PUBLICATIONS DE L'INSTITUT MATHÉMATIQUE
Nouvelle série, tome 16 (30), 1973 pp. 147—157

LAMINAR SLIP-FLOW THROUGH A UNIFORM CIRCULAR PIPE WITH SMALL SUCTION

V. M. Soundalgekar and
V. G. Divekar

(Received June 28, 1972)

SUMMARY

A steady two-dimensional flow of an incompressible viscous fluid in a circular porous pipe is investigated by the method of small perturbation theory. The Navier-Stokes equations are reduced to the third and second order non-linear differential equations and are solved by perturbation method under the first order slip flow boundary conditions. Solutions to the axial and radial velocity profiles, the axial pressure drop, skin-friction and the mass flow are derived to $0(\lambda)^2$, where λ is the suction Reynolds number assumed very small. The results are shown on graphs. It is observed that the separation is delayed due to the rarefaction of the gas. The maximum of the radial velocity approaches to the wall in the slip flow regime. The pressure gradient increases as h_1 (the rarefaction parameter) increases, and hence the back-flow is delayed. The skin-friction decreases with increasing h_1 The mass flow coefficient also increases with h_1.

1. Introduction.

Choudhary and Sinha (1964) presented an analysis of the flow of a viscous, incompressible fluid through a circular pipe of uniform cross-secton with small uniform suction under the assumption that the pressure is uniform over a cross-section and that the axial pressure gradient is also uniform throughout the channel. This assumption is not correct as due to uniform small suction, there must occur a change in the pressure gradient along the axis of the pipe. This problem was again attempted by Bansal (1966) in his recent paper and he modified this assumption on the pressure gradient. The laminar flow in a porous pipe was also attempted by Yuan and Finkelstein (1956) under the assumption that the maximum velocity of the Hagen-Poiseuille flow occurs at the centre of the mouth of the pipe. Their treatment of the problem was in terms of a stream function. By following a method of perturbation, Bansal (1966) showed that at the entrance of the pipe, the pressure gradient along the axis is different from the one in the Poisseulle flow.

A similar problem was also attempted by Varma and Bansal (1966) in case of flow between the porous parallel plates in relative motion with suction at stationary plate wherein they again modified the problem of Sinha and Choudhary (1965). In this paper, a continuum flow under no-slip boundary conditions was assumed. However, where the fluid is flowing at high temperature or low pressure, there is slight reduction in the density of the fluid. Under these conditions, though the fluid medium is in continuum state, it has been observed that no-slip boundary conditions do not help us to describe the motion. Hence the first order velocity slip boundary conditions along with the momentum equations of continuous gas dynamics are suggested, the details about which are described by Schaaf and Chambre' (1958).

The object of the present paper is to study the modifications due to introduction of first order velocity slip boundary conditions. Such an attempt was made earlier by Soundalgekar and Divekar (1968) to study the Couette flow with suction at the stationary plate. It was observed that due to rarefaction there is a decrease in the axial pressure and the shearing stress at the stationary plate. Also, initially there is an increase in the flow rate, but far downstream, it decreases due to rarefaction.

In section 2, the two-dimensional flow is described by taking Navier-Stokes equations in polar co-ordinates. They are non-dimensionalised and solved by perturbation method. The solutions are presented for the axial and radial velocities to $0(\lambda^2)$, where λ is the suction Reynolds number assumed $\ll$ unity. Also expressions for the axial pressure drop, skin-friction and mass flow coefficient are derived to $0(\lambda^2)$. They are shown graphically for different values of λ, h_1, the rarefaction parameter, R_e, the Reynolds number.

In section 3, the effects of rarefaction on the flow-field are discussed in detail.

2. Mathematical Analysis:

The two dimensional flow is assumed to be governed by the continuum form of the momentum equations which in cylindrical polar co-ordinates are

(1)
$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial r}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}+\frac{\partial^2 u}{\partial x^2}\right)$$

(2)
$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial r}=-\frac{1}{\rho}\frac{\partial p}{\partial r}+\nu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial r^2}+\frac{1}{r}\cdot\frac{\partial v}{\partial r}-\frac{v}{r^2}\right)$$

the equation of continuity is

(3)
$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial r}+\frac{v}{r}=0.$$

Here the x — axis is chosen along the axis of the pipe and r is measured at right angles to it.

