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RELATIONS BETWEEN INDUCED CURVATURE TENSORS OF FINSLER HYPERSURFACE F_{n-1} AND CURVATURE TENSORS OF IMBEDDING SPACE F_n.

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SUMMARY

It is well known that F_n has three curvature tensors: R^i_{jhk}, P^i_{jhk} and S^i_{jhk} [1], which are formed by means of connection coefficients Γ^{*i}_{jk} and tensor A^i_{jk}. On each hypersurface of Finsler space there exist two kinds of connections corresponding to the induced and intrinsic differentiations respectively. In the present paper we give six relations between six tensors of hypersurface formed by means of the induced connection coefficients and tensors R, P and S (indices suppressed). Two of these relations are known as the equations of *Gauss* and *Codazzi* for hypersurface of F_n; four others are new.

§ 1. Introduction

Consider a Finsler space F_n, where the metric function is given by $\mathscr{L}(x, \dot{x})$. The hypersurface of F_n is determined by the equations

(1.1) $$x^i = x^i(u^\alpha) \quad (i, j, k, \ldots = 1, 2, \ldots, n;\ \alpha, \beta, \gamma, \delta, \ldots, = 1, 2, \ldots, n-1),$$

where the Jacobian Matrix $(X^i_\alpha) = \left(\dfrac{\partial x^i}{\partial u^\alpha}\right)$ is of rank $n-1$. Vectors $X^i_\alpha(u_0)$ considered as vectors of imbedding space are tangent vectors of hypersurface at point $u = u_0$. The metric function $L(u, \dot{u})$ of hypersurface (1.1) is determined by

(1.2) $$L(u, \dot{u}) = \mathscr{L}\left(x(u), \frac{\partial x}{\partial u}\dot{u}\right),$$

because of

(1.3) $$\dot{x}^i = X^i_\alpha \dot{u}^\alpha.$$

The normal vector $\underset{n}{N}^i(u. \dot{u})$ of (1.1) is defined by the equations [2]:

(1.4) $$\overset{n}{N}_i X^i_\alpha = 0, \quad \underset{n}{N}^i = g^{ij}(x, \dot{x}) \overset{n}{N}_j, \quad g_{ij}(x, \dot{x}) \underset{n}{N}^i \underset{n}{N}^j = 1,$$

where $g_{ij}(x, \dot{x})$ is the metric tensor of F_n.

If we express the absolute differentials of X^i_α and $\underset{n}{N}^i$ linearly in terms of X^i_α and $\underset{n}{N}^i$, we get [3]

$$(1.5)\qquad D X^i_\alpha = w^\delta_\alpha X^i_\delta + \theta^n_\alpha \underset{n}{N}^i$$

$$(1.6)\qquad D \underset{n}{N}^i = -\theta^\delta_n X^i_\delta,$$

where w^δ_α and θ^n_α are Pfaffs forms with respect to du^β and $\bar{D}l^\beta$:

$$(1.7)\qquad w^\delta_\alpha = \bar{\Gamma}^{*\delta}_{\alpha\beta}\, du^\beta + \theta^\delta_{\alpha\beta}\, \bar{D} l^\beta$$

$$(1.8)\qquad \theta^n_\alpha = \underset{1}{\bar{\theta}}{}^{*n}_{\alpha\beta}\, du^\beta + \underset{2}{\bar{\theta}}{}^{*n}_{\alpha\beta}\, \bar{D} l^\beta.$$

Here $\bar{D}$ denotes the induced differentiation ([4] p. 159), and $l^\beta = \dfrac{\dot{u}^\beta}{L(u, \dot{u})}$. It is known ([5], [6]), that

$$(1.9)\qquad \bar{\Gamma}^*_{\alpha\gamma\beta} = g_{ir} X^r_\gamma (X^i_{\alpha\beta} + \Gamma^{*i}_{jk} X^{jk}_{\alpha\beta} + A^i_{jk} X^j_\alpha O^n_\beta \underset{n}{N}^k),$$

$$(1.10)\qquad \theta_{\alpha\gamma\beta} = A_{\alpha\gamma\beta},$$

$$(1.11)\qquad \underset{1}{\bar{\theta}}{}^{*n}_{\alpha\beta} = g_{ir} \underset{n}{N}^r (X^i_{\alpha\beta} + \Gamma^{*i}_{jk} X^{jk}_{\alpha\beta} + A^i_{jk} X^j_\alpha O^n_\beta \underset{n}{N}^k),$$

$$(1.12)\qquad \underset{2}{\bar{\theta}}{}^{*n}_{\alpha\beta} = g_{ir} \underset{n}{N}^r A^i_{jk} X^{jk}_{\alpha\beta},$$

where

$$O^n_\beta = \underset{1}{\bar{\theta}}{}^{*n}_{\alpha\beta} l^\alpha = g_{ir} \underset{n}{N}^r (X^i_{\alpha\beta} + \Gamma^{*i}_{jk} X^{jk}_{\alpha\beta}) l^\alpha.$$

