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ON THE FLOW OF A RAREFIED GAS PAST AN INFINITE POROUS WALL WITH TIME-DEPENDENT SUCTION

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Summary

A two-dimensional flow of a viscous, incompressible rarefied gas along an infinite flat plate is analysed under slip boundary conditions and the suction velocity varying periodically with time. Separate solutions are developed in following two cases:

i) Constant free stream velocity with suction varying periodically

ii) Both the suction velocity and the free stream velocity vary periodically with time and with the same frequency.

It is observed that tha shape of the mean velocity profile is changed due to the periodic variation of the suction. The mean value of the wall shearing stress is also altered in both the cases.

1. Introduction

An oscillatory flow of an incompressible viscous fluid past an infinite plate with constant suction velocity, has been considered by Stuart [1] under no-slip boundary conditions. But in case of the flow of the low-density gases, the no-slip boundary conditions are replaced by the slip-velocity boundary conditions. These have been discussed by Schaaf and Chambre's [2]. Stuart's problem was later-on studied by Reddy [3] under slip-flow boundary conditions and it was found that in the slip-flow regime i) the amplitude of the skin-friction never exceeds a finite limit ii) the phase-lead of the skin-friction fluctuations over the main stream tends to zero for large frequencies and iii) the back-flow at the plate may be avoided.

Recently, Messiha [4] extended Stuart's problem to the case of variable suction velocity under no-slip boundary conditions. Soundalgekar [5] studied Messiha's problem under slip-flow boundary conditions. The hydromagnetic flow corresponding to Stuart's problem was investigated by Suryaprakasarao [6,7] and corresponding to those in references [3], [4] and [5], was investigated recently by Soundalgekar [8, 9, 10]. In these last three papers, the induced magnetic field was, however, neglected.

In a recent paper, Kelly [11] investigated under no-slip boundary conditions, the effects of time-dependent suction on the flow of an incompressible, viscous fluid past an infinite plate. In the case of periodic variation of the suction velocity, Kelly has observed that the shape of the mean velocity profile is affected which is absent in all the earlier studies.

Hence the object of the present invesigation is to study the modifications in Kelly's problem when his no-slip boundary conditions are replaced by the first- order velocity-slip boundary conditions. In the case of subsonic flows of relatively hot gases, the assumption of incompressibility is physically realizable. The more generally accepted method of analysis for slip-flows is utilised here viz., the continuum equations of motion are used throughout, together with the first-order slip velocity boundary conditions at the plate. In section **2**, the problem is posed under suitable assumptions, Kelly's method of solution is closely followed and the problem is solved for suction velocity varying periodically with time. The results have been compared with Kelly's results wherever necessary to bring out the effects of fluid rarefaction. In section **3**, a comprehensive summary of results is presented.

2. Mathematical Analysis

Here a two-dimensional flow of an incompressible, viscous, rarefied gas along an infinite wall is considered. The X-axis is taken along the wall in the direction of flow and Y-axis is normal to the wall. Let u and v be the velocity components along x and y directions respectively, t the time, p the pressure, ρ the density and ν the kinematic viscosity. Along an infinite wall, the flow is independent of x. Then the Navier-Stokes equations governing the flow are

(1) $$\frac{\partial u}{\partial t}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\frac{\partial^2 u}{\partial y^2}$$

(2) $$\frac{\partial v}{\partial t}=-\frac{1}{\rho}\frac{\partial p}{\partial y}$$

The boundary conditions are the first order velocity slip condition as given in (2) (neglecting the thermal creep term)

(3) $$u=\frac{2-f_1}{f_1}L\frac{\partial u}{\partial y}=L_1\frac{\partial u}{\partial y} \quad \text{at} \quad y=0$$

and

(4) $$u=U(t) \quad \text{as} \quad y\to\infty$$

Here $L=\mu(\pi/(2p\rho))^{1/2}$ is the mean free path and is a constant for an incompressible gas. Hence L_1 is also a constant. f_1 is the Maxwell's reflection coefficient and $U(t)$ is the velocity at a large distance from the plate. Then one can show that

