. For every object t_i' of $\sigma_{i+1}(s_i)$, $s_i \in q_{i+1}(s_{i+1})$ there exists a p_{i+1}-rule $F_{i+1}(s_i) \wedge t_i' \to t_i$. We shall omit, otherwise, to emphasize the dependence of σ_{i+1} on the funhom F_{i+1} always when it is obvious from the context. A funhom σ_{i+1} relative to an object t_i of $p_{i+1}(t_{i+1})$ in the above sense we shall denote by $\sigma_{i+1}^{t_i}$. The class of relative many-valued funhoms $\sigma_{i+1}^{t_i}$ of $q_{i+1}(s_{i+1})$ to $p_{i+1}(t_{i+1})$ for all $t_i \in p_{i+1}(t_{i+1})$ we shall denote by $\mathfrak{S}_{i+1}$. This class is of the $(i+1)$th level if we assume that $p_{i+1}(t_{i+1})$ is not the $(i+1)$th universe.

Now we shall make this class to be capable of our purposes. Because of that we have to specify a class of rules in it. Since the images of objects of $q_{i+1}(s_{i+1})$ under elements of $\mathfrak{S}_{i+1}$ are subclasses of $p_{i+1}(t_{i+1})$ then we can involve inclusion rules into $\mathfrak{S}_{i+1}$. We specify the rules in the following manner. Let $\sigma_{i+1}^{t_i}$ and $\sigma_{i+1}^{t_i'}$ be two elements of $\mathfrak{S}_{i+1}$ i. e. two many-valued funhoms of $q_{i+1}(s_{i+1})$ to $p_{i+1}(t_{i+1})$ relative respectively to objects t_i and t_i' of $p_{i+1}(t_{i+1})$. If there exists a rule $p_i: t_i \to t_i'$ then we require that $\sigma_{i+1}^{t_i}(s_i) \subseteq \sigma_{i+1}^{t_i'}(s_i)$, $s_i \in q_{i+1}(s_{i+1})$, i. e. that $\sigma_{i+1}^{t_i}$ is a subfunhom of $\sigma_{i+1}^{t_i'}$. In what follows we shall always regard that the class $\mathfrak{S}_{i+1}$ is endowed with such a class of rules.

The class $\mathfrak{S}_{i+1}$ is fundamental for the definition of intuitionistic topology. To define this topology we need one more notion. It is the last selection funhom. For a funhom $\sigma_{i+1}^{t_i}$ we say to have a *sequent funhom* if there is a single-valued funhom $S_{i+1}^{t_i}$ and a natural rule $\eta_{i+1}^{t_i}: \sigma_{i+1}^{t_i} \to S_{i+1}^{t_i}$ such that $\mathfrak{F}_{i+1}^{t_i}(s_i) = (\sigma_{i+1}^{t_i}, \eta_{i+1}^{t_i}, S_{i+1}^{t_i})(s_i)$ is *fc* in $p_{i+1}(t_{i+1})$, for all $s_i \in q_{i+1}(s_{i+1})$. If, moreover, the funhom $S_{i+1}^{t_i}$ is such that $S_{i+1}^{t_i}(s_i) \in \sigma_{i+1}^{t_i}(s_i)$ then we shall call it the *last selection funhom*, and for $\sigma_{i+1}^{t_i}$ we shall say to have the last selection funhom. If the funhom $\sigma_{i+1}^{t_i}$ has such a funhom then the class $\sigma_{i+1}^{t_i}(s_i)$, $s_i \in q_{i+1}(s_{i+1})$ will have a last object. Such a class of $p_{i+1}(t_{i+1})$ we shall call a constructively closed class. Having the above notion in mind we define intuitionistic topology as follows.

Definition 1. Let $q_{i+1}(s_{i+1})$ be an arbitrary fundamental semigroupoid, $p_{i+1}(t_{i+1})$ a fundamental semigroupoid allowing presequent formations on all pairs of its objects without regard to classes of rules between them and F_{i+1} a single-valued funhom of $q_{i+1}(s_{i+1})$ to $p_{i+1}(t_{i+1})$. By an *intuitionistic topology* on the fundamental semigroupoid $p_{i+1}(t_{i+1})$ determined by the fundamental semigroupoid $q_{i+1}(s_{i+1})$ and the funhom F_{i+1} we mean the class $\mathfrak{S}_{i+1}$ of relative many-valued funhoms $\sigma_{i+1}^{t_i}$ of $q_{i+1}(s_{i+1})$ to $p_{i+1}(t_{i+1})$, $t_i \in p_{i+1}(t_{i+1})$, such that each funhom $\sigma_{i+1}^{t_i}$ has a last selection funhom. By an *intuitionistic topological space* we mean the pair $(p_{i+1}(t_{i+1}); \mathfrak{S}_{i+1})$.