§ 2. The exterior differential of vector X^i_α

Let d and δ be two commuting symbols of differentiation and D and Δ the corresponding absolute differentials. Because of (1.5) and (1.6) we have [7]

$$\Delta D X^i_\alpha = \delta w^\delta_\alpha (d) X^i_\delta + w^\delta_\alpha (d) [w^\varepsilon_\delta (\delta) X^i_\varepsilon + \theta^n_\delta (\delta) \underset{n}{N}^i] +$$
$$+ \delta \theta^n_\alpha (d) \underset{n}{N}^i - \theta^n_\alpha (d) \theta^\varepsilon_n (\delta) X^i_\varepsilon,$$

and so

$$(2.1)\qquad (\Delta D - D \Delta) X^i_\alpha = A^\varepsilon_\alpha X^i_\varepsilon + B^n_\alpha \underset{n}{N}^i,$$

where

$$(2.2)\qquad A^\varepsilon_\alpha = w^\delta_\alpha (d)\, w^\varepsilon_\delta (\delta) - w^\delta_\alpha (\delta)\, w^\varepsilon_\delta (d) -$$
$$-[d\, w^\varepsilon_\alpha (\delta) - \delta\, w^\varepsilon_\alpha (d)] - \theta^n_\alpha (d)\, \theta^\varepsilon_n (\delta) + \theta^n_\alpha (\delta)\, \theta^\varepsilon_n (d)$$

and

$$(2.3)\qquad B^n_\alpha = \delta\, \theta^n_\alpha (d) - d\, \theta^n_\alpha (\delta) + w^\delta_\alpha (d)\, \theta^n_\delta (\delta) - w^\delta_\alpha (\delta)\, \theta^n_\delta (d).$$

Theorem 1. *The necessary and sufficient conditions that the exterior differential of tangent vector X^i_α of hypersurface of Finsler space has the direction of the normal vector $\underset{n}{N}^i$ are:*

$$(2.4)\qquad \bar{R}^\varepsilon_{\alpha\beta\gamma} = \underset{1}{\bar{\theta}}{}^{*n}_{\alpha\beta}\, g^{\varepsilon\delta}\, \underset{1}{\bar{\theta}}{}^{*n}_{\delta\gamma} - \underset{1}{\bar{\theta}}{}^{*n}_{\alpha\gamma}\, g^{\varepsilon\delta}\, \underset{1}{\bar{\theta}}{}^{*n}_{\delta\beta},$$

(2.5) $$\overline{P}^{\varepsilon}_{\alpha\beta\gamma} = \underset{1}{\overline{\theta}}{}^{*n}_{\alpha\beta}\, g^{\varepsilon\delta}\, \underset{2}{\overline{\theta}}{}^{*n}_{\delta\gamma} - \underset{2}{\overline{\theta}}{}^{*n}_{\alpha\gamma}\, g^{\varepsilon\delta}\, \underset{1}{\overline{\theta}}{}^{*n}_{\delta\beta},$$

(2.6) $$\overline{S}^{\varepsilon}_{\alpha\beta\gamma} = \underset{2}{\overline{\theta}}{}^{*n}_{\alpha\beta}\, g^{\varepsilon\delta}\, \underset{2}{\overline{\theta}}{}^{*n}_{\delta\gamma} - \underset{2}{\overline{\theta}}{}^{*n}_{\alpha\gamma}\, g^{\varepsilon\delta}\, \underset{2}{\overline{\theta}}{}^{*n}_{\delta\beta},$$

where

(2.7) $$\overline{R}^{\varepsilon}_{\alpha\beta\gamma} = 2\,\overline{\Gamma}{}^{*\delta}_{\alpha[\beta}\,\overline{\Gamma}{}^{*\varepsilon}_{|\delta|\gamma]} + 2\,\partial_{[\gamma}\overline{\Gamma}{}^{*\varepsilon}_{|\alpha|\beta]} - 2\,\partial_{\dot{u}^{\delta}}\overline{\Gamma}{}^{*\varepsilon}_{\alpha[\beta}\,\overline{\Gamma}{}^{*\delta}_{\gamma]} +$$
$$+ 2\,C^{\varepsilon}_{\alpha\delta}\,(\partial_{[\gamma}\overline{\Gamma}{}^{*\delta}_{\beta]} - \partial_{\dot{u}^{\varkappa}}\overline{\Gamma}{}^{*\delta}_{[\beta}\,\overline{\Gamma}{}^{*\varkappa}_{\gamma]}),$$