(5) $$-\frac{1}{\rho}\frac{\partial p}{\partial x}=\frac{\partial U}{\partial t}$$

Hence from (1) and (5), we get

(6) $$\frac{\partial u}{\partial t}+v\frac{\partial u}{\partial y}=\frac{\partial U}{\partial t}+\nu\frac{\partial^2 u}{\partial y^2}$$

As the suction velocity is to be considered as a function of time, we assume $v(t)$ in the form of a periodic function given by

$$v(t) = v_0 [1 + \delta (e^{i\omega t} + e^{-i\omega t})]. \tag{7}$$

Two different forms for free stream velocities are considered during the course of discussion:

i) Constant free stream velocity.

ii) Oscillating free stream velocity.

In the latter case, it is assumed that both $v(t)$ and $U(t)$ vary slowly.

2.1: *Suction varies periodically and the free stream velocity $U(t)$ is constant.*

We now assume, following the method of Kelly, that $u(y, t)$ is of the form

$$u(y, t) = u_0(y) + \sum_{n=1}^{\infty} u_n(y) e^{in\omega t} + \sum_{n=1}^{\infty} \tilde{u}_n(y) e^{-in\omega t} \tag{8}$$

where the symbol $\sim$ denotes a complex conjugate.

Substituting (7) and (8) in equations (6), (3) and (4), we obtain on comparing non-harmonic and harmonic terms, the following set of equations and the boundary conditions:

$$\begin{cases} v_0 \dfrac{du_0}{dy} + v_0 \delta \left(\dfrac{du_1}{dy} + \dfrac{d\tilde{u}_1}{dy} \right) = \nu \dfrac{d^2 u_0}{dy^2} \\ y = 0: \ u_0 = L_1 \dfrac{du_0}{dy} \\ y \to \infty: \ u_0 \to U_0 \end{cases} \tag{9}$$

$$\begin{cases} i\omega u_1 + v_0 \dfrac{du_1}{dy} + v_0 \delta \left(\dfrac{du_0}{dy} + \dfrac{du_2}{dy} \right) = \nu \dfrac{d^2 u_1}{dy^2} \\ y = 0: \ u_1 = L_1 \dfrac{du_1}{dy} \\ y \to \infty: \ u_1 \to 0 \end{cases} \tag{10}$$

$$\begin{cases} in\omega u_n + v_0 \dfrac{du_n}{dy} + v_0 \delta \left(\dfrac{du_{n-1}}{dy} + \dfrac{du_{n+1}}{dy} \right) = \nu \dfrac{d^2 u_n}{dy^2} \\ y = 0: \ u_n = L_1 \dfrac{du_n}{dy} \\ y \to \infty: \ u_n = 0 \end{cases} \tag{11}$$

We also obtain a set of equations similar to (10) and (11) for $\tilde{u}_1$ and $\tilde{u}_n$.

These equations are the same as those obtained by Kelly (11) but the boundary conditions are different. It is evident from equations in (9) that the mean flow of a rarefied gas is affected by the oscillations through u_1 and $\tilde{u}_1$ and also through all the higher harmonic terms. The distortion of the mean velocity profile is due to the time-dependent suction which is absent in problems considered by Stuart, Reddy, Messiha, Soundalgekar.

