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SOLUTION OF THE PARABOLIC PARTIAL DIFFERENTIAL EQUATION*)

$$\lambda^2 \frac{\partial^2 u}{\partial x^2} - \mu^2 \frac{\partial u}{\partial y} = f(x, y)$$

BY MEANS OF ALGEBRAIC OPERATIONAL CALCULUS OF DISTRIBUTIONS WITH SUPPORT IN R_+^2

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Summary

In the present paper we give a solution to the parabolic partial differential equation

(1) $$\lambda^2 \frac{\partial^2 u}{\partial x^2} - \mu^2 \frac{\partial u}{\partial y} = f(x, y)$$

by using algebraic operational calculus of distributions as described in [1], [2], [3], [4], [5], [6].

1. *Fundamental solution of the parabolic differential equation* (1): Let $(\mathcal{D}'_{\boldsymbol{R}_+^2})$ be the space of distributions of support in $\boldsymbol{R}_+^2 = [0, \infty[^2$. (cf. [2]). Let us consider in $(\mathcal{D}'_{\boldsymbol{R}_+^2})$ the convolution equation

(2) $$\{\lambda\delta'(x)\otimes\delta(y) + \mu\delta(x)\}\otimes\frac{\delta}{\delta y}\left\{\frac{1}{\sqrt{\pi y}}\right\}\overset{xy}{*}F^{(\lambda,\mu)}(x, y)\} = \delta(x)\otimes\delta(y)$$

where λ, μ are real or complex parameters.

In [3], formulas (45) and (48), we have given fundamental solution of (2) as follows:

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$$
(3)\qquad \{F^{(\lambda,\mu)}(x,y)=\begin{cases}
\dfrac{1}{\lambda p}\dfrac{1}{1+\dfrac{\mu}{\lambda}\dfrac{1}{p}\sqrt{q}}=\dfrac{1}{\lambda}\exp\left(\dfrac{-\mu}{\lambda}x\sqrt{q}\right)=\\
\quad =Y(x)\dfrac{1}{\lambda}\dfrac{\mu x}{2\lambda y\sqrt{\pi y}}\exp\dfrac{-\mu^2x^2}{4\lambda^2y^2}=F_1^{(\lambda,\mu)}(x,y)\\
\text{as function of } x\geqslant 0, \text{ with values in the space } (\mathcal{D}'_{Y_+}) \text{ of distributions of support contained in } Y_+=[0,\infty[_y;\\
\dfrac{1}{\mu\sqrt{q}}\dfrac{1}{1+\dfrac{\lambda}{\mu}p\dfrac{1}{\sqrt{q}}}=\dfrac{1}{\mu}\sum_{n\in N}(-1)^n\left(\dfrac{\lambda}{\mu}\right)^n p^n(\sqrt{q})^{-(n+1)}\\
\quad =\dfrac{1}{\mu}\sum_{n\in N}(-1)^n\left(\dfrac{\lambda}{\mu}\right)^n\delta^{(n)}(x)\otimes\\
\quad \otimes\left\{\dfrac{1}{\sqrt{\pi y}}\right\}^{\overset{*}{(n+1)}}=\{F_1^{(\lambda,\mu)}\}.\\
\text{as functions of } y>0, \text{ with values in the algebra } [[\mathcal{D}'_{X_+})^N]] \text{ of formal power series with coefficients in the space } (\mathcal{D}'_{X_+}) \text{ of distributions of support contained in } X_+=[0,\infty[_x.
\end{cases}
$$

and $\{F^{(\lambda,\mu)}(x,y)=\dfrac{1}{\lambda p+\mu\sqrt{q}}$.

Under these conditions, the fundamental solution $\{E\}$ of the parabolic differential equations

$$\lambda^2\frac{\partial^2\{E\}}{\partial x^2}-\mu^2\frac{\partial\{E\}}{\partial y}=\delta(x)\otimes\delta(y)$$

is given by

$$
(4)\quad \{E\}=\frac{1}{\lambda^2p^2-\mu^2q}=\frac{1}{\lambda p+\mu\sqrt{q}}\cdot\frac{1}{\lambda p-\mu\sqrt{q}}
$$

$$
\begin{cases}
=\dfrac{1}{\lambda^2}\exp\left(\dfrac{-\mu}{\lambda}x\sqrt{q}\right)\overset{x}{*}\exp\left(\dfrac{\mu}{\lambda}x\sqrt{q}\right)=\{E_1\}\\
\text{as function of } x>0 \text{ with values in the algebra } [[\mathcal{D}'_{Y_+}]^N] \text{ of formal power series with coefficients in } (\mathcal{D}'_{Y_+}); \text{ and:}\\
\dfrac{-1}{\mu^2}\cdot\dfrac{1_y}{q}\left(\dfrac{\delta(x)\otimes 1_y}{1-\dfrac{\lambda}{\mu}\delta'(x)\dfrac{1}{\sqrt{q}}}\right)\overset{x}{*}\left(\dfrac{\delta(x)\otimes 1_y}{1+\dfrac{\lambda}{\mu}\delta'(x)\dfrac{1}{\sqrt{q}}}\right)=\{E_2\}
\end{cases}
$$

as function of $y>0$ with values in the algebra $[[\mathcal{D}'_{X_+})^N]]$ of formal power series with coefficients in $(\mathcal{D}'_{X_+})$.