(2.8) $$\overline{P}^{\varepsilon}_{\alpha\beta\gamma} = L\,\frac{\partial\overline{\Gamma}{}^{*\varepsilon}_{\alpha\beta}}{\partial\dot{u}^{\gamma}} - A^{\varepsilon}_{\alpha\gamma|\overline{\beta}} + A^{\varepsilon}_{\alpha\delta}\,\frac{\partial\overline{\Gamma}{}^{*\delta}_{\varkappa\beta}}{\partial\dot{u}^{\gamma}}\,\dot{u}^{\varkappa},$$

(2.9) $$\overline{S}^{\varepsilon}_{\alpha\beta\gamma} = A^{\delta}_{\alpha\gamma}\,A^{\varepsilon}_{\delta\beta} - A^{\delta}_{\alpha\beta}\,A^{\varepsilon}_{\delta\gamma},$$
$$\overline{\Gamma}{}^{*\delta}_{\beta} = \overline{\Gamma}{}^{*\delta}_{\alpha\beta}\,\dot{u}^{\alpha},$$

and $A^{\varepsilon}_{\alpha\gamma|\overline{\beta}}$ denotes the covariant differential of $A^{\varepsilon}_{\alpha\gamma}$ formed by means of induced connection coefficients (1.9).

P r o o f. The vector $(A\,D - D\,\Delta)\,X^{i}_{\alpha}$ has the direction of the normal vector $\underset{n}{N^{i}}(u, \dot{u})$ if and only if

(2.10) $$A^{\varepsilon}_{\alpha} = 0,$$

where A^{ε}_{α} is given by (2.2). Therefore:

(2.11) $$A^{\varepsilon}_{\alpha} = (\overline{\Gamma}{}^{*\delta}_{\alpha\beta}\,du^{\beta} + A^{\delta}_{\alpha\beta}\,\overline{D}l^{\beta})\,(\overline{\Gamma}{}^{*\varepsilon}_{\delta\gamma}\,\delta u^{\gamma} + A^{\varepsilon}_{\delta\gamma}\,\overline{\Delta}\,l^{\gamma}) -$$
$$-(\overline{\Gamma}{}^{*\delta}_{\alpha\gamma}\,\delta u^{\gamma} + A^{\delta}_{\alpha\gamma}\,\overline{\Delta}\,l^{\gamma})\,(\overline{\Gamma}{}^{*\varepsilon}_{\delta\beta}\,du^{\beta} + A^{\varepsilon}_{\delta\beta}\,\overline{D}\,l^{\beta}) +$$
$$+\left(\frac{\partial\overline{\Gamma}{}^{*\varepsilon}_{\alpha\beta}}{\partial\dot{u}^{\gamma}}\,\delta\dot{u}^{\gamma}\,du^{\beta} + \frac{\partial\overline{\Gamma}{}^{*\varepsilon}_{\alpha\beta}}{\partial u^{\gamma}}\,\delta u^{\gamma}\,du^{\beta} + \overline{\Gamma}{}^{*\varepsilon}_{\alpha\beta}\,\delta\,du^{\beta} +\right.$$
$$+\frac{\partial A^{\varepsilon}_{\alpha\beta}}{\partial\dot{u}^{\gamma}}\,\delta\dot{u}^{\gamma}\,\overline{D}l^{\beta} + \frac{\partial A^{\varepsilon}_{\alpha\beta}}{\partial u^{\gamma}}\,\delta u^{\gamma}\,\overline{D}l^{\beta} + A^{\varepsilon}_{\alpha\beta}\,\delta\,\overline{D}l^{\beta}) -$$
$$-\left(\frac{\partial\overline{\Gamma}{}^{*\varepsilon}_{\alpha\gamma}}{\partial\dot{u}^{\beta}}\,d\dot{u}^{\beta}\,\delta u^{\gamma} + \frac{\partial\overline{\Gamma}{}^{*\varepsilon}_{\alpha\gamma}}{\partial u^{\beta}}\,du^{\beta}\,\delta u^{\gamma} + \overline{\Gamma}{}^{*\varepsilon}_{\alpha\gamma}\,d\,\delta u^{\gamma} +\right.$$
$$\left.+\frac{\partial A^{\varepsilon}_{\alpha\gamma}}{\partial\dot{u}^{\beta}}\,d\dot{u}^{\beta}\,\overline{\Delta}\,l^{\gamma} + \frac{\partial A^{\varepsilon}_{\alpha\gamma}}{\partial u^{\beta}}\,du^{\beta}\,\overline{\Delta}\,l^{\gamma} + A^{\varepsilon}_{\alpha\gamma}\,d\,\overline{\Delta}\,l^{\gamma}\right) -$$
$$-(\underset{1}{\overline{\theta}}{}^{*n}_{\alpha\beta}\,du^{\beta} + \underset{2}{\overline{\theta}}{}^{*n}_{\alpha\beta}\,\overline{D}l^{\beta})\,g^{\varepsilon\delta}\,(\underset{1}{\overline{\theta}}{}^{*n}_{\delta\gamma}\,\delta u^{\gamma} + \underset{2}{\overline{\theta}}{}^{*n}_{\delta\gamma}\,\overline{\Delta}\,l^{\gamma}) +$$
$$+(\underset{1}{\overline{\theta}}{}^{*n}_{\alpha\gamma}\,\delta u^{\gamma} + \underset{2}{\overline{\theta}}{}^{*n}_{\alpha\gamma}\,\overline{\Delta}\,l^{\gamma})\,g^{\varepsilon\delta}\,(\underset{1}{\overline{\theta}}{}^{*n}_{\delta\beta}\,du^{\beta} + \underset{2}{\overline{\theta}}{}^{*n}_{\delta\beta}\,\overline{D}l^{\beta}).$$