Equations (9) and (11) reduce to the following nondimensional form:

$$\frac{v_0}{|v_0|}\frac{d\Phi_0}{d\eta}+\frac{\delta v_0}{|v_0|}\left(\frac{d\Phi_1}{d\eta}+\frac{d\tilde{\Phi}_1}{d\eta}\right)=\frac{d^2\Phi_0}{d\eta^2} \tag{12}$$

$$\eta=0:\ \Phi_0=h_1\frac{d\Phi_0}{d\eta},\quad \eta\to\infty,\ \Phi_0\to 1$$

in

$$\lambda\Phi_n+\frac{v_0}{|v_0|}\frac{d\Phi_n}{d\eta}+\frac{\delta v_0}{|v_0|}\left(\frac{d\Phi_{n-1}}{d\eta}+\frac{d\Phi_{n+1}}{d\eta}\right)=\frac{d^2\Phi_n}{d\eta^2} \tag{13}$$

$$\eta=0:\ \Phi_n=h_1\frac{d\Phi_n}{d\eta},\quad \eta\to\infty,\ \Phi_n\to 0$$

where

$$y=\frac{\nu}{|v_0|}\eta,\quad u_n=U_0\Phi_n,\quad \lambda=\frac{\omega\nu}{|v_0|^2} \tag{14}$$

and $h_1=\frac{|v_0|L_1}{\nu}$ is the rarefaction parameter.

To solve the infinite set of equations in (12) and (13), we assume $\delta\ll 1$, and expand Φ's in powers of δ. This assumption leads to the solution of weakly coupled equations. Now let

$$\Phi_n(\eta)=\sum_{j=0}^{\infty}\Phi_{nj}(\eta)\,\delta^j. \tag{15}$$

From (12) and (15), we obtain the following set of equations for Φ_{00}:

$$\frac{d^2\Phi_{00}}{d\eta^2}-\frac{v_0}{|v_0|}\frac{d\Phi_{00}}{d\eta}=0$$

$$\eta=0:\quad \Phi_{00}=h_1\frac{d\Phi_{00}}{d\eta},\quad \eta\to\infty,\quad \Phi_{00}\to 1 \tag{16}$$

The solution for this system exists only if $v_0<0$ i.e. for suction at the wall. Hence the solution of (16) is

$$\Phi_{00}=1-\frac{e^{-\eta}}{1+h_1}. \tag{17}$$

For $\delta>1$, there is a strong coupling because of the presence of blowing occurring at certain times during the cycle of oscillations.

The equations for $\Phi_{10}(\eta)$ are

$$\frac{d^2\Phi_{10}}{d\eta^2}+\frac{d\Phi_{10}}{d\eta}-i\lambda\Phi_{10}=0$$

$$\eta=0:\ \Phi_{10}=h_1\frac{d\Phi_{10}}{d\eta},\quad \eta\to\infty;\ \Phi_{10}\to\infty. \tag{18}$$

This system leads to $\Phi_{10}\equiv 0$. We can also show that $\Phi_{no}\equiv 0$ for $n\geq 1$. Also $\Phi_{10}\equiv 0$ implies that $\Phi_{01}\equiv 0$.

The equations for $\Phi_{11}(\eta)$ are

$$\frac{d^2\Phi_{11}}{d\eta^2}+\frac{d\Phi_{11}}{d\eta}-i\lambda\,\Phi_{11}=-\frac{e^{-\eta}}{1+h_1}$$

(19)

$$\eta=0:\ \Phi_{11}=h_1\frac{d\Phi_{11}}{d\eta},\quad \eta\to\infty;\ \Phi_{11}\to 0.$$

This system leads to following solution for $\Phi_{11}(\eta)$

(20) $$\Phi_{11}(\eta)=\frac{i}{\lambda(1+h_1)^2}e^{-h\eta}-\frac{i}{\lambda(1+h_1)}e^{-\eta}=\frac{\sin h_i\eta}{\lambda(1+h_1)^2}e^{-h_r\eta}$$

where

$$h=h_r+ih_i=\frac{1}{2}[1+(1+4i\lambda)^{1/2}]$$

$$=\frac{1}{2}+\frac{1}{2}\left[\frac{1+(1+16\lambda^2)^{1/2}}{2}\right]^{1/2}+\frac{i}{2}\left[\frac{-1+(1+16\lambda^2)^{1/2}}{2}\right]^{1/2}.$$