The first-order slip-flow boundary condition which permits a slip velocity u_s at the wall $(r=R)$ is

(4)
$$u=-\xi_u\frac{\partial u}{\partial r},\quad v=v_0 \text{ at } r=R$$
$$\frac{\partial u}{\partial r}=0,\qquad v=0 \text{ at } r=0.$$

According to Maxwell, the slip coefficient ξ_u is given by

$$\xi_u = \frac{2-f^*}{f^*} \lambda^*$$

where λ^* is the mean free path and f^* is termed Maxwell's reflection coefficient. Thus ξ_u depends upon f^* and λ^*. According to table (10.1). of Eckert and Drake (1959), f^* values for a wide variety of gas surface combinations fall between 0.9 and 1.0 with perhaps more lying closer to 1. For $f^* \simeq 1$, it follows that $\xi_u \simeq \lambda^*$. The mean free path λ^* can be expressed in terms of thermodynamic variables by utilising equation (10.23) of Eckert and Drake (1959). Thus

$$\lambda^* = \frac{\sqrt{\pi/8}}{0 \cdot 499} \frac{\mu}{P} \sqrt{RT}. \tag{5}$$

If uniform suction is assumed at the boundary, then we have

$$\frac{\partial v}{\partial x} = 0 \tag{6}$$

which leads to v a function of r only. Hence from (3) and (6) equations (1) and (2) reduce to the following.

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial r} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + v\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r}\right) \tag{7}$$

$$v \frac{\partial v}{\partial r} = -\frac{1}{\rho} \frac{\partial p}{\partial r} + v\left(\frac{\partial^2 v}{\partial r^2} + \frac{1}{r} \frac{\partial v}{\partial r} - \frac{v}{r^2}\right). \tag{8}$$

We now introduce the following non-dimensional quantities:

$$\left.\begin{array}{ll} \bar{u} = u/u_m, & \bar{v} = v/v_0 \\ \bar{x} = x/R, & \eta = r/R \quad \bar{p} = p/\rho U_m^2 \\ \lambda = v_0 R/\nu & \text{Suction Reynolds number} \\ R_e = \dfrac{U_m R}{v}, & \text{Reynolds number} \\ h_1 = \dfrac{\xi_u}{R}, & \end{array}\right\} \tag{9}$$

Here v_0 is the uniform suction velocity and U_m is the maximum velocity of the Poiseuille slip-flow in the absence of suction.

In view of (9), equations (7), (8) and (3) reduce to the following non-dimensional form.

$$\bar{u} \frac{\partial \bar{u}}{\partial x} + \frac{\lambda}{R_e} \bar{v} \frac{\partial \bar{u}}{\partial \eta} = -\frac{\partial \bar{p}}{\partial \bar{x}} + \frac{1}{R_e} \left(\frac{\partial^2 \bar{u}}{\partial \eta^2} + \frac{1}{\eta} \frac{\partial \bar{u}}{\partial \eta}\right) \tag{10}$$

(11) $$\bar{v}\frac{\partial \bar{v}}{\partial \eta}=-\frac{R_e^2}{\lambda^2}\frac{\partial \bar{p}}{\partial \eta}+\frac{1}{\lambda}\left(\frac{\partial^2 \bar{v}}{\partial \eta^2}+\frac{1}{\eta}\frac{\partial \bar{v}}{\partial \eta}-\frac{\bar{v}}{\eta^2}\right)$$

(12) $$\frac{\partial \bar{u}}{\partial \bar{x}}+\frac{\lambda}{R_e}\left(\frac{\partial \bar{v}}{\partial \eta}+\frac{\bar{v}}{\eta}\right)=0$$

and the boundary conditions (4) reduce to

(13) $$\left.\begin{aligned}&\bar{u}=-h_1\frac{\partial \bar{u}}{\partial \eta},\ \bar{v}=1 \text{ at } \eta=1;\\ &\frac{\partial \bar{u}}{\partial \eta}=0,\ \bar{v}=0 \text{ at } \eta=0.\end{aligned}\right\}$$

We represent the perturbations caused by the suction in the following way:

(14) $$\bar{p}\ (\bar{x},\ \eta)=P_0+p'\ (\bar{x},\eta);\ \bar{u}\ (\bar{x},\eta)=u_0+u'\ (\bar{x},\eta);\ \bar{v}=v'\ (\eta)$$

Here P_0 and u_0 are the known quantities for the flow without suction which is described in Sparrow and Lin (1962). They satisfy the following relations.