More precisely, we have

(5) $$\{E_1\}=\frac{1}{\lambda^2}\exp\left(\frac{-\mu x}{\lambda}\sqrt{q}\right)\overset{x}{*}\exp\left(\frac{\mu x}{\lambda}\sqrt{q}\right)=$$

$$=\frac{1}{2\lambda\mu\sqrt{q}}\left[\exp\left(\frac{\mu x}{\lambda}\sqrt{q}\right)-\exp\left(\frac{-\mu x}{\lambda}\sqrt{q}\right)=\right.$$

$$=\frac{1}{\lambda\mu}\left[\left(\frac{\mu}{\lambda}\right)\{\overset{*}{Y}(x)\}^2\otimes\delta(y)+\left(\frac{\mu}{\lambda}\right)^3\{\overset{*}{Y}(x)\}^4\otimes\delta'(y)+\cdots+\right.$$

$$\left.+\cdots+\left(\frac{\mu}{\lambda}\right)^{2k+1}\{\overset{*}{Y}(x)\}^{2k+2}\otimes\delta^{(x)}(y)+\cdots\right]$$

$$=\frac{1}{\lambda\mu}\left[\left(\frac{\mu}{\lambda}\right)\left\{\frac{x}{1!}\right\}\otimes\delta(y)+\left(\frac{\mu}{\lambda}\right)^3\left\{\frac{x^3}{3!}\right\}\otimes\delta'(y)+\cdots+\cdots\right.$$

$$\left.\cdots+\left(\frac{\mu}{\lambda}\right)^{2k+1}\cdot\left\{\frac{x^{2k+1}}{(2k+1)!}\right\}\otimes\delta^{(x)}(y)+\cdots\right]$$

and

(6) $$\{E_2\}=\frac{-1}{\mu^2}\left[\delta(x)\otimes Y(y)+\left(\frac{\lambda}{\mu}\right)^2\delta''(x)\otimes\{\overset{*}{Y}(y)\}^2+\cdots\right.$$

$$\left.\cdots+\left(\frac{\lambda}{\mu}\right)^{2k}\delta^{(2k)}(x)\otimes\{\overset{*}{Y}(y)\}^{k+1}+\cdots\right]$$

$$=\frac{-1}{\mu^2}\left[\delta(x)\otimes Y(y)+\left(\frac{\lambda}{\mu}\right)^2\delta''(x)\otimes\left\{\frac{y}{1!}\right\}+\cdots\right.$$

$$\left.\cdots+\left(\frac{\lambda}{\mu}\right)^{2k}\delta^{(2k)}(x)\otimes\left\{\frac{y^k}{k!}\right\}+\cdots\right],$$

where $\{\overset{*}{Y}(x)\}^k$ (resp. $\{\overset{*}{Y}(y)\}^k$) is the convolution of order k of the Heaviside function $Y(x)$ (resp. $Y(y)$}. The formulas (5) and (6) show that for $y>0$ (resp. $x>0$) we have $E_1(x, y)=0$ (resp. $E_2(x, y)=0$). On the other hand, formula (5) (resp. (6)) shows that $\{E_1)$ (resp. $\{E_2\}$) is a formal power series in $x\in X_+$ (resp. $y\in Y_+$) whose coefficients belong to $(\mathcal{D}'_{Y_+})$ (resp. $(\mathcal{D}'_{X_+})$).

2. *Parabolic differential equations* $(\mathcal{D}'_{R^2_+})$.

Consider the parabolic differential eqution

(7) $$\lambda^2\frac{\partial^2\{u\}}{\partial x^2}-\mu^2\frac{\partial\{u\}}{\partial y}=T(x, y)$$

where λ, μ are real or complex parameters and $T(x, y)\in(\mathcal{D}'_{R^2_+})$, the derivatives being taken in the sense of distributions.

The solution of (7) in $[[\mathcal{D}'_{R^2_+})^N]]$ is given by

$$(8)\qquad \{u\}=\begin{cases}\{u_1\}=\{E_1\}\overset{xy}{*}T(x,y) \text{ as a function of } x \text{ for } x\geqslant 0\\ \{u_2\}=\{E_2\}\overset{xy}{*}T(x,y) \text{ as a function of } y \text{ for } y\geqslant 0,\end{cases}$$

where $\overset{xy}{*}$ signifies convolution in $(\mathcal{D}'_{R^2_+}$.

$\{E_1\}$ (resp. $\{E_2\}$) is given by (5) (resp. (6)). Therefore, we have

$$(9)\qquad \{u\}=\begin{cases}\dfrac{1}{\lambda\mu}\sum\limits_{k=0}^{\infty}\left(\dfrac{\mu}{\lambda}\right)^{2k+1}\left\{\dfrac{x}{(2k+1)!}\right\}^{2k+1}\overset{x}{*}\dfrac{\partial^k T(x,y)}{\partial y^k} \text{ for } x\geqslant 0.\\ \dfrac{1}{\mu^2}\sum\limits_{k=0}^{\infty}\left(\dfrac{\lambda}{\mu}\right)^{2k}\dfrac{\partial^{2k} T(x,y)}{\partial x^{2k}}.\overset{y}{*}\left\{\dfrac{y^k}{k!}\right\} \text{ for } y>0.\end{cases}$$

Let us now consider the formal power series

$$(10)\qquad \{u_1\}=\sum_{k=0}^{\infty}\left(\frac{\mu}{\lambda}\right)^{2k+1}\left\{\frac{x^{2k+1}}{(2k+1)!}\right\}\overset{x}{*}\frac{\partial^k T(x,y)}{\partial y^k}.$$

For any $\Phi(x,y)\in\mathcal{D}_{(-\Gamma)}$, where $(\mathcal{D}_{-\Gamma})$ is the locally convex space of infinitely differentiable functions of support limited to the right in $\boldsymbol{R}^2$ (cf. [2], chap. II, § 2), we may write

$$<\left\{\frac{x^{2k+1}}{(2k+1)!}\right\}\overset{x}{*}\frac{\partial^k T(x,y)}{\partial y^k},\quad \Phi(x,y)>=$$

$$=<\left\{\frac{\xi^{2k+1}}{(2k+1)!}\right\},<\frac{\partial^k T(\eta,y)}{\partial y^k},\quad \Phi(\xi+\eta,y)_{\eta}^{>}>.$$

But (cf. [2], chap. II, § 3, no 2)
$<\dfrac{\partial^k T(\eta,y)}{\partial y^k},\ \Phi(\xi+\eta,y)_{\eta}^{>}$ is an infinitely differentiable function, of ξ, and distribution of y.

Suppose

$$(11)\qquad \sup\left|<\frac{\partial^k T(\eta,y)}{\partial y^k},\ \Phi(\xi+\eta,y)_{\eta}^{>}\right|<M^{2k+2}.k!$$

for each $k\in\boldsymbol{N}$, $\xi+\eta\leqslant a$, $a>0$, $\forall y\in[0,b]$, $b>0$.