The equations for $\Phi_{02}(\eta)$ are

$$\frac{d^2\Phi_{02}}{d\eta^2}+\frac{d\Phi_{02}}{d\eta}=\frac{h_r\sin h_i\eta-h_i\cos h_i\eta}{\lambda(1+h_1)^2}e^{-h_r\eta}$$

(21)

$$\eta=0:\ \Phi_{02}=h_1\frac{d\Phi_{02}}{d\eta},\quad \eta\to\infty;\ \Phi_{02}\to 0$$

which leads to the following solution:

(22) $$\Phi_{02}(\eta)=\frac{1}{\lambda(1+h_1)^2}\left[\frac{-h_i}{h_i^2+(1-h_r)^2}e^{-\eta}\right]+$$

$$+\frac{e^{-h_r\eta}}{\lambda(1+h_1)^2}\left[\frac{h_i\cos h_i\eta+(h_r-1)\sin h_i\eta}{h_i^2+(1-h_r)^2}\right].$$

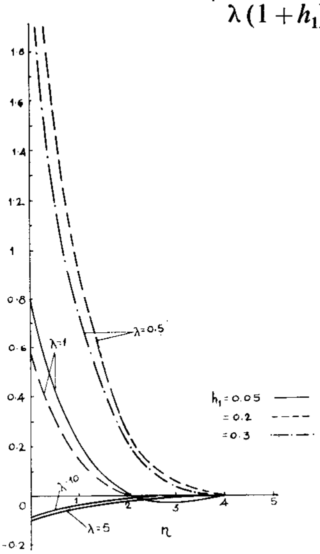

FIG 2 ϕ_{02} VS η

Φ_{11} and Φ_{02} are plotted in Figs. 1 and 2 respectively for different values of λ, h_1 and η. From Fig. 1, one can conclude that for the same λ, an increase in h_1 leads to a decrease in Φ_{11}. Φ_{11} also decreases with increasing λ. The nature of Φ_{02} is completely modified in slip-flow regime in the presence of time-dependent suction. Kelly has observed that Φ_{02} is completely negative for moderately small values of λ. But in slip-flow

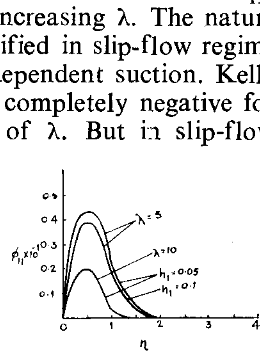

FIG. 1 - ϕ_{11} VS DISTANCE FROM THE WALL

regime, for small values of $\lambda=0.5$, 1 there is a sudden rise in Φ_{02} near the wall for all h_1. It is negative only for very small h_1 and moderately large λ.

The solution for the mean velocity profile to $0(\delta^2)$ can now be expressed in the following form:

$$u_0(\lambda, h_1, \eta) = U_0\left[1 - \frac{e^{-\eta}}{1+h_1} + \delta^2 \Phi_{02}(\lambda, h_1, \eta)\right]. \tag{23}$$

As $\lambda \to \infty$, h_r and h_i still tend to $(\lambda/2)^{1/2}$ and Φ_{02} decays as $[1/2\lambda^{3/2}(1+h_1)^2]$. Also for small λ

$$\Phi_{02} \to -\frac{\eta^2 e^{-\eta}}{2(1+h_1)^2} \quad \text{as} \quad \eta \to 0. \tag{24}$$

As in no-slip case, in slip-flow regime, the mean value of the wall shear stress is not affected by terms of $0(\delta^2)$ for $\Phi_{02}'(0)=0$. An increase in h_1 leads to a decrease in the mean value of the wall shear stress.