From the above definition we have that an intuitionistic world $(p_{i+1}(t_{i+1}); \mathfrak{S}_{i+1})$ organized by means of the fundamental semigroupoid $q_{i+1}(s_{i+1})$ and the funhom F_{i+1} has constructively closed parts. However, we have not yet ensured it to have this property as a whole. To do this we have to impose certain conditions upon the fundamental semigroupoids $q_{i+1}(s_{i+1})$ and $p_{i+1}(t_{i+1})$ and the funhom F_{i+1}. We claim that $q_{i+1}(s_{i+1})$ and $p_{i+1}(t_{i+1})$ possess the strict first objects, denoted respectively by o_i' and o_i, and that F_{i+1} is such to preserve the object o_i', i. e. that $F_{i+1}(o_i') = o_i$. In that case we have that $\sigma_{i+1}^{o_i}(o_i)$ is all $p_{i+1}(t_{i+1})$ and $S_{i+1}^{o_i}(o_i)$ its last object. We denote it by 1_i. From this discussion we have

Proposition 2. *An intuitionistic topological space* $(p_{i+1}(t_{i+1}); \mathfrak{S}_{i+1})$ *formed by means of a fundamental semigroupoid* $q_{i+1}(s_{i+1})$ *and a funhom* F_{i+1} *will have a constructive end if* $q_{i+1}(s_{i+1})$ *and* $p_{i+1}(t_{i+1})$ *have strict first objects and if* F_{i+1} *preserves the first object of* $q_{i+1}(s_{i+1})$.∎

An intuitionistic topological space possessing the object 1_i we shall call a *constructively complete space*. In such a space $(p_{i+1}(t_{i+1}); \mathfrak{S}_{i+1})_{q_{i+1}(s_{i+1}), F_{i+1}}$, where $q_{i+1}(s_{i+1})$ and F_{i+1} denote respectively the fundamental semigroupoid and the funhom by means of which it is formed, we have that $S_{i+1}^{F_{i+1}(s_i)}(s_i)$ is equal to 1_i for all $s_i \in q_{i+1}(s_{i+1})$. Thus an intuitionistic topology on the semigroupoid $p_{i+1}(t_{i+1})$ enables it to be constructively complete and to possesses this property on certain chosen subclasses of itself.

In what follows we shall be concerned with properties of an intuitionistic space $(p_{i+1}(t_{i+1}); \mathfrak{S}_{i+1})_{q_{i+1}(s_{i+1}), F_{i+1}}$. First, if we regard a class of constant funhoms $c_{i+1}^{t_i}$ of $q_{i+1}(s_{i+1})$ to $p_{i+1}(t_{i+1})$, $t_i \in p_{i+1}(t_{i+1})$ and natural rules $c_{i+1}^{p_i}$, $p_i \in p_{i+1}(t_{i+1})$ such that $c_{i+1}^{t_i}(s_i) = t_i$ and $c_{i+1}^{p_i}(s_i) = p_i$, for every $s_i \in q_{i+1}(s_{i+1})$, then we can prove that a rule J_{i+1} of $p_{i+1}(t_{i+1})$ to $\mathfrak{S}_{i+1}$ defined by $t_i \mapsto c_{i+1}^{t_i}$ and $p_i \mapsto c_{i+1}^{p_i}$ is a choice embedding funhom. Namely, in that case the statements p_i, $t_i \in p_{i+1}(t_{i+1})$ and $c_{i+1}^{t_i}$ is a selection funhom of $\sigma_{i+1}^{t_i}$ and $c_{i+1}^{p_i}$ a natural rule between two such funhoms, are equivalent. Hence we have the following

Proposition 3. *Let* $(p_{i+1}(t_{i+1}); \mathfrak{S}_{i+1})$ *be an intuitionistic topological space. Then there is an embedding of the fundamental semigroupoid* $p_{i+1}(t_{i+1})$ *into the topology* $\mathfrak{S}_{i+1}$ *as a choice subsemigroupoid.*∎

Thus $p_{i+1}(t_{i+1})$ can be regarded as a choice subsemigroupoid of $\mathfrak{S}_{i+1}$. The next such subsemigroupoid is the semigroupoid $\mathbf{S}_{i+1}$ consisting of the funhoms $S_{i+1}^{t_i}$ and the natural rules $S_{i+1}^{p_i} : S_{i+1}^{t_i} \rightarrow S_{i+1}^{t_i'}$ for $p_i : t_i \rightarrow t_i'$ of $p_{i+1}(t_{i+1})$. The pair $(p_{i+1}(t_{i+1}); \mathbf{S}_{i+1})$ we shall call a choice subspace of the space $(p_{i+1}(t_{i+1}); \mathfrak{S}_{i+1})$ or a space with intuitionistic choice topology.