Then

$$\left|<\left\{\frac{\xi^{2k+1}}{(2k+1)!}\right\},<\frac{\partial^k T(\eta,y)}{\partial y^k},\quad \Phi(\xi+\eta,y)_{\eta}^{>}>\right|=$$

$$=\left|\int_0^a\frac{\xi^{2k+1}}{(2k+1)!}<\frac{{}^k T(\eta,y)}{\partial y^k},\quad \Phi(\xi+\eta,y)_{\eta}^{>}d\xi\right|\leqslant M^{2k+2}.k!\int_0^a\frac{\xi^{2k+2}}{(2k+1)!}d\xi$$

$$\text{But}\int_0^a\frac{\xi^{2k+1}}{(2k+1)!}d\xi=\frac{a^{2k+2}}{(2k+2)!}.$$

Hence:

$$\left|<\left\{\frac{x^{2k+1}}{(2k+1)!}\right\}\overset{x}{*}\frac{\partial^k T(x,y)}{\partial y^k},\ \Phi(x,y)>\right|\leqslant (M^2a^2)^{2+1}\frac{1}{(k+1)(k+2)\cdots(2k+2)}$$

and

$$\lim_{k\to\infty}\left|<\left\{\frac{x^{2k+1}}{(2k+1)!}\right\}\overset{x}{*}\frac{\partial^k T(x,y)}{\partial y^k},\ \Phi(x,y)>\right|=0.$$

It is easy to prove that for each $\Phi\in(\mathcal{D}'_{-\Gamma})$, the series

$$\sum_{k=0}^{\infty}\frac{\mu}{\lambda}^{2k+1}\cdot<\left\{\frac{x^{2k+1}}{(2k+1)!}\right\}\overset{x}{*}\frac{\partial^k T(x,y)}{\partial y^k},\ \Phi(x,y)>$$

is absolutely and uniformely convergent for $(x, y)\in K$, where K is an arbitrary compact subset of $\boldsymbol{R}^2_+$.

Therefore the series of the right hand of (10) converges in the topology of $(\mathcal{D}'_{R^2_+})$, for each $T\in(\mathcal{D}'_{R^2_+})$ satisfying the condition (11).

Under these conditions we have $\{u_1\}\in(\mathcal{D}'_{R^2_+})$. Likewise, consider the formal power series

(12) $$\{u_2\}=\frac{-1}{\mu^2}\sum_{k=0}^{\infty}\left(\frac{\lambda}{\mu}\right)^{2k}\frac{\partial^{2k}T(x,y)}{\partial x^{2k}}\overset{y}{*}\left\{\frac{y^k}{k!}\right\}\text{ for } y>0;$$

We have:

$$<\frac{\partial^{2k}T(x,y)}{\partial x^{2k}}\overset{y}{*}\left\{\frac{y^k}{k!}\right\},\ \Phi(x,y)>=<\left\{\frac{\eta^k}{k!}\right\},<\frac{{}^{2k}T(x,\varkappa)}{\partial x^{2k}},\ \Phi(x,\eta+\varkappa)\gg_y,$$

where

$$<\frac{\partial^{2k}T(x,\varkappa)}{\partial x^{2k}},\ \Phi(x,\eta+\varkappa)\overset{>}{x}$$

is an infinitely differentiable function of η and distribution of x.

Suppose

(13) $$\sup\left|<\frac{\partial^{2k}T(x,\varkappa)}{\partial x^{2k}},\ \Phi(x,\eta+\varkappa)\overset{>}{x}\right|<N^{k+1},$$

for each $k\in N$, $\eta+\varkappa=b$, $b>0$, $\forall x\in[0, a]$ $a>0$.

Then, for each $\Phi\in(\mathcal{D}_{-\Gamma})$, we have

$$\left|<\left\{\frac{y^k}{k!}\right\}\overset{y}{*}\frac{\partial^{2k}T(x,y)}{\partial x^k},\ \Phi(x;y)>\right|\leqslant\frac{(Nb)^{k+1}}{(k+1)!},\text{ whence:}$$

$$\lim_{k\to\infty}\left|<\left\{\frac{y^k}{k!}\right\}\overset{y}{*}\frac{\partial^{2k}T(x,y)}{\partial x^{2k}},\ \Phi(x,y)>\right|=0.$$

Under these conditions, it is easy to show that the series of the right hand side of (12) converges in the topology $(\mathcal{D}'_{R^2_+})$. In short we can state the following

Theorem 1. *The parabolic differential equation* (7) *where* λ, μ *are complex parameters possesses a solution in* $(\mathcal{D}'_{R^2_+})$, *if* $T(x, y)$ *satisfies the conditions* (11) *and* (12). *Then* $\{u\}=\{u_1\}$ *for* $x>0$ *and* $\{u\}=\{u_2\}$ *for* $y>0$.

3. *Boundary value problem for the parabolic differential equation.*

Consider the parabolic differential equation

$$\lambda^2 \frac{\partial^2 u}{\partial x^2} - \mu^2 \frac{\partial u}{\partial y} = f(x, y) \tag{14}$$

where λ, μ are complex parameters and $f(x, y)$ an integrable function on each compact subset of $\boldsymbol{R}^2$.

We shall determine a solution of (14) satisfying the following boundary value conditions:

$$\lim_{\substack{y\to 0+\\ x>0}} u(x, y) = A(x); \quad \lim_{\substack{x\to 0+\\ y>0}} u(x, y) = B(y); \quad \lim_{\substack{x\to 0+\\ y>0}} \frac{\partial u}{\partial x} = C(y). \tag{15}$$

To do this, we transfer (g) into $(\mathcal{D}'_{R^2_+})$, then we have:

$$\lambda^2 \left\{\frac{\partial^2 u}{\partial x^2}\right\} - \mu^2 \left\{\frac{\partial u}{\partial y}\right\} = \{f\}. \tag{16}$$

On the other hand, keeping in mind the general formulas of [2], chap. III, § 1, no 10, we obtain in $(\mathcal{D}'_{R^2_+})$ the equation

$$\lambda^2 \frac{\partial^2 \{u\}}{\partial x^2} - u^2 \frac{\partial \{u\}}{\partial y} = \{f\} + \lambda^2 \delta'(x) \otimes \{B(y)\} + \lambda^2 \delta(x) \otimes \{C(y)\} - \\ - \mu^2 \delta(y) \otimes \{A(x)\}, \tag{17}$$

the formal solution of which is given by

$$\{u\} = \begin{matrix} \{u_1\} = \{E_1\} \overset{xy}{*} \{T(x, y)\} \\ \{u_2\} = \{E_2\} \overset{xy}{*} \{T(x, y)\} \end{matrix} \tag{18}$$

where

$$\{T(x, y)\} = \{f(x, y) + \lambda^2 \delta'(x) \otimes B(y) + \lambda^2 \delta(x) \otimes C(y) - \mu^2 \delta(y) \otimes A(x)\} \tag{19}$$

is an element of $(\mathcal{D}'_{R^2_+})$.