2.2. *Periodic variation of suction with a periodic free Stream velocity:*

We now assume $v(t)$ to be given by (7) and assume the velocity of the external flow to oscillate with a frequency equal to that of $v(t)$, viz ω and having an arbitrary phase angle α. Hence we assume

$$U = U_0[1 + 2\varepsilon \cos(\omega t + \alpha)] \tag{25}$$

or

$$U = U_0[1 + \varepsilon_1 e^{i\omega t} + \tilde{\varepsilon}_1 \varepsilon^{-i\omega t}]$$

where

$$\varepsilon_1 = \varepsilon e^{i\alpha}. \tag{26}$$

The relations (8) and (9) i.e. (12) still hold good and for $n \geq 2$, relation (11) i.e. (13) remains the same. Equations (10) in virtue of (25), (26) and (14) become

$$i\lambda\Phi_1 - \frac{d\Phi_1}{d\eta} - \delta\left(\frac{d\Phi_0}{d\eta} + \frac{d\Phi_2}{d\eta}\right) = i\lambda\varepsilon_1 + \frac{d^2\Phi_1}{d\eta^2} \tag{27}$$

with a corresponding equation for $\tilde{\Phi}_1(\eta)$. From (27) and (15), we obtain for $\Phi_{10}(\eta)$ the following

$$i\lambda\Phi_{10} - \frac{d\Phi_{10}}{d\eta} = i\lambda\varepsilon_1 + \frac{d^2\Phi_{10}}{d\eta^2} \tag{28}$$

with boundary conditions

$$\eta = 0: \Phi_{10} = h_1 \frac{d\Phi_{10}}{d\eta}, \quad \eta \to \infty: \ \Phi_{10} \to \varepsilon_1 \tag{29}$$

We have again $\Phi_{20} \equiv 0$ and hence equations (19) and (21) are respectively valid for $\Phi_{11}(\eta)$ and $\Phi_{02}(\eta)$. Hence the terms of order δ^2 distorting the mean profiles remain the same. In the case of periodic free stream, the terms of order δ, which are non-zero terms, are additional terms distorting the mean profile.

Now the solution of (28) subject to the conditions (27) is given by

$$\Phi_{10}(\eta) = \varepsilon_1\left(1 - \frac{e^{-h\eta}}{1+hh_1}\right) \tag{30}$$

In case of constant free stream velocity, is was shown above that $\Phi_{10}(\eta)\equiv 0$. This is not true now as can be seen from (30). We now determine $\Phi_{01}(\eta)$ from the following equation:

$$\frac{d^2\Phi_{01}}{d\eta^2}+\frac{d\Phi_{01}}{d\eta}=-\left(\frac{d\Phi_{10}}{d\eta}+\frac{d\tilde{\Phi}_{10}}{d\eta}\right)$$

$$\eta=:\Phi_{01}=h_1\frac{d\Phi_{01}}{d\eta},\quad \eta\to\infty,\ \Phi_{01}\to 0. \tag{31}$$

We substitute for $\Phi_{10}(\eta)$ and its conjugate form (30) in (31) and obtain after taking the real part,

$$\frac{d^2\Phi_{01}}{d\eta^2}+\frac{d\Phi_{01}}{d\eta}=-\frac{d}{d\eta}\left[2\varepsilon_{1r}\left\{1-\frac{(1+h_1h_r)\cos h_i\eta-h_1h_i\sin h_i\eta}{(1+h_1h_r)^2+(h_1h_i)^2}\right\}e^{-h_r\eta}\right.$$

$$\left.-2\varepsilon_{1i}\frac{(1+h_1h_r)\sin h_i\eta+h_1h_i\cos h_i\eta}{(1+h_1h_i)^2+(h_1h_i)^2}e^{-h_r\eta}\right] \tag{32}$$

where

$$\varepsilon_1=\varepsilon_{1r}+i\varepsilon_{1i}.$$

Following Kelly, we now split Φ_{01} into two components viz.,

$$\Phi_{01}=\varepsilon_{1r}\Phi_{01,1}+\varepsilon_{1i}\Phi_{01,2}. \tag{33}$$

Then $\Phi_{01,1}$ is determined by

$$\frac{d^2\Phi_{01,1}}{d\eta^2}+\frac{d\Phi_{01,1}}{d\eta}=-\frac{d}{d\eta}\left[2-\frac{2\{(1+h_1h_r)\cos h_i\eta-h_1h_i\sin h_i\eta\}}{(1+h_1h_r)^2+(h_1h_i)^2}e^{-h_r\eta}\right] \tag{34}$$