Let us regard now the space $(p_{i+1}(t_{i+1}); \mathbf{S}_{i+1})_{q_{i+1}(s_{i+1}), F_{i+1}}$ and the object $S_{i+1}^{o_i} \in \mathbf{S}_{i+1}$. This funhom has the following property. For every $s_i \in q_{i+1}(s_{i+1})$, $S_{i+1}^{o_i}(s_i)$ is a unique $p_{i+1}(t_{i+1})$-object such that $F_{i+1}(s_i) \wedge S_{i+1}^{o_i}(s_i) = o_i$. Thus $S_{i+1}^{o_i}(s_i)$

is a pseudocomplement of the object $F_{i+1}(s_i)$. If $q_{i+1}(s_{i+1}) = p_{i+1}(t_{i+1})$, then composition $S^{oi}_{i+1} \circ S^{oi}_{i+1}$ is a closure operator on $p_{i+1}(t_{i+1})$. We could deal now with properties of these operators. However, we shall not do this because we are much more interested in relationships of intuitionistic and other introduced topologies. We shall show that a topological organization is an example of intuitionistic organization. We shall prove the following

Proposition 4. *A topology τ_{i+1} on a space $(s_{i+1}, \tau_{i+1}, p_{i+1}(t_{i+1}))$ is an intuitionistic topology.*

Proof. We regard an equivalent definition of topology given in [1]. Thus a topology on this space is a subclass $p_{i+1}(t_{i+1})^{c\beta}$ of $p_{i+1}(t_{i+1})$ consisting of all objects and rules of the filters $\tau_{i+1}(s_i)$, $s_i \in s_{i+1}$ and satisfying the conditions of the Proposition 1 of [1]. Let t_i and t_i' be two arbitrary objects of $p_{i+1}(t_{i+1})$ and $r_{i+1}(a_{i+1})$ a subclass of $p_{i+1}(t_{i+1})^{c\beta}$ the class of objects of which consists of all those objects a_i of $p_{i+1}(t_{i+1})^{c\beta}$ such that $t_i \wedge a_i \rightarrow t_i'$. Assume $t_i \wedge a_i = o_i^s$ for all $a_i \in r_{i+1}(a_{i+1})$ Denote the common sequent of the class $r_{i+1}(a_{i+1})$ and the object t_i' by c_i. It obviously exists in $p_{i+1}(t_{i+1})$ and is equanl to $t_i' \vee \mathscr{C}_{i+1}(t_i)$, where $\mathscr{C}_{i+1}(t_i)$ is the last object of $r_{i+1}(a_{i+1})$. If we take its interior then there is a unique *lcc* $\mathbf{O}_{i+1}(c_i) \rightarrow c_i$. Hence we have further the existence of unique rules $a_i \rightarrow \mathbf{O}_{i+1}(c_i)$ for all $a_i \in r_{i+1}(a_{i+1})$ and also a unique rule $t_i' \rightarrow \mathbf{O}_{i+1}(c_i)$. If t_i, $t_i' \in p_{i+1}(t_{i+1})^{c\beta}$ then we have got a subclass of $p_{i+1}(t_{i+1})^{c\beta}$ of all those objects a_i satisfying the condition $t_i \wedge a_i \rightarrow t_i'$ and which has the object $\mathbf{O}_{i+1}(c_i)$ as its last object. Thus we can define a class $\mathfrak{S}_{i+1}$ of relative many-valued funhoms $\sigma^{t_i}_{i+1}$ from $p_{i+1}(t_{i+1})^{c\beta}$ to itself such that each its object has a last selection funhom. Moreover we have that the existence of a rule $p_i: t_i \rightarrow t_i'$ of $p_{i+1}(t_{i+1})^{c\beta}$ implies that $\sigma^{t_i}_{i+1}$ is a subfunhom of $\sigma^{t'_i}_{i+1}$.∎

Hence we have that a topological organization is a particular case – a stronger form of an intuitionistic organization. A pretopological organization is also such an organization. Thus in a world formed by means of a topology or a pretopology we can find constructively complete wholes. However, all these wholes are only by the way steps up to the ultimate construction — the whole space. Hence we have that the object 1_i is, in fact, the proper constructive end in a topological world, for the difference from an intuitionistic world in general for which this is not the case.