Let us prove that $\{u\}$ given by (18) satisfies the conditions (15).

We first note that $\{u\}$ is a function of x defined by $\{E_1\}$ and a function of y defined by $\{E_2\}$.

Then we must take

$$\lim_{\substack{x\to 0+\\ x>0}} u(x, y) = \lim_{\substack{x\to 0+\\ x>0}} u_1(x, y) \quad \text{and} \quad \lim_{\substack{x\to 0+\\ x>0}} \frac{\partial u}{\partial x} = \lim_{\substack{x\to 0+\\ x>0}} \frac{\partial u_1}{\partial x}$$

since $\{E_2(x, y)\}$ vanishes for $x>0$. On the other hand we must take

$$\lim_{\substack{y\to 0+\\ y>0}} u(x, y) = \lim_{\substack{y\to 0+\\ y>0}} u_2(x, y)$$

since $\{E_1(x, y)\}$ vanishes for $y>0$.

Therefore, for $x>0$, we have

$$\text{(20)} \qquad \{u\}=u_1\}=\int_0^x f(\xi, y) \overset{y}{*} E_1(x-\xi, y)\, d\xi+\lambda^2\{B(y)\} \overset{y}{*} \frac{\partial\{E_1\}}{\partial x}+$$

$$+\lambda^2\}C(y)\} \overset{y}{*} \{E_1\}-\mu^2\left\{\int_0^x A(\xi)\, E_1(x-\xi, y)\, d\xi\right\}.$$

Whence,

$$\lim_{\substack{x\to 0+\\ x>0}} \{u_1(x, y)\}=\lambda^2\{B(y) \overset{y}{*} \lim_{\substack{x\to 0+\\ x>0}} \frac{\partial\{E_1\}}{\partial x}=\{B(y)\} \overset{y}{*} \{\delta(y)\}=B(y),$$

because (5) $\Rightarrow$ $\{E_1(0, y)\}=\lim\limits_{\substack{x\to 0+\\ x>0}} (E_1(x, y)\}=0$ and

$$\lim_{\substack{x\to 0+\\ x>0}} \frac{\partial\{E_1\}}{\partial x}=\frac{1}{\lambda^2}\delta(y).$$

Moreover (20) yields

$$\text{(21)} \qquad \frac{\partial\{u_1\}}{\partial x}=\int_0^x f(\xi, y) \overset{y}{*} \frac{\partial\{E_1(x-\xi, y)\}}{\partial x} d\xi+$$

$$+\lambda^2\{B(y)\} \overset{y}{*} \frac{\partial^2\{E_1\}}{\partial x^2}+\lambda^2\{C(y)\} \overset{y}{*} \frac{\partial\{E_1\}}{\partial x}-\int_0^x A(\xi)\frac{\partial\{E_1(x-\xi, y)\}}{\partial x} d\xi,$$

since $E_1(0, y)=0$.

Hence

$$\lim_{x\to 0+} \frac{\partial\{u_1\}}{\partial x}=\lambda^2\{B(y)\} \overset{y}{*} \lim_{x\to 0+} \frac{\partial^2\{E_1\}}{\partial x^2}+\lambda^2\{C(y)\} \overset{y}{*} \lim_{x\to 0+} \frac{\partial\{E_1\}}{\partial x}.$$

But

$$\{B(y)\} \overset{y}{*} \lim_{x\to 0+} \frac{\partial^2\{E_1\}}{\partial x^2}=\{B(y)\} \underset{x>0}{\otimes} \delta(x)=0$$

and

$$\lambda^2\{C(y)\} \overset{y}{*} \lim_{\substack{x\to 0+\\ x>0}} \frac{\partial\{E_1\}}{\partial x}=\{C(y)\}.$$

Therefore

$$\lim_{x\to 0+} \frac{\partial\{u\}}{\partial x}=\lim_{x\to 0+} \frac{\partial\{u_1)}{\partial x}=\{C(y)\}.$$

Likewise

$$\lim_{y\to 0+} u(x, y)=\lim_{y\to 0+} u_2(x, y), \text{ since } E_1(x, y)=0 \text{ for } y>0.$$

But $u_2(x, y)$ as a function of $y>0$ is given by

$$(22)\qquad \{u_2(x, y)\} = \int_0^y \{f(x, \eta) \overset{x}{*} \{E_2(x, y-\eta)\} d\eta +$$

$$+\lambda^2 \int_0^y B(\eta) \frac{\partial\{E_2(x, y-\eta)\}}{\partial x} d\eta + \lambda^2 \int_0^y C(\eta)\{E_2(x, y-\eta)\} d\eta -$$

$$-\mu^2 (A(x)\} \overset{x}{*} \{E_2(x, y)\}.$$

Whence

$$\lim_{y\to 0+} \{u(x, y)\} = \lim_{y\to 0+} u_2(x, y)\} = -\mu^2\{A(x)\} \overset{x}{*} \lim_{y\to 0+} \{E_2(x, y)\} = \{A(x)\},$$

since

$$\lim_{y\to 0+} \{E_2(x, y)\} = -\frac{1}{\mu^2}\delta(x).$$

Hence

$$\lim_{y\to 0+} \{u(x, y)\} = \{A(x)\}.$$

Let us now prove that $\{u_1(x, y)\}$ given by (20) satisfies the equation

$$(23)\qquad \lambda^2 \frac{\partial^2\{u_1\}}{\partial x^2} - \mu^2 \frac{\partial\{u_1\}}{\partial x} = f(x, y)\} - \mu^2\{A(x)\}\otimes\delta(y)$$

for $x>0$, where the derivatives are taken with respect to x in the sense of functions and with respect to y in the sense of distributions.