(15) $$\frac{\partial p_0}{\partial \eta}=0,\ \frac{\partial u_0}{\partial \bar{x}}=0 \text{ and } \frac{\partial p_0}{\partial \bar{x}}=\frac{1}{R_e}\left(\frac{\partial^2 u_0}{\partial \eta^2}+\frac{1}{\eta}\frac{\partial u_0}{\partial \eta}\right)$$

Here P_0 is independent of η and u_0 is given by

(16) $$u_0=1-\eta^2/(1+2h_1).$$

We now substitute (14) in (10), (11) and (12) and we obtain in view of (15),

(17) $$u_0\frac{\partial u'}{\partial \bar{x}}+\frac{\lambda}{R_e}v'\frac{\partial u_0}{\partial \eta}+u'\frac{\partial u'}{\partial \bar{x}}+\frac{\lambda}{R_e}v'\frac{\partial u'}{\partial \eta}=-\frac{\partial p'}{\partial \bar{x}}+\frac{1}{R_e}\left(\frac{\partial^2 u'}{\partial \eta^2}+\frac{1}{\eta}\frac{\partial u'}{\partial \eta}\right)$$

(18) $$v'\frac{\partial v'}{\partial \eta}=-\frac{R_e^2}{\lambda^2}\frac{\partial p'}{\partial \eta}+\frac{1}{\lambda}\left(\frac{\partial^2 v'}{\partial \eta^2}+\frac{1}{\eta}\frac{\partial v'}{\partial \eta}-\frac{v'}{\eta^2}\right)$$

and

(19) $$\frac{\partial u'}{\partial \bar{x}}+\frac{\lambda}{R_e}\left(\frac{\partial v'}{\partial \eta}+\frac{v'}{\eta}\right)=0$$

and the boundary conditions on the perturbed quantities are

(20) $$u'\ (1)=-h_1\left.\frac{\partial u'}{\partial \eta}\right|_{\eta=1}\ v'\ (1)=1;\ \left.\frac{\partial u'}{\partial \eta}\right|_{\eta=0}=0,\ v'\ (0)=0$$

To solve the non-linear equations (17) and (18), we follow the perturbation method.

(21) $$\text{Assume } v'=\frac{1}{\eta}f(\eta)$$

the function $f(\eta)$ is to be chosen in such a way that Lt $(f(\eta)/\eta) \to 0$ as $\eta \to 0$. Then from (19) and (21), we get

$$u' = -\frac{\lambda}{R_e}\,\bar{x}\,\frac{1}{\eta} f'(\eta) + F(\eta) \tag{22}$$

where $f(\eta)$ and $F(\eta)$ are the unknown functions to be determined.

Substituting (21) and (22) in (17) and (18) there follows,

$$\frac{\partial p'}{\partial \bar{x}} = -\frac{1}{R_e}\left[F'' + \frac{1}{\eta}F' + \frac{\lambda}{\eta}\left\{\left(1 - \frac{\eta^2}{1+2h_1}\right)f' + \frac{2}{1+2h_1}\eta f + Ff' - fF'\right\}\right] - \frac{\lambda}{R_e^2}\,\bar{x}\,\frac{1}{\eta^3}\left[\eta^2 f''' - \eta f'' + f' + \lambda\left(\eta f'^2 \eta ff'' + ff'\right)\right] \tag{23}$$

and

$$\frac{\partial p'}{\partial \eta} = \frac{\lambda}{R_e^2}\,\frac{1}{\eta^3}\left[\eta^{2''} - \eta f' - \lambda(\eta ff' - f^2)\right]. \tag{24}$$

Following Bansal (1966), we assume that at

$$\bar{x} = 0,\ \frac{\partial p'}{\partial \bar{x}} = 0 \tag{25}$$

which means that the perturbed pressure gradient is zero at the mouth of the channel. Hence from (25) and (23), we obtain.

$$F'' + \frac{1}{\eta}F' + \frac{\lambda}{\eta}\left\{\left(1 - \frac{\eta^2}{1+2h_1}\right)f' + \frac{2}{1+2h_1}\eta f + Ff' - fF'\right\} = 0. \tag{26}$$

From (24), we get after differentiation with respect to $\bar{x}$

$$\frac{\partial^2 p'}{\partial \bar{x}\,\partial \eta} = 0. \tag{27}$$

We differentiate (23) with respect to η and we obtain in view of (26) and (27).

$$\frac{d}{d\eta}\left\{\frac{1}{\eta^3}\left[\eta^2 f''' - \eta f'' + f' + \lambda\left(\eta f'^2 - \eta ff'' + ff'\right)\right]\right\} = 0 \tag{28}$$

which is true for all $\bar{x}$.