We need to calculate $\delta \overline{D} l^\beta$ and $d \overline{\Delta} l^\beta$. Because of

$$\overline{D} l^\beta = dl^\beta + \frac{1}{L} \overline{\Gamma}^{*\beta}_{\gamma} du^\gamma \tag{2.12}$$

and

$$d\dot{u}^\beta = L \overline{D} l^\beta + \dot{u}^\beta \frac{dL}{L} - \overline{\Gamma}^{*\beta}_{\gamma} du^\gamma . \tag{2.13}$$

we have

$$\delta \overline{D} l^\beta = \delta \, dl^\beta + \frac{1}{L} \frac{\partial \overline{\Gamma}^{*\beta}_{\delta}}{\partial u^\gamma} \delta u^\gamma du^\delta + \frac{1}{L} \frac{\partial \overline{\Gamma}^{*\beta}_{\delta}}{\partial \dot{u}^\gamma} du^\delta (L \overline{\Delta} l^\gamma + l^\gamma \overline{\Delta} L - \overline{\Gamma}^{*\gamma}_{\varepsilon} \delta u^\varepsilon) -$$
$$- \frac{1}{L^2} \delta L \, \overline{\Gamma}^{*\beta}_{\gamma} du^\gamma + \frac{1}{L} \overline{\Gamma}^{*\beta}_{\delta} \delta \, du^\delta .$$

Since $\overline{\Gamma}^{*\beta}_{\delta}$ is positively homogeneous of degree one in $\dot{u}$, and since $\overline{\Delta} L = \delta L$, we get for $\delta \overline{D} l^\beta$:

$$\delta \overline{D} l^\beta = \delta \, dl^\beta + \frac{1}{L} \frac{\partial \overline{\Gamma}^{*\beta}_{\delta}}{\partial u^\gamma} \delta u^\gamma du^\delta + \frac{\partial \overline{\Gamma}^{*\beta}_{\delta}}{\partial \dot{u}^\gamma} du^\delta \overline{\Delta} l^\gamma -$$
$$- \frac{1}{L} \frac{\partial \overline{\Gamma}^{*\beta}_{\delta}}{\partial \dot{u}^\gamma} \overline{\Gamma}^{*\gamma}_{\varepsilon} \delta u^\varepsilon du^\delta + \frac{1}{L} \overline{\Gamma}^{*\beta}_{\delta} \delta \, du^\delta . \tag{2.14}$$

Notice that $\overline{\Gamma}^{*\varepsilon}_{\alpha\beta}$ and $A^\varepsilon_{\alpha\beta}$ are positively homogeneous of degree zero in $\dot{u}$:

$$\frac{\partial \overline{\Gamma}^{*\varepsilon}_{\alpha\beta}}{\partial \dot{u}^\gamma} l^\gamma = 0, \quad \frac{\partial A^\varepsilon_{\alpha\beta}}{\partial \dot{u}^\gamma} l^\gamma = 0. \tag{2.15}$$