Indeed, (21) implies

$$\frac{\partial^2\{u_1)}{\partial x^2} = \frac{1}{\lambda^2} f(x, y) + \int_0^x f(\xi, y) \overset{x}{*} \frac{\partial^2\{E_1(x-\xi, y)\}}{\partial x^2} d\xi +$$

$$+\lambda^2\{B(y)\} \overset{y}{*} \frac{\partial^3\{E_1\}}{\partial x^3} + \lambda^2\{C(y)\} \overset{y}{*} \frac{\partial^2\{E_1\}}{\partial x^2} -$$

$$-\mu^2 \int_0^x A(\xi) \frac{\partial^3\{E_1(x-\xi, y)\}}{\partial x^2} d\xi - \mu^2\{A(x)\}\otimes\frac{1}{\lambda^2}\delta(y)$$

since

$$\left.\frac{\partial\{E_1(x-\xi, y)\}}{\partial x}\right|_{\xi=x} = \frac{1}{\lambda^2}\delta(y).$$

Likewise, it follows from (20) that

$$\frac{\partial\{u_1\}}{\partial y} = \int_0^x f(\xi, y) \overset{y}{*} \frac{\partial\{E_1(x-\xi, y)\}}{\partial y} d\xi + \lambda^2\{B(y)\} \overset{y}{*} \frac{\partial^2\{E_1\}}{\partial x\,\partial y} +$$

$$+\lambda^2\{C(y)\} \overset{y}{*} \frac{\partial\{E_1\}}{\partial y} - \mu^2 \int_0^x A(\xi) \frac{\partial E_1(x-\xi, y)}{\partial y} d\xi,$$

whence

$$\lambda^2 \frac{\partial^2 \{u_1\}}{\partial x^2} - \mu^2 \frac{\partial \{u_1\}}{\partial y} = f(x, y) - \mu^2 \{A(x)\} \otimes \delta(y).$$

Since $\lambda^2 \frac{\partial^2 \{E_1\}}{\partial x^2} - \mu^2 \frac{\partial \{E_1\}}{\partial y} = 0$, for $x > 0$.

In the same way one can show that $\{u_2\}$ given by (22) satisfies the equation:

$$(24) \quad \lambda^2 \frac{\partial^2 \{u_2\}}{\partial x^2} - \mu^2 \frac{\partial \{u_2\}}{\partial y} = \{f(x, y)\} + \lambda^2 \{B(y) \otimes \delta'(x) + \lambda^2 \{C(y)\} \otimes \delta(x),$$

where the derivatives are taken with respect to x in the sense of distributions and with respect to $y > 0$ in the sense of functions.

Indeed, we have:

$$\frac{\partial^2 \{u_2\}}{\partial x^2} = \int_0^y \{f(x, \eta) \overset{x}{*} \frac{\partial^2 \{E_2(x, y-\eta)\}}{\partial x^2} d\eta +$$

$$+ \lambda^2 \int_0^y B(\eta) \frac{\partial^3 \{E_2(x, y-\eta)\}}{\partial x^3} d\eta + \lambda \int_0^y C(\eta) \frac{\partial^2 \{E_2(x, y-\eta)\}}{\partial x^2} d\eta -$$

$$- \mu^2 \{A(x)\} \overset{x}{*} \frac{\partial^2 \{E_2\}}{\partial x^2}$$

and

$$\frac{\partial \{u_2\}}{\partial y} = f(x, y) \overset{x}{*} E_2(x, 0)\} + \int_0^y \{f(x, \eta)\} \overset{x}{*} \frac{\partial \{E_2(x, y-\eta)\}}{\partial y} d\eta +$$

$$+ \lambda^2 B(y) \frac{\partial \{E_2(x, 0)\}}{\partial x} + \lambda^2 \int_0^y B(\eta) \frac{\partial^2 \{E_2(x, y-\eta)\}}{\partial x \partial y} d\eta +$$

$$+ \lambda^2 \{C(y)\} \otimes \{E_2(x, 0)\} - \int_0^y C(\eta) \frac{\partial \{E_2(x, y-\eta)\}}{\partial y} d\eta -$$

$$- \mu^2 \{A(x)\} \overset{x}{*} \frac{\partial \{E_2(x, y)\}}{\partial y}.$$

But

$$(25) \quad \begin{cases} E_2(x, 0) = -\dfrac{1}{\mu^2} \delta(x) \\ \dfrac{\partial \{E_2 x, 0)\}}{\partial x} = -\dfrac{1}{\mu^2} \delta'(x). \end{cases}$$

Whence

$$\lambda^2 \frac{\partial^2 \{u_2\}}{\partial x^2} - \mu^2 \frac{\partial \{u_2\}}{\partial y} = f(x, y) + \int_0^y f(x, \eta) \overset{x}{*} \left\{ \lambda^2 \frac{\partial \{E_2(x, y-\eta)\}}{\partial x^2} - \right.$$

$$\left. - \mu^2 \frac{\partial \{E_2(x, y-\eta)\}}{\partial y} \right\} d\eta + \lambda^2 \int_0^y c(\eta) \left\{ \lambda^2 \frac{\partial^2 \{E_2(x, y-\eta)\}}{\partial x^2} - \right.$$

$$\left. - \mu^2 \frac{\partial \{E_2(x, y-\eta)\}}{\partial y} \right\} d\eta + \lambda^2 \{B(y)\} \otimes \delta'(x) \, \lambda^2 \{c(y)\} \otimes \delta(x).$$

But

$$\lambda^2 \frac{\partial^2 E_2\}}{\partial x^2} - \mu^2 \frac{\partial \{E_2\}}{\partial y} = 0 \text{ for } y \geqslant 0, \text{ therefore } \{u_2\}$$

satisfies (24).

4. *Problems of convergence.*

The solution $\{u_1\}$ (resp. $\{u_2\}$ of the equation (17), given by (20) (resp. (22)) is a formal solution i.e. element of $[[(\mathcal{D}'_{Y_+})^N]]$ (resp. $[[(\mathcal{D}'_{X_+})^N]]$).

Let us consider $\{u_1\}$ given by (20). i.e.