We could mention some examples of involving intuitionistic topologies and accordingly intuitionistic spaces. For instance, let F_{i+1} be a funhom of $q_{i+1}(s_{i+1})$ to $p_{i+1}(t_{i+1})$ and $Preseq_{i+1}$ a funhom of $p_{i+1}(t_{i+1})^2$ to $p_{i+1}(t_{i+1})$ which assigns to each $F_{i+1}(s_i)$ and t_i of $p_{i+1}(t_{i+1})$ their presequent. Then the class of funhoms $S^{t_i}_{i+1}$ of $q_{i+1}(s_{i+1})$ to $p_{i+1}(t_{i+1})$ which assigns to each object $s_i \in q_{i+1}(s_{i+1})$ an object $S^{t_i}_{i+1}(s_i) \in p_{i+1}(t_{i+1})$, $t_i \in p_{i+1}(t_{i+1})$, in such a way that there is an isomorphism

$$[t_i', S^{t_i}_{i+1}(s_i)] \cong [Preseq_{i+1}(t_i', F_{i+1}(s_i)), t_i]$$

defines a choice intuitionistic topology on $p_{i+1}(t_{i+1})$. In that case, $S^{t_i}_{i+1}(s_i)$ is the last object of the class of all t_i' of $p_{i+1}(t_{i+1})$ for which there exist rules $Preseq_{i+1}(t_i', F_{i+1}(s_i)) \rightarrow t_i$. At this the objects s_i and t_i are regarded to be fixed.

Furthermore, we could also say that Lawvere's toposes are a special case of an intuitionistic organization. To show this it is enough to make specifications of choices and rules in such a sense.

At the end of this paper we mention that we could make some generalizations of intuitonistic topology presented here. For instance we could regard three fundamental semigroupoids $q_{i+1}(s_{i+1})$, $p_{i+1}(t_{i+1})$ and $r_{i+1}(a_{i+1})$ of a class of the $(i+2)$th level and a funhom $F_{i+1}: q_{i+1}(s_{i+1}) \rightarrow p_{i+1}(t_{i+1})$. A generalization would be then in the sense of formations of intuitionistic spaces by the funhom F_{i+1} and the fundamental semigroupoid $q_{i+1}(s_{i+1})$ and with respect to the semigroupoid $r_{i+1}(a_{i+1})$. We shall not deal here with this as well as some other possible generalisations of the topology. However we shall take them into account in the context of general considerations of the problem of formations of spatial wholes in $\mathcal{U}$.

4. Conclusion.

In this paper we have dealt with some organizations of spatial wholes in $\mathcal{U}$. First we regarded some weaker forms of topological organisations and then a more general form that we called intuitionistic. The basic features of this organization, as we saw, are choices of subclasses of a fundamental semigroupoid which is to be organized and certain creations on them. We shall regard now these features in several details. First there arises the following question. What are criterions for choices? To answer this question we have firstly to be clear what we intend to form from these subclasses and also which means we have at our disposal to do it. If we specify the first then we determine our creative goal on the regarded fundamental semigroupoid. Specification of the second determines creations needed for reaching this goal. If we are acquainted with these two things then the next we have to do is to choose subclasses of the fundamental semigroupoid on which we shall make adopted creations. All creations we are interested in have a constructive character. Hence there are certain operations which we want to make on chosen subclasses of the regarded fundamental semigroupoid. All operations that we know we can devide into two types. First we have operations of inductive type. They are operations by means of which we obtain from a class its sequent. The second are coinductive operations, i. e. operations by means of which we obtain from a class its presequent. We proceed further as follows. We utilize one type of operations for purposes of choices and then claim that chosen subclasses allow the other type of operations. We could adopt this as a general criterion for choices of subclasses. Further we have that a choice can be either without or with respect to objects of a fundamental semigroupoid. Finally we have that it can depend on certain funhoms from a fundamental semigroupoid to the fundamental semigroupoid which is to be organized. All these questions we shall regard once more and in more details at a general considerations of organizational problems of $\mathcal{U}$.

REFERENCES

[1] Đurić, M., *Topological structures on classes I*, Publ. Inst. Math., 15 (29), 55—73.

[2] Đurić, M., *Topological structures on classes II*, Publ. Inst. Math., 15 (29), 75—83.

[3] Đurić, M., *Some fundamental structures on classes*, Publ. Inst. Math., 14 (28), 49—66.

[4] Heyting, A., *Intuitionism*, North Holland Publ. Company, Amsterdam, 1956.

[5] Lawvere, W., *Toposes, Algebraic Geometry and Logic*, Springer-Verlag, Berlin-Heidelberg-New York, 1972.