If we substitute (2.13), (2.14), (2.15) and the similar expression for $\delta \dot{u}^\beta$ and $d \overline{\Delta} l^\beta$ into (2.11), taking into account that d and δ are commuting differential symbols we obtain:

$$A^\varepsilon_\alpha = [2 \overline{\Gamma}^{*\delta}_{\alpha[\beta} \overline{\Gamma}^{*\varepsilon}_{|\delta|\gamma]} + 2 \partial_{[\gamma} \overline{\Gamma}^{*\varepsilon}_{|\alpha|\beta]} - 2 \partial \dot{u}^\delta \overline{\Gamma}^{*\varepsilon}_{\alpha[\beta} \overline{\Gamma}^{*\delta}_{\gamma]} +$$
$$+ 2 C^\varepsilon_{\alpha\delta} (\partial_{[\gamma} \overline{\Gamma}^{*\delta}_{\beta]} - 2 \partial \dot{u}^\varkappa \overline{\Gamma}^{*\delta}_{[\beta} \overline{\Gamma}^{*\varkappa}_{\gamma]}) - \underset{1}{\overline{\theta}}{}^{*n}_{\alpha\beta} g^{\varepsilon\delta} \underset{1}{\overline{\theta}}{}^{*n}_{\delta\gamma} + \underset{1}{\overline{\theta}}{}^{*n}_{\alpha\gamma} g^{\varepsilon\delta} \underset{1}{\overline{\theta}}{}^{*n}_{\delta\beta}] du^\beta \delta u^\gamma +$$
$$+ \Big(\overline{\Gamma}^{*\delta}_{\alpha\beta} A^\varepsilon_{\delta\gamma} - \overline{\Gamma}^{*\varepsilon}_{\delta\beta} A^\delta_{\alpha\gamma} + L \frac{\partial \overline{\Gamma}^{*\varepsilon}_{\alpha\beta}}{\partial \dot{u}^\gamma} - \frac{\partial A^\varepsilon_{\alpha\gamma}}{\partial u^\beta} + \frac{\partial A^\varepsilon_{\alpha\gamma}}{\partial \dot{u}^\delta} \overline{\Gamma}^{*\delta}_{\beta} +$$
$$+ A^\varepsilon_{\alpha\delta} \frac{\partial \overline{\Gamma}^{*\delta}_{\beta}}{\partial \dot{u}^\gamma} - \underset{1}{\overline{\theta}}{}^{*n}_{\alpha\beta} g^{\varepsilon\delta} \underset{2}{\overline{\theta}}{}^{*n}_{\delta\gamma} + \underset{1}{\overline{\theta}}{}^{*n}_{\delta\beta} g^{\varepsilon\delta} \underset{2}{\overline{\theta}}{}^{*n}_{\alpha\gamma} \Big) [du^\delta, \overline{D} l^\gamma] +$$
$$+ \Big(A^\delta_{\alpha\beta} A^\varepsilon_{\delta\gamma} - A^\delta_{\alpha\gamma} A^\varepsilon_{\delta\beta} + L \frac{\partial A^\varepsilon_{\alpha\beta}}{\partial \dot{u}^\gamma} - L \frac{\partial A^\varepsilon_{\alpha\gamma}}{\partial \dot{u}^\beta} -$$
$$- \underset{2}{\overline{\theta}}{}^{*n}_{\alpha\beta} g^{\varepsilon\delta} \underset{2}{\overline{\theta}}{}^{*n}_{\delta\gamma} + \underset{2}{\overline{\theta}}{}^{*n}_{\alpha\gamma} g^{\varepsilon\delta} \underset{2}{\overline{\theta}}{}^{*n}_{\delta\beta} \Big) \overline{D} l^\beta \overline{\Delta} l^\gamma . \tag{2.16}$$

Further, because of

$$l_\beta \overline{D} l^\beta = 0, \quad l_\gamma \overline{\Delta} l^\gamma = 0 \tag{2.17}$$

and

$$\frac{\partial g^{\varepsilon\delta}}{\partial \dot{u}^{\gamma}} = -2\, g^{\varkappa\delta} C^{\varepsilon}_{\varkappa\gamma},$$

we get

(2.18) $$\left(L \frac{\partial A^{\varepsilon}_{\alpha\beta}}{\partial \dot{u}^{\gamma}} - L \frac{\partial A^{\varepsilon}_{\alpha\gamma}}{\partial \dot{u}^{\beta}}\right) \overline{D} l^{\beta}\, \overline{\Delta} l^{\gamma} = -2\, A^{\delta}_{\alpha\beta} A^{\varepsilon}_{\delta\gamma} + 2\, A^{\delta}_{\alpha\gamma} A^{\varepsilon}_{\delta\beta}.$$