with

$$\eta=0:\Phi_{01,1}=h_1\frac{d\Phi_{01,1}}{d\eta},\quad \eta\to\infty,\quad \Phi_{01,1}\to 0$$

and $\Phi_{01,2}$ is given by

$$\frac{d^2\Phi_{01,2}}{d\eta^2}+\frac{d\Phi_{01,2}}{d\eta}=-\frac{d}{d\eta}\left[\frac{2\{(1+h_1h_r)\sin h_i\eta+h_1h_i\cos h_i\eta\}}{(1+h_1h_r)^2+(h_1h_i)^2}e^{-h_r\eta}\right] \tag{35}$$

with

$$\eta=0:\Phi_{01,2}=h_1\frac{d\Phi_{01,2}}{d\eta};\quad \eta\to\infty,\quad \Phi_{01,2}\to 0.$$

From (31), we can conclude that when the free stream velocity is either in phase or directly out of phase with the suction, $\varepsilon_{1r}\Phi_{01,1}$ is the solution and when the free stream velocity is 90° out of phase with the suction velocity, $\varepsilon_{1i}\Phi_{01,2}$ is the solution. Also in the no-slip case, Kelley has observed that

$$\Phi_{01,2}(\eta)=-\lambda\Phi_{02}(\eta).$$

This is not true in case of slip-flow regime, which is evident from equation (35).

Hence the solutions of the equations in (34) and (35) are now given by

$$\Phi_{01,1}(\eta)=\frac{2[(1+h_1h_r)\{h_r-1+h_1(h_r^2-h_i^2-h_r)\}-2h_1h_i^2(h_1(1-2h_r)-1)]}{(1+h_1)[(1+h_1h_r)^2+(h_1h_i)^2][h_i^2+(1-h_r)^2]}e^{-\eta}$$

$$-\frac{2[\{h_r-1+h_1(h_r^2-h_i^2-h_r)\}\cos(h_i\eta)+h_i(h_1(1-2h_r)-1)\sin(h_i\eta)]}{[(1+h_1h_r)^2+(h_1h_i)^2][h_i^2+(1-h_r)^2]}e^{-h_r\eta} \tag{36}$$

$$\Phi_{01,2}(\eta) = \frac{2\,[h_1 h_i \{1 - h_r + h_i h_i (1 + h_i - h_r)\} - h_i (1 + h_1 h_r)(h_1 (1 - h_r - h_i) - 1)]}{(1 + h_1)\,[(1 + h_1 h_r)^2 + (h_1 h_i)^2]\,[h_i^2 + (1 - h_r)^2]}\, e^{-\eta}$$

(37)

$$+ \frac{2\,[h_i (h_1 (1 - h_r - h_i) - 1)\cos(h_i \eta) + \{1 - h_r + h_1 h_i (1 + h_i - h_r)\}\sin(h_i \eta)]}{[(1 + h_1 h_r)^2 + (h_1 h_i)^2]\,[h_i^2 + (1 - h_r)^2]}\, e^{-h_r \eta}$$

The functions $\Phi_{01,1}(\eta)$ and $\Phi_{01,2}(\eta)$ are shown graphically in figures 3 and 4. Kelly has observed that an increase in λ leads to a decrease in $\Phi_{01,1}$, which is also true in slip-flow regime. $\Phi_{01,1}$ increases near the wall with increasing h_1. $\Phi_{01,2}$ is shown in Fig. 4. An increase in λ leads to a decrease in $\Phi_{01,2}$ for h_1 constant, but it decreases with increasing h_1.