(26)
$$\{u_1\} = \frac{1}{\lambda\mu} \sum_{k=0}^{\infty} \left(\frac{\mu}{\lambda}\right)^{2k+1} \left\{ \int_0^x \frac{(x-\xi)^{2k+1}}{(2k+1)!} \frac{\partial^k \{f(\xi, y)\}}{\partial y^k} d\xi + \right.$$
$$+ \frac{\lambda}{\mu} \sum_{k=0}^{\infty} \frac{\partial}{\partial x} \left\{ \frac{x^{2k+1}}{(2k+1)!} \right\} \left(\frac{\mu}{\lambda}\right)^{2k+1} \frac{d^k \{B(y)\}}{dy^k} +$$
$$+ \frac{\lambda}{\mu} \sum_{k=0}^{\infty} \left(\frac{\mu}{\lambda}\right)^{2k+1} \left\{ \frac{x^{2k+1}}{(2k+1)!} \right\} \frac{d^k \{C(y)\}}{dy^k} -$$
$$\left. - \frac{1}{\lambda} \sum_{k=0}^{\infty} \left(\frac{\mu}{\lambda}\right)^{2k+1} \left\{ \int_0^x \frac{(x-\xi)^{2k+1}}{(2k+1)!} A(\xi) \, d\xi \otimes \delta^k(y) \right\} \right. .$$

We have:

$$\left\{ \int_0^x \frac{(x-\xi)^{2k+1}}{(2k+1)!} \frac{\partial^k \{f(\xi, y)\}}{\partial y^k} d\xi \right\} = \left\{ \frac{x^{2k+1}}{(2k+1)!} \right\} \overset{x}{*} \frac{\partial^k \{f(x, y)\}}{\partial y^k} \Rightarrow$$

$$\Rightarrow \left\{ \frac{x^{2k+1}}{(2k+1)!} \right\} \overset{x}{*} \frac{\partial^k \{f\}}{\partial y^k} = \left\langle \frac{\xi^{2k+1}}{(2k+1)!}, \left\langle \frac{\partial^k \{f\}}{\partial y^k}, \Phi(\xi+\eta, y) \right\rangle_\eta \right\rangle =$$

$$= \left\langle \frac{\xi^{2k+1}}{(2k+1)!}, \int_0^b \frac{\partial^k f(\eta, y)}{\partial y^k} \Phi(\xi+\eta, y) \, d\eta \right\rangle_\xi =$$

$$= \int_0^a \frac{\xi^{2k+1}}{(2k+1)!} \left(\int_0^b \frac{\partial^k f(\eta, y)}{\partial y^k} \Phi(\xi+\eta, y) \, d\eta \right) d\xi; \qquad a > 0, \; b > 0$$

where, for each $y \in Y_+ = [0, \infty[_u = (\boldsymbol{R}_+)_y$, the function

$$\int_0^b \frac{\partial^k f(\eta, y)}{\partial y^k} \Phi(\xi+\eta, y)\, d\eta \text{ belongs to } (\mathcal{D}_{-\Gamma_\xi}).$$

Therefore,

$$\left| \int_0^a \frac{\xi^{2k+1}}{(2k+1)!} \left(\int_0^b \frac{\partial^k f(\eta, y)}{\partial y^k} \Phi(\xi+\eta, y)\, d\eta \right) d\xi \right| \leqslant$$

$$\leqslant \int_0^a \frac{\xi^{2k+1}}{(2k+1)!} \left(\int_0^b \left| \frac{\partial^k f(\eta, y)}{\partial y^k} \right| |\Phi(\xi+\eta, y)|\, d\eta \right) d\xi.$$

But $\displaystyle\int_0^b \left| \frac{\partial kf(\eta, y)}{\partial y^k} \right| |\Phi(\xi+\eta, y)|\, d\eta \leqslant bA_\Phi$. Sup $\displaystyle\left| \frac{\partial^k f(x, y)}{\partial y^k} \right|$

where $A_\Phi = \sup |\Phi(\xi+\eta, y)|$

$((\xi+\eta), y) \in K$, K compact subset of $\boldsymbol{R}^2_+$.

Suppose

$$\left| \frac{\partial kf(x, y)}{\partial y^k} \right| \leqslant M^{2k+2} k! \qquad \forall k \in \boldsymbol{N} \tag{27}$$

for $(x, y) \in K$, K arbitrary compact subset of $\boldsymbol{R}^2_+$.

Then

$$\left| < \left\{ \frac{x^{2k+1}}{(2k+1)} \right\} \overset{x}{*} \frac{\partial^k \{f\}}{\partial y^k},\ \Phi(x, y) \right| \leqslant A_\Phi\, b \cdot \frac{(a^2 M^2)^{2k+2}}{(k+1)(2k+2)\cdots(2k+2)}$$

and the series

$$\frac{1}{\lambda\mu} \sum_{k=0}^{\infty} \left(\frac{\mu}{\lambda} \right)^{2k+1} \left\{ \frac{x^{2k+1}}{(2k+1)!} \right\} \overset{x}{*} \left\{ \frac{\partial^k \{f\}}{\partial y^k} \right\} \text{ converges for the topology of } (\mathcal{D}'_{\boldsymbol{R}^2_+}).$$

On the other hand, for $x \geqslant 0$, we have:

$$\left| \left\langle \left\{ \frac{x^{2k}}{(2k)!} \right\} \otimes \frac{d^k \{B(y)\}}{dy^k},\ \Phi(x, y) \right\rangle \right| \leqslant \frac{a^{2k} \cdot b}{(2k)!} \cdot A_\Phi \cdot \sup \left| \frac{d^k B}{dy^k} \right| \text{ for } a > 0$$

and if

$$\left| \frac{dkB(y)}{dy^k} \right| < M_B^{2k} \cdot k! \qquad \forall k \in \boldsymbol{N} \tag{28}$$

for $y \in K_y$, K_y compact subset of Y_+, then the series

$$\frac{\partial}{\partial x} \left\{ \frac{x^{2k+1}}{(2k+1)!} \right\} \otimes \frac{d^k \{B(y)\}}{\partial y^k} \text{ converges for the topology of } (\mathcal{D}'_{\boldsymbol{R}^2_+}).$$

Likewise, one can prove that the series

$$\sum_{k=0}^{\infty}\left(\frac{\mu}{\lambda}\right)^{2k+1}\left\{\frac{x^{2k+1}}{(2k+1)!}\right\}\otimes\frac{d^k\{C(y)\}}{dy^k} \text{ converges in } (\mathcal{D}'_{R^2_+}) \text{ if}$$

(29)
$$\left|\frac{d^k C(y)}{dy^k}\right| < M_C^{2k+1}\cdot k!$$

$$y\in K_y.$$

Consequently, if the functions $f(x, y)$, $B(y)$, $C(y)$ are infinitely differentiable with respect to $y\in Y_+$ and satisfy respectively the conditions (27), (28), (29), then the first three series of the right hand side of $\{u_1\}$ given by (26) are elements of $(\mathcal{D}'_{R^2_+})$.