The coefficients of $du^{\beta}\,\delta u^{\gamma}$ and $\overline{D} l^{\beta}\,\overline{\Delta}\, l^{\gamma}$ in (2.16) are skew-symmetric in β and γ. In virtue of (2.18) and the last remark, the equation (2.16) becomes:

(2.19) $$A^{\varepsilon}_{\alpha} = \frac{1}{2}\,[\overline{R}^{\varepsilon}_{\alpha\beta\gamma} - (\underset{1}{\overline{\theta}}{}^{*n}_{\alpha\beta}\, g^{\varepsilon\delta}\, \underset{1}{\overline{\theta}}{}^{*n}_{\delta\gamma} - \underset{1}{\overline{\theta}}{}^{*n}_{\alpha\gamma}\, g^{\varepsilon\delta}\, \underset{1}{\overline{\theta}}{}^{*n}_{\delta\beta})]\,[du^{\beta}, du^{\gamma}] +$$

$$+ [\overline{P}^{\varepsilon}_{\alpha\beta\gamma} - (\underset{1}{\overline{\theta}}{}^{*n}_{\alpha\beta}\, g^{\varepsilon\delta}\, \underset{2}{\overline{\theta}}{}^{*n}_{\delta\gamma} - \underset{1}{\overline{\theta}}{}^{*n}_{\delta\beta}\, g^{\varepsilon\delta}\, \underset{2}{\overline{\theta}}{}^{*n}_{\alpha\gamma})]\,[du^{\beta}, \overline{D} l^{\gamma}] +$$

$$+ \frac{1}{2}\,[\overline{S}^{\varepsilon}_{\alpha\beta\gamma} - (\underset{2}{\overline{\theta}}{}^{*n}_{\alpha\beta}\, g^{\varepsilon\delta}\, \underset{2}{\overline{\theta}}{}^{*n}_{\delta\gamma} - \underset{2}{\overline{\theta}}{}^{*n}_{\alpha\gamma}\, g^{\varepsilon\delta}\, \underset{2}{\overline{\theta}}{}^{*n}_{\delta\beta})]\,[\overline{D} l^{\beta}, \overline{D} l^{\gamma}],$$

where $\overline{R}^{\varepsilon}_{\alpha\beta\gamma}$, $\overline{P}^{\varepsilon}_{\alpha\beta\gamma}$ and $\overline{S}^{\varepsilon}_{\alpha\beta\gamma}$ are defined by (2.7), (2.8) and (2.9) respectively. Since the bivectors $[du^{\beta}, du^{\gamma}]$, $[du^{\beta}, \overline{D} l^{\gamma}]$, $[\overline{D} l^{\beta}, \overline{D} l^{\gamma}]$ are chosen arbitrarily, from (2.10) and (2.19) follows (2.4), (2.5) and (2.6), which proves the theorem. We next prove:

Theorem 2. *The necessary and sufficient conditions that the exterior differential of the tangent vector* X^{i}_{α} *of hypersurface of a Finsler space belongs to its tangent hyperplane are:*

(2.20) $$\overline{R}^{n}_{\alpha\beta\gamma} = 0,$$

(2.21) $$\overline{P}^{n}_{\alpha\beta\gamma} - A^{\delta}_{\alpha\gamma}\, \underset{1}{\overline{\theta}}{}^{*n}_{\delta\beta} = 0,$$

(2.22) $$\overline{S}^{n}_{\alpha\beta\gamma} = 0,$$

where we put:

(2.23) $$\overline{R}^{n}_{\alpha\beta\gamma} = 2\, \overline{\Gamma}^{*\delta}_{\alpha[\beta}\, \underset{1}{\overline{\theta}}{}^{*n}_{|\delta|\gamma]} + 2\, \partial_{[\gamma}\, \underset{1}{\overline{\theta}}{}^{*n}_{|\alpha|\beta]} - 2\, \partial \dot{u}^{\delta}\, \underset{1}{\overline{\theta}}{}^{*n}_{\alpha[\beta}\, \overline{\Gamma}^{*\delta}_{\gamma]} +$$

$$+ 2\, \frac{\underset{2}{\overline{\theta}}{}^{*n}_{\alpha\delta}}{L}\, (\partial_{[\gamma}\, \overline{\Gamma}^{*\delta}_{\beta]} - \partial\, \dot{u}^{\varepsilon}\, \overline{\Gamma}^{*\delta}_{[\varepsilon}\, \overline{\Gamma}^{*\varepsilon}_{\gamma]}),$$