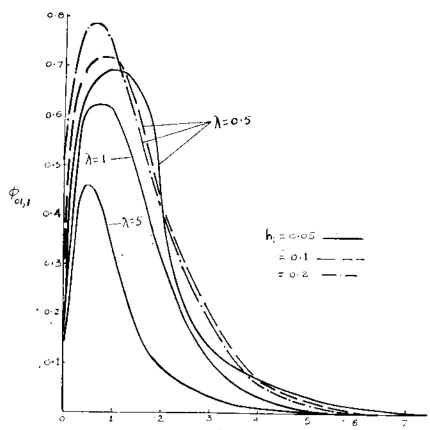

Fig. 3. $\Phi_{01,1}$ VS DISTANCE FROM THE WALL.

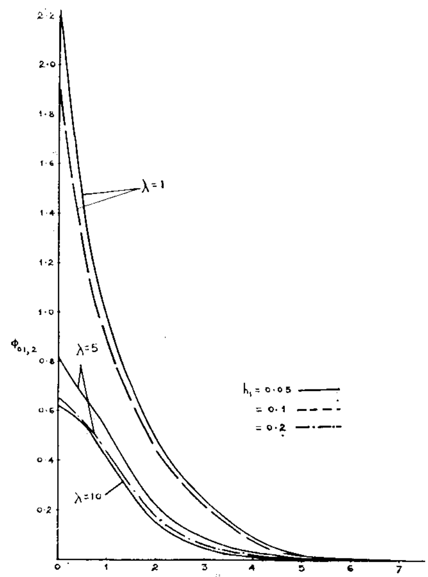

Fig. 4. $\Phi_{01,2}$ VS DISTANCE FROM THE WALL.

The solution for the mean velocity can now be written in the form:

$$u_0(\eta) = U_0\left[1 - \frac{e^{-\eta}}{1 + h_1} + \delta\,(\varepsilon_{1r}\Phi_{01,1} + \varepsilon_{1i}\Phi_{01,2}) + \delta^2 \Phi_{02}\right] \tag{38}$$

to order δ^2.

According to Kelly, in no-slip case because $\Phi_{01,2}(\eta)$ is a multiple of $\Phi_{02}(\eta)$, the component $\Phi_{01,1}(\eta)$ which is in phase or directly out of phase with the suction velocity dominates at low frequencies. This is not the case in the slip-flow regime. One can observe from Fig. 3 and Fig. 4 that the component $\Phi_{01,2}(\eta)$ is more dominating than the component $\Phi_{01,1}(\eta)$ at low frequency in slip flow regime. Also $\Phi_{01,2}(\eta)$ is not zero at the plate. Hence the mean value of the wall shearing stress is affected by both the components $\Phi_{01,1}(\eta)$, $\Phi_{01,2}(\eta)$ in the slip-flow regime. To obtain the mean value of the wall shearing stress, we have from (38),

$$\tau_w = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0} = \frac{\mu U_0 |v_0|}{\nu}\left[\frac{1}{1+h_1} + \right.$$

$$(39) \qquad + 2\delta\left\{\varepsilon_{1r}\frac{h_i^2(h_1(2h_r-1)+1)+(1-h_r)^2+h_1^2(1-h_r)(h_r^2-h_i^2-h_r)}{(1+h_1)[(1+h_1h_r)^2+(h_1h_i)^2][h_i^2+(1-h_r)^2]} - \right.$$

$$\left.\left. - \varepsilon_{1i}\frac{h_1h_r}{(1+h_1)[(1+h_1h_r)^2+(h_1h_i)^2]}\right\}\right]$$