On integrating (28), we get

$$\eta^3 f''' - \eta^2 f'' \psi\, \eta f' + \lambda\left(\eta^2 f'^2 - \eta^2 ff'' + \eta ff'\right) = C\eta^4 \tag{29}$$

where C is the constant of integration to be determined.

We now solve equations (26) and (29) under the following boundary conditions:

$$f(1)=1,\ f'(1)=\frac{h_1}{h_1-1}f''(1),\ F=\frac{h_1}{h_1-1}F'(1)$$

(30)

$$f(0)=0,\ \underset{\eta\to 0}{\mathrm{Lt}}\frac{d}{d\eta}\left(\frac{f'(\eta)}{\eta}\right)=0,\ F'(0)=0\cdot$$

To solve the two coupled equations (26) and (29), we follow the method of perturbation. We expand f and C in powers of λ about $\lambda=0$ where λ is the suction Reynolds number assumed less than unity. Thus

(31) $$f=f_0+\lambda f_1+\lambda^2 f_2+\ \ldots\ldots\ldots+\lambda^n f_n$$

(32) $$C=C_0+\lambda C_1+\lambda^2 C_2+\ \ldots\ldots+\lambda^n C_n$$

where f_n 's and C_n 's are assumed to be independent of λ.

We now substitute equations (31) and (32) in (29) and equate the coefficients of like powers of λ. This leads to the following system of equations:

(33) $$\eta^3 f_0''' -\eta^2 f_0''+\eta f_0'=C_0\eta^4$$

(34) $$\eta^3 f_1''' -\eta^2 f_1''+\eta f_1'+\eta^2 f_0'^2-\eta^2 f_0 f_0''+\eta f_0 f_0'=C_1\eta^4$$

(35) $$\eta^3 f_2''' -\eta^2 f_2''+\eta f_2'+2\eta^2 f_0' f_1'-\eta^2 f_1 f_0''-\eta^2 f_0 f_1''+$$
$$+\eta f_1 f_0'+\eta f_0 f_1'=C_2\eta^4.$$

From (30), (31) the boundary conditions on f_n 's are now given as

(36) $$f_n(0)=0,\ f_n'(1)=\frac{h_1}{h_1-1}f_n''(1),\ \underset{\eta\to 0}{\mathrm{Lt}}\frac{d}{d\eta}\left(\frac{f_n'}{\eta}\right)=0 \text{ for } n\geqslant 0$$

and

$$f_0(1)=1,\ f_n(1)=0 \text{ for } n\geqslant 1$$

Solving equations (33), (34) and (35) under the boundary conditions (36), substituting in (31) and (32) we get the solution for f to $0(\lambda^2)$ as

$$f(\eta)=\frac{2(1+2h_1)}{1+4h_1}\eta^2-\frac{1}{1+4h_1}\eta^4+\lambda\left\{\frac{1+6h_1}{9(1+4h_1)^2}\eta^2-\right.$$
$$\left.-\frac{1}{4(1+4h_1)}\eta^4+\frac{1+2h_1}{6(1+4h_1)^2}\eta^6-\frac{1}{36(1+4h^1)^2}\eta^8\right\}+$$
$$+\lambda^2\left\{\frac{166+2324h_1{}^2+10080h_1{}^2+14400h_1{}^3}{5400(1+4h_1)^4}\eta^2-\right.$$
$$-\frac{19+288h_1+870h_1{}^2+1080h_1{}^3}{270(1+4h_1)^4}\eta^4+\frac{11+66h_1+72h_1{}^2}{216(1+4h_1)^3}\eta^6-$$

(37) $$\left.-\frac{1}{72(1+4h_1)^2}\eta^8+\frac{1+2h_1}{360(1+4h_1)^3}\eta^{10}-\frac{1}{5400(1+4h_1)}\eta^{12}\right\}$$

and

(38) $$C = -\frac{16}{1+4h_1} + \frac{4(3+12h_1+16h_1^2)}{(1+4h_1)^2}\lambda + \frac{8(11+132h_1+450h_1^2+360h_1^3)}{135(1+4h_1)^4}\lambda^2$$

Following Bansal (1966), we now assume the solution for F in the form.