On the other hand, the fourth series of (26) is an element of $[[(\mathcal{D}'_{X_+}\otimes\otimes\mathcal{D}'_{Y_+})^N]]$, that is, of the algebra of convolution of formal series whose terms are elements of $\mathcal{D}'_{X_+}\otimes\mathcal{D}'_{Y_+}$),

But for $y\geqslant 0$, this element of $[[\mathcal{D}'_{X_+}\otimes\mathcal{D}'_{Y_+})^N]]$ vanishes. Theorefore, for $x>0$, $y>0$ the solution $\{u_1\}$ given by (26) is a function of the form

(30)
$$\{u_1\}=\frac{1}{\lambda\mu}\sum_{k=0}^{\infty}\left(\frac{\mu}{\lambda}\right)^{2k+1}\int_0^x\frac{(x-\xi)^{2k+1}}{(2k+1)!}\frac{\partial^k f(\xi, y)}{dy^k}d\xi+$$
$$+\frac{\lambda}{\mu}\sum_{k=0}^{\infty}\left(\frac{\mu}{\lambda}\right)^{2k+1}\frac{x^{2k}}{(2k)!}\frac{d^k B(y)}{dy^k}$$
$$+\frac{\lambda}{\mu}\sum_{k=0}^{\infty}\left(\frac{\mu}{\lambda}\right)^{2k+1}\frac{x^{2k+1}}{(2k+1)!}\frac{d^k C(y)}{dy^k}=u_1(x, y)$$

and

$$\lim_{\substack{x\to 0\\ x>0}} u_1(x, y)=B(y)$$

From (26) we get for the derivative of the distribution $\{u_1\}\in(\mathcal{D}'_{R^2_+})$ with respect to x and for $y>0$:

$$\frac{\partial\{u_1\}}{\partial x}=\frac{1}{\lambda\mu}\sum_{k=0}^{\infty}\left(\frac{\mu}{\lambda}\right)^{2k+1}\frac{\partial}{\partial x}\left\{\frac{x^{2k+1}}{(2k+1)!}\right\}\overset{x}{*}\left\{\frac{\partial^k f(x, y)}{\partial y^k}\right\}+$$
$$+\frac{\lambda}{\mu}\sum_{k=0}^{\infty}\left(\frac{\mu}{\lambda}\right)^{2k+1}\frac{\partial^2}{\partial x^2}\left\{\frac{x^{2k+1}}{(2k+1)!}\right\}\oplus\frac{d^k B(y)}{y^k}+$$
$$+\frac{\lambda}{\mu}\sum_{k=0}^{\infty}\left(\frac{\mu}{\lambda}\right)^{2k+1}\left\{\frac{x^{2k+1}}{(2k+1)!}\right\}\oplus\frac{d^k C(y)}{dy^k}.$$

But

$$\frac{\partial^2}{\partial x^2}\left\{\frac{x^{2k+1}}{(2k+1)!}\right\}=\frac{\partial^2}{\partial x^2}\{Y(x)\}^{\overset{*}{2k+2}}=\{Y(x)\}^{\overset{*}{2k}}$$

whence

$$\frac{\partial^2}{\partial x^2}\left\{\frac{x^{2k+1}}{(2k+1)!}\right\}_{k=0}=\delta(x),$$

and

$$\frac{\partial^2}{\partial x^2}\left\{\frac{x^{2k+1}}{(2k+1)!}\right\}_{k\neq 0}=0 \text{ for } x\geq 0.$$

Therefore, for $x\geq 0$, $y\geq 0$, we have:

$$\frac{\partial\{u_1\}}{\partial x}=\frac{\partial u_1}{\partial x}=\frac{1}{\lambda\mu}\sum_{k=0}^{\infty}\left(\frac{\mu}{\lambda}\right)^{2k+1}\int_0^x\frac{(x-\xi)^{2k}}{2k}\frac{\partial^k f(\xi,y)}{\partial y^k}d\xi+$$

$$+\frac{\lambda}{\mu}\sum_{k=1}^{\infty}\left(\frac{\mu}{\lambda}\right)^{2k+1}\frac{x^{2k-1}}{(2k-1)!}\frac{d^k B(y)}{dy^k}+\frac{\lambda}{\mu}\sum_{k=0}\left(\frac{\mu}{\lambda}\right)^{2k+1}\frac{x^{2k}}{(2k)!}\frac{d^k C(y)}{dy^k}$$

whence:

$$\lim_{x\to 0+}\frac{\partial u_1}{\partial x}=C(y). \quad \text{for } (x,y)\in R_+^2.$$

Consider likewise the solution $\{u_2\}$ given by (22), i.e.

$$\{u_2\}=\frac{-1}{\mu^2}\sum_{k=0}^{\infty}\left(\frac{\lambda}{\mu}\right)^{2k}\left\{\int_0^y\frac{\partial^{2k}\{f(x,\eta)\}}{\partial x^{2k}}\frac{(y-\eta)^k}{k!}d\eta\right\}+ \tag{31}$$

$$+\left\{\left(\frac{-1}{\mu^2}\right)\sum_{k=0}^{\infty}\left(\frac{\lambda}{\mu}\right)^{2k}\delta^{(2k)}(x)\int_0^y\frac{(y-\eta)^k}{k!}\right\}d\eta+$$

$$+\lambda^2\left(\frac{-1}{\mu^2}\right)\sum_{k=0}^{\infty}\left(\frac{\lambda}{\mu}\right)^{2k}\delta^{(2k)}(x)\int_0^y\frac{(y-\eta)^k}{k!}d\eta+$$