which reduces to Kelly's result (4.17) when h_1 is zero. Thus the mean value of the wall shearing stress is completely modified in the slip-flow regime when both the free stream velocity and the suction velocity oscillate with the same frequency. It has been observed by Kelly that the wall shear stress is increased if the two oscillations are in phase and decreased if they are 180° out of phase. This is because of the vanishing of the coefficient of ε_{1i} in no-slip case. In the present case, the derivatives of $\Phi_{01,1}$ and $\Phi_{01,2}$ are different from unity and zero respectively. Hence to study the effects of the term of $0(\delta)$ on the wall shear stress, the numerical values of $\Phi'_{01,1}|_{\eta=0}$ and $\Phi'_{01,2}|_{\eta=0}$ (prime denotes differentiation with respect to η) are calculated for different values of h_1 and λ. They are entered in Table I. A close study of the table reveals that $\Phi'_{01,1}|_{\eta=0}$ increases with increasing λ for the same value of the rarefaction parameter h_1 whereas for the same value of λ, the frequency parameter, an increase in h_1 leads to a decrease in $\Phi'_{01,1}|_{\eta=0}$. In the case of $\Phi'_{01,2}|_{\eta=0}$, for same h_1, it increases with increasing λ, but for small values of λ, say ≤ 5, an increase in h_1 leads to an increase in $\Phi'_{01,2}|_{\eta=0}$ whereas for moderately large values of λ, it first increases and then decays as h_1 increases.

Values of $\Phi'_{01,1}|_{\eta=0}$

Table 1

h_1/λ	0.5	1	5	10	15
0.05	0.9008	0.8934	0.8568	0.8289	0.8078
0.1	0.8153	0.8020	0.7380	0.6915	0.6579
0.2	0.6762	0.6549	0.5589	0.4964	0.4551
0.3	0.5693	0.5434	0.4352	0.3717	0.3324

Values of $\Phi'_{01,2}|_{\eta=0}$

0.05	0.0167	0.0262	0.0597	0.0804	0.0943
0.1	0.0287	0.0443	0.0938	0.1196	0.1347
0.2	0.0433	0.0649	0.1210	0.1412	0.1496
0.3	0.0500	0.0732	0.1230	0.1346	0.1368

3. Conclusions

When the first order slip boundary conditions are imposed, some interesting features in the shape of the mean velocity profiles and the mean value of the wall shearing stress are observed. They are summarised as follows:

(i) In case of constant free stream velocity, the mean velocity profile is still not affected by terms of order δ. However, the function which modifies the shape of the mean velocity profile viz. $\Phi_{02}(\eta)$, behaves in a quite different way in slip flow regime. For very small λ, $\Phi_{02}(\eta)$ suddenly increases for all values of h_1, the rarefaction parameter. It is negative only for small h_1 and large λ.

(ii) An increase in h_1 leads to a decrease in the mean value of the wall shear stress, in case of constant free stream velocity.

(iii) In the slip-flow regime, when the free stream velocity varies periodically, the mean velocity profile is still affected by terms of order δ and δ^2, as in no-slip case. But the function $\varepsilon_{1i}\Phi_{01,2}$, which is the solution when the free stream velocity is 90° out of phase with the suction, is completely different from the one in no-slip case. Hence in the slip-flow regime, both these functions are dominating in addition to Φ_{02}. In the present case, $\Phi_{01,1}$ decays early for small h_1 and large λ, but for small λ, an increase in h_1 leads to an increase in $\Phi_{01,1}$ near the wall. Also the function $\Phi_{01,2}$ increases suddenly near the wall when h_1 and λ are both very small. It decreases with increasing λ and h_1.

(iv) The functions dominating the mean value of the wall shearing stress are $\Phi'_{01,1}|_{\eta=0}$ and $\Phi'_{01,2}|_{\eta=0}$, where prime denotes differentiation with respect to η. The former decreases with increasing λ as well as h_1 whereas the latter increases with increasing h_1 when λ is small (≤ 5) and for large λ, it increases first and then decreases as h_1 increases.

(v) The rarefaction parameter h_1 involves $|v_0|$, which is the mean suction velocity. Hence the variation of this parameter does not correspond to only the variation in the level of rarefaction, but also to the variation in the mean suction velocity imposed at the plate.

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