(39) $$F(\eta) = -\left(1 - \frac{\eta}{1+2h_1}\right) + \frac{\Phi(\eta)}{\eta}$$

F where the boundary conditions on Φ are

(40) $$\Phi(1) = \frac{h_1}{h_1-1}\Phi'(1), \ \underset{\eta\to 0}{\mathrm{Lt}}\ \frac{d}{d\eta}\left(\frac{\Phi(\eta)}{\eta}\right) = 0.$$

From (39) and (26), we get

(41) $$\frac{1}{1+2h_1}\eta^3 + \eta^2\Phi'' - \eta\Phi' + \Phi + \lambda\ [\eta\Phi f' - \eta\Phi' f + \Phi f] = 0.$$

From (29) and (41), we can show that

(42) $$\Phi(\eta) = -\frac{4}{(1+2h_1)C} f'(\eta).$$

Thus (39) and (42) now lead to

(43) $$F(\eta) = -\left(1 - \frac{\eta^2}{1+2h_1}\right) - \frac{4}{(1+2h_1)C}\frac{f'(\eta)}{\eta}$$

Knowing $f(\eta)$ and $F(\eta)$, we now obtain the expression for axial pressure gradient from (14), (15), (23) (26) and (29) as

(44) $$-\frac{\partial\bar{p}}{\partial\bar{x}} = \frac{4}{(1+2h_1)R_e} + \frac{\lambda\bar{x}}{R_e^2}C$$

where C is given by (38)

We can now write from the equations (14), (16), (21), (22), (37), (39) and (44) the axial and radial components of the velocity in the form

(45), (46) $$\bar{u} = \frac{R_e}{C}\frac{\partial\bar{p}}{\partial\bar{x}}\frac{f'_{(n)}}{\eta};\ \bar{v} = \frac{f(n)}{\eta}$$

where $f'(\eta)$ and $\frac{\partial\bar{p}}{\partial\bar{x}}$ can be obtained from (37) and (44) respectively.

We now derive the pressure distribution in the axial and the radial directions by substituting (26) and (29) in (23) and (24).

Thus an integration leads to

(47) $$\bar{p}(0,0) - \bar{p}(\bar{x},\eta) = \frac{4\bar{x}}{(1+2h_1)R_e} + \frac{\lambda C\bar{x}^2}{2R_e^2} - \frac{\lambda}{R_e^2}\frac{f'}{\eta} + \frac{\lambda^2}{2R_e^2}\frac{f^2}{\eta^2} + \frac{\lambda}{R_e^2}\left[\frac{4(1+2h_1)}{1+4h_1} + \frac{2(1+6h_1)}{9(1+4h_1)^2}\lambda + \frac{166+2324h_1+10080h_1^2+14400h_1^3}{2700(1+2h_1)^4}\lambda.\right]$$

which reduces to equation (3—17) of Bansal (1966) when $h_1 = 0$ From (47), the axial pressure drop is now derived as

$$\bar{p}(0, \eta) - \bar{p}(\bar{x}, \eta) = \frac{4\bar{x}}{(1+2h_1)R_e} + \frac{\lambda C \bar{x}}{2R_e^2}. \tag{48}$$

The skin-friction at the wall is given by

$$\tau_0 = \frac{\mu U_m}{R}\left(\frac{\partial \bar{u}}{\partial \eta}\right)_{\eta=1} \tag{49}$$

Hence in non-dimensional form, we get from (49), (14), (16) and (22), the skin-friction as

$$C_f = \frac{\tau_0}{\frac{1}{2}\rho U_m^2} = \frac{2}{C}\frac{\partial \bar{p}}{\partial \bar{x}}\left[\frac{8}{1+4h_1} - \frac{2}{3(1+4h_1)^2}\lambda - \frac{26+186h_1+360h_1^2}{135(1+4h_1)^4}\lambda^2\right] \tag{50}$$

The mass flow across a section at any $\bar{x}$ is given by

$$Q = 2\pi R^2 U_m \int_0^R \bar{u}\,\eta\, d\eta = 2\pi R^2 U_m \frac{R_e}{C}\frac{\partial \bar{p}}{\partial \bar{x}} \tag{51}$$