$$+\sum_{k=0}^{\infty}\left(\frac{\lambda}{\mu}\right)^{2k}\frac{d^{2k}\{A(x)\}}{dx2k}\otimes\frac{\{yk\}}{k!};$$

we have

$$\left\{\int_0^y\frac{\partial^{2k}\{f(x,\eta)\}}{\partial x^{2k}}\frac{(y-\eta)^k}{k!}d\eta\right\}=\frac{\partial^{2k}\{f(x,y)\}}{\partial x^{2k}}\underset{*}{y}\left\{\frac{y^k}{k!}\right\}\Rightarrow$$

$$<\frac{\partial^{2k}\{f(x,y)\}}{\partial x^{2k}}\underset{*}{y}\left\{\frac{y^k}{k!}\right\},\ \Phi(x,y)>=<\frac{\eta^k}{k!},<\frac{\partial^{2k}f(x,\chi)}{\partial x^{2k}},$$

$$\Phi(x,y+\chi)>_\chi>_\eta=<\frac{\eta^k}{k!},\int_0^a\frac{\partial^{2k}f(x,\chi)}{\partial x^{2k}}\Phi(x,\eta+\chi)d\chi>=$$

$$=\int_0^b\frac{\eta^k}{k!}\left(\int_0^a\frac{\partial^{2k}f(x,\chi)}{\partial x^{2k}}\Phi(x,\eta+\chi)d\chi\right)d\eta,\ a>0,\ b>0,$$

where for each $x\in X_+$, the function

$$\int_0^a\frac{\partial^{2k}f(x,\chi)}{\partial x^k}\Phi(x,\eta+\chi)d\chi \text{ belongs to } (\mathcal{D}\ )_\eta$$

Therefore

$$\left|\int_0^b \frac{\eta^k}{k!}\left(\int_0^a \frac{\partial^{2k} f(x,\chi)}{\partial x^{2k}} \Phi(x,\eta+\chi)\,d\chi\right) d\eta\right| \leq$$

$$\leq \int_0^b \frac{\eta^k}{k!}\left(\int_0^a \left|\frac{\partial^{2k} f(x,\chi)}{\partial x^{2k}}\right| |\Phi(x,\eta+\chi)|\,d\chi\right) d\eta.$$

But

$$\int_0^a \left|\frac{\partial^{2k} f(x.\chi)}{\partial x^{2k}}\right| |\Phi(x,\eta+\chi)|\,d\chi)\,d\eta \leq$$

$$\leq a \cdot A_\Phi \sup\left|\frac{\partial^{2k} f(x,y)}{\partial x^{2k}}\right|$$

where

$$A_\Phi = \sup|\Phi(x,\eta+\chi) \qquad (x,\eta+\chi)\in K,\ K \text{ compact subset of } \boldsymbol{R}^2_+.$$

Suppose

$$\left|\frac{\partial^{2k} f(x,y)}{\partial x^{2k}}\right| \leq N^{k+1}, \quad \forall k\in \boldsymbol{N} \tag{32}$$

for $(x,y)\in K$, K arbitrary compact subset of $\boldsymbol{R}^2_+$. Then, we have:

$$\left|<\frac{\partial^2\{f(x,y)\}}{\partial x^{2k}} \overset{y}{*} \left\{\frac{y^k}{k!}\right\},\quad \Phi(x,y)>\right| \leq \frac{(Nb)^{k+1}}{(k+1)!} a\cdot A_\Phi.$$

Under these conditions, it is easy to show that the first term of the right hand side of (31) is a convergent series in the topology of $(\mathcal{D}'_{R^2_+})$.

Likewise one can prove that the series

$$\sum_{k=0}^{\infty}\left(\frac{\lambda}{\mu}\right)^{2k} \frac{d^{2k}\{A(x)\}}{dx^{2k}} \otimes \left\{\frac{y^k}{k!}\right\}$$

converges in the topology of $(\mathcal{D}'_{R^2_+})$, if $A(x)$ is an infinitely differentiable function which satisfies the following condition:

$$\left|\frac{d^{2k} A(x)}{dx^{2k}}\right| \leq N_A^K \cdot \text{for each } k\in \boldsymbol{N} \text{ and for } x\in K_x, \tag{33}$$

K_x arbitrary compact subset of $\boldsymbol{R}_x$. On the other hand, the second and third terms of (31) are formal series, i.e. elemens of $[[(\mathcal{D}'_{X_+})^N]]$, which vanish for $x>0$. Thus, for $x>0$, $y>0$ $u_2(x,y)$ is a function, and $\lim\limits_{\substack{y\to 0\\ y>0}} u_2(x,y) = A(x)$.

In brief, one can state the following.

Theorem 2. *The parabolic differential equation* (14), *where* λ, μ *are complex parameters possesses a solution in* $\boldsymbol{R}^2_+$, *which satisfies the boundary value conditions* (15) *if the functions* $f(x,y)$, $A(x)$, $B(y)$, $C(y)$ *are infinitely* *differentiable and satisfy respectively the conditions:*

$$(\alpha)\quad \left|\frac{\partial^k f(x,y)}{\partial y^k}\right| \leq M_f^{2k+2}\cdot k! \qquad \left|\frac{\partial^{2k} f(x,y)}{\partial x^{2k}}\right| < N^{k+1}, \quad \forall k \in N,$$

for $(x, y) \in K$, *with* K *arbitrary compact subset of* R^2_+;

$$(\beta)\quad \left|\frac{d^{2k} A(x)}{d x^{2k}}\right| \leq N_A^k, \quad \forall k \in N$$

and $x \in K_x$, *with* K_x *arbitrary compact subset of* X_+;

$$(\gamma)\quad \left|\frac{d^k B(y)}{dy^k}\right| \leq M_B^{2k}\cdot k!, \quad \left|\frac{dk\, C(y)}{dy^k}\right| \leq M_C^{2k+1}\cdot k!, \quad \forall k \in N$$

for $y \in K_y$, K_y *arbitrary compact subset of* Y_+.

For $x>0$ (resp. $y>0$) *the solution given by* (20) (resp. (22)) *satisfies the differential equation* (23) (resp. 24), *where the derivatives are taken with respect to* $x>0$ (resp. $y>0$) *in the sense of functions and with respect to* y (resp. x) *in the sense of distributions.*

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