Also, in case of impermeable wall ($\lambda = 0$), we have the mass flow as

$$Q_0 = \pi R^2 U_m \frac{1+4h_1}{2(1+2h_1)} \tag{52}$$

From (51) and (52), we can write down the discharge coefficient as

$$C_Q = \frac{Q}{Q_0} = \frac{4(1+2h_1)}{1+4h_1}\frac{R_e}{C}\frac{\partial \bar{p}}{\partial \bar{x}} \tag{53}$$

Numerical values of $\bar{u}$, $\bar{v}$, pp $(= \bar{p}(0,0) - \bar{p}(\bar{x}, \eta))$, C_f and C_Q are calculated for $\lambda = 0.1$, 0.2; $h_1 = 0$, 0.1, 0.2, $R_e = 1000$ and are shown on graphs in figures 1 — 5.

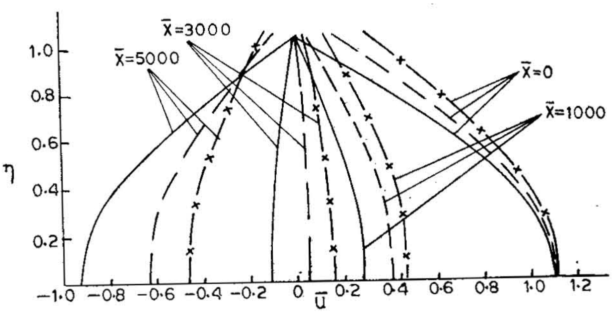

FIG. 1. AXIAL VELOCITY PROFILES $\lambda = 0.1$
$h_1 = 0$ ———
$= 0.1$ — — —
$= 0.2$ —x—x—

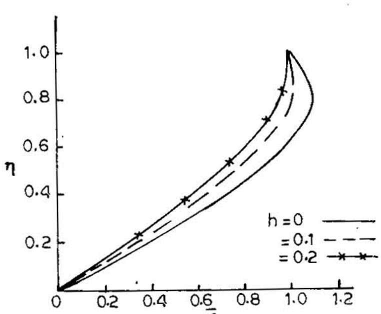

FIG. 2. RADIAL VELOCITY PROFILE
$\lambda = 0.1$

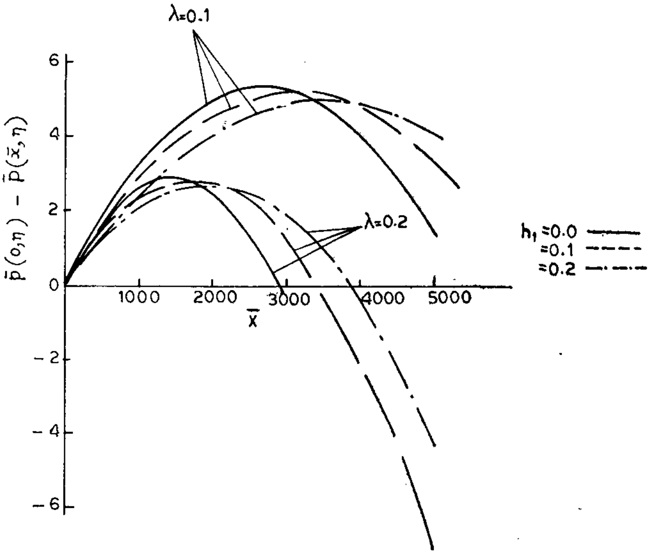

FIG. 3. AXIAL PRESSURE DROP VS $\bar{x}$. λ=0.1, 0.2

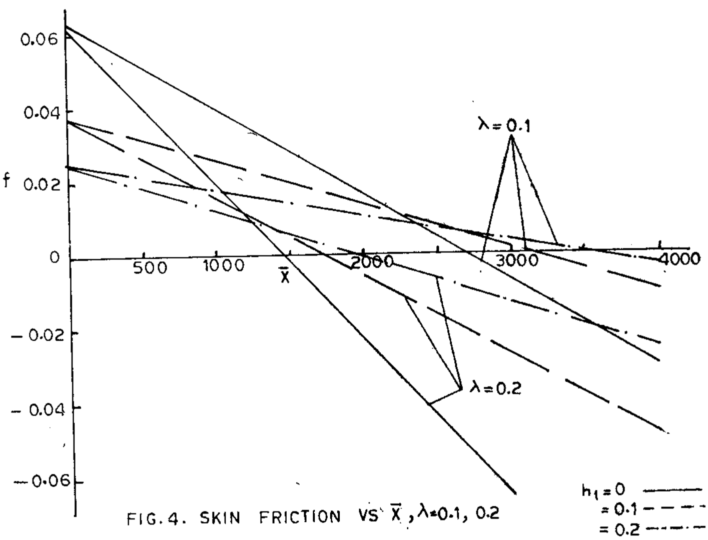

FIG. 4. SKIN FRICTION VS $\bar{x}$, λ=0.1, 0.2

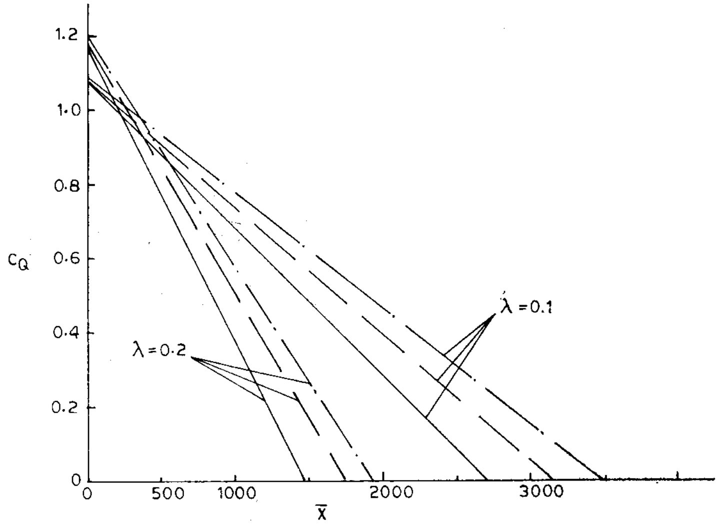

FIG. 5. MASS COEFFICIENT VS $\bar{X}$, $\lambda = 0.1, 0.2$.
$h_1 = 0$ ———
$= 0.1$ - - - -
$= 0.2$ —·—·—

3. Conclusions:

1. Velocity profiles:

The nature of the velocity profiles in the slip-flow regime is the same as that in no-slip case. This has been described by Bansal (1966). However the magnitude of the velocity changes due to the gas-rarefaction. It can be observed from the axial velocity profiles shown in fig. 1 that an increase in h_1 leads to an increase in the velocity of the gas. The velocity, however, still decreases as the axial distance increases. Bansal (1966) has observed that for $\lambda = 0.1$, the axial velocity vanishes at $\bar{x} = 2702.7$ in the no-slip case. From fig. 1, we can observe that the velocity is positive for $\bar{x} = 3000$ when $h_1 = 0.1$, 0.2 whereas it is negative for $h_1 = 0$. Hence one can conclude that the point of separation is delayed as the rarefaction parameter h_1 increases. Also this leads to the conclusion that the adverse pressure gradient which causes the back flow, is developed far away in case of slip-flow regime than in case of no-slip regime.

In fig. 2, radial velocity profiles are shown. As observed by Bansal (1966), in no-slip case, the maximum radial velocity occurs at $\eta = 0.8$ which can be seen from fig. 2. for $h_1 = 0$. But from the other two radial velocity profiles for $h_1 = 0.1$, 0.2, we can conclude that the maximum radial velocity approaches to $\eta = 1$. Hence in slip-flow regime, as h_1 increases, the maximum radial velocity is always at the wall.

In fig. 3, the axial pressure drop is plotted for $\lambda = 0.1$ and 0.2. On comparing the two sets of curves for $\lambda = 0.1$ and $\lambda = 0.2$, we observe that an increase in λ leads to a decrease in the pressure drop in both no-slip and slip-flow. However, for the same value of λ, as h_1 increases, the pressure drop decreases in the initial stages, but far away from the mouth, an increase in h_1 leads to an increase in the pressure drop which causes the back-flow to be delayed. As the suction velocity increases, the pressure gradient also decreases early and hence there is back-flow.

From fig. 4, one can observe that in general the skin friction decreases as the suction parameter λ increases. In the initial stages, an increase in h_1 leads to a decrease in C_f, but the rate of decrease far away is less as h_1 increases.

In fig. 5, the mass coefficient is plotted. For $\bar{x} < 300$, C_Q increases with increasing λ, but it falls rapidly with increasing λ for $\bar{x} > 300$. An increase in h_1 leads to an increase in C_Q which is due to an increase in the magnitude of the velocity.

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