(2.24) $$\overline{P}^{n}_{\alpha\beta\gamma} = L\, \frac{\partial\, \underset{1}{\overline{\theta}}{}^{*n}_{\alpha\beta}}{\partial \dot{u}^{\gamma}} - \underset{2}{\overline{\theta}}{}^{*n}_{\alpha\gamma|\beta} + \underset{2}{\overline{\theta}}{}^{*n}_{\alpha\delta}\, \frac{\partial \overline{\Gamma}^{*\delta}_{\varepsilon\beta}}{\partial \dot{u}^{\gamma}}\, \dot{u}^{\varepsilon},$$

(2.25) $$\overline{S}^{n}_{\alpha\beta\gamma} = A^{\delta}_{\alpha\beta}\, \underset{2}{\overline{\theta}}{}^{*n}_{\delta\gamma} - A^{\delta}_{\alpha\gamma}\, \underset{2}{\overline{\theta}}{}^{*n}_{\delta\beta} + L\, \frac{\partial\, \underset{2}{\overline{\theta}}{}^{*n}_{\alpha\beta}}{\partial \dot{u}^{\gamma}} - L\, \frac{\partial\, \underset{2}{\overline{\theta}}{}^{*n}_{\alpha\gamma}}{\partial \dot{u}^{\beta}}.$$

Proof. The exterior differential of X^i_α lies completely in tangent hyperplane of F_{n-1}, if and only if

$$B^n_\alpha = 0, \tag{2.26}$$

where B^n_α is given by (2.3).

Taking into account (2.3), (2.13), (2.14) and (2.15) we get:

$$\begin{aligned}
B^n_\alpha = {} & \frac{\partial \underset{1}{\overline{\theta}}{}^{*n}_{\alpha\beta}}{\partial u^\gamma} du^\beta \delta u^\gamma + \frac{\partial \underset{1}{\overline{\theta}}{}^{*n}_{\alpha\beta}}{\partial \dot{u}^\gamma} du^\beta L \overline{\Delta} l^\gamma - \frac{\partial \underset{1}{\overline{\theta}}{}^{*n}_{\alpha\beta}}{\partial \dot{u}^\delta} \overline{\Gamma}^{*\delta}_\gamma du^\beta \delta u^\gamma + \frac{\partial \underset{1}{\overline{\theta}}{}^{*n}_{\alpha\beta}}{\partial \dot{u}^\gamma} l^\gamma (\overline{\Delta} L) du^\beta + \\
& + \frac{\partial \underset{2}{\overline{\theta}}{}^{*n}_{\alpha\gamma}}{\partial u^\beta} \delta u^\beta \overline{D} l^\gamma + \frac{\partial \underset{2}{\overline{\theta}}{}^{*n}_{\alpha\beta}}{\partial \dot{u}^\gamma} L \overline{\Delta} l^\gamma \overline{D} l^\beta - \frac{\partial \underset{2}{\overline{\theta}}{}^{*n}_{\alpha\gamma}}{\partial \dot{u}^\delta} \overline{\Gamma}^{*\delta}_\beta \delta u^\beta \overline{D} l^\gamma + \frac{\partial \underset{2}{\overline{\theta}}{}^{*n}_{\alpha\beta}}{\partial \dot{u}^\gamma} l^\gamma (\overline{\Delta} L) \overline{D} l^\beta + \\
& + \underset{2}{\overline{\theta}}{}^{*n}_{\alpha\delta} \frac{1}{L} \frac{\partial \overline{\Gamma}^{*\delta}_\beta}{\partial u^\gamma} \delta u^\gamma du^\beta - \underset{2}{\overline{\theta}}{}^{*n}_{\alpha\beta} \frac{1}{L} \frac{\partial \overline{\Gamma}^{*\delta}_\beta}{\partial \dot{u}^\varepsilon} \overline{\Gamma}^{*\varepsilon}_\gamma du^\beta \delta u^\gamma + \underset{2}{\overline{\theta}}{}^{*n}_{\alpha\delta} \frac{\partial \overline{\Gamma}^{*\delta}_\beta}{\partial \dot{u}^\gamma} du^\beta \overline{\Delta} l^\gamma - \\
& - \frac{\partial \underset{1}{\overline{\theta}}{}^{*n}_{\alpha\beta}}{\partial u^\gamma} \delta u^\beta du^\gamma - \frac{\partial \underset{1}{\overline{\theta}}{}^{*n}_{\alpha\beta}}{\partial \dot{u}^\gamma} L \overline{D} l^\gamma \delta u^\beta + \frac{\partial \underset{1}{\overline{\theta}}{}^{*n}_{\alpha\beta}}{\partial \dot{u}^\delta} \overline{\Gamma}^{*\delta}_\gamma \delta u^\beta du^\gamma - \frac{\partial \underset{1}{\overline{\theta}}{}^{*n}_{\alpha\beta}}{\partial \dot{u}^\gamma} l^\gamma \delta u^\beta \overline{D} L - \\
& - \frac{\partial \underset{2}{\overline{\theta}}{}^{*n}_{\alpha\gamma}}{\partial u^\beta} du^\beta \overline{\Delta} l^\gamma - \frac{\partial \underset{2}{\overline{\theta}}{}^{*n}_{\alpha\beta}}{\partial \dot{u}^\gamma} L \overline{D} l^\gamma \overline{\Delta} l^\beta + \frac{\partial \underset{2}{\overline{\theta}}{}^{*n}_{\alpha\gamma}}{\partial \dot{u}^\delta} \overline{\Gamma}^{*\delta}_\beta du^\beta \overline{\Delta} l^\gamma - \frac{\partial \underset{2}{\overline{\theta}}{}^{*n}_{\alpha\beta}}{\partial \dot{u}^\gamma} l^\gamma \overline{\Delta} l^\beta \overline{D} L - \\
& - \underset{2}{\overline{\theta}}{}^{*n}_{\alpha\delta} \frac{1}{L} \frac{\partial \overline{\Gamma}^{*\delta}_\beta}{\partial u^\gamma} du^\gamma \delta u^\beta + \underset{2}{\overline{\theta}}{}^{*n}_{\alpha\delta} \frac{1}{L} \frac{\partial \overline{\Gamma}^{*\delta}_\beta}{\partial \dot{u}^\varepsilon} \overline{\Gamma}^{*\varepsilon}_\gamma \delta u^\beta du^\gamma - \underset{2}{\overline{\theta}}{}^{*n}_{\alpha\delta} \frac{\partial \overline{\Gamma}^{*\delta}_\beta}{\partial \dot{u}^\gamma} \delta u^\beta \overline{D} l^\gamma + \\
& + (\overline{\Gamma}^{*\delta}_{\alpha\beta} du^\beta + A^\delta_{\alpha\beta} \overline{D} l^\beta)(\underset{1}{\overline{\theta}}{}^{*n}_{\delta\gamma} \delta u^\gamma + \underset{2}{\overline{\theta}}{}^{*n}_{\delta\gamma} \overline{\Delta} l^\gamma) - \\
& - (\overline{\Gamma}^{*\delta}_{\alpha\gamma} \delta u^\gamma + A^\delta_{\alpha\gamma} \overline{\Delta} l^\gamma)(\underset{1}{\overline{\theta}}{}^{*n}_{\delta\beta} du^\beta + \underset{2}{\overline{\theta}}{}^{*n}_{\delta\beta} \overline{D} l^\beta).
\end{aligned} \tag{2.27}$$

Because of

$$\frac{\partial \overline{\Gamma}^{*\delta}_\beta}{\partial \dot{u}^\gamma} = \overline{\Gamma}^{*\delta}_{\gamma\beta} + \frac{\partial \overline{\Gamma}^{*\delta}_{\alpha\beta}}{\partial \dot{u}^\gamma} \dot{u}^\alpha, \tag{2.28}$$

taking into account that $\underset{1}{\overline{\theta}}{}^{*n}_{\alpha\beta}$ and $\underset{2}{\overline{\theta}}{}^{*n}_{\alpha\beta}$ are positively homogeneous of degree zero in $\dot{u}^\gamma$, (2.27) takes the form:

$$\begin{aligned}
B^n_\alpha = \Big[& 2 \overline{\Gamma}^{*\delta}_{\alpha[\beta} \underset{1}{\overline{\theta}}{}^{*n}_{|\delta|\gamma]} + 2 \partial_{[\gamma} \underset{1}{\overline{\theta}}{}^{*n}_{|\alpha|\beta]} - 2 \partial \dot{u}^\delta \underset{1}{\overline{\theta}}{}^{*n}_{\alpha[\beta} \overline{\Gamma}^{*\delta}_{\gamma]} + \\
& \frac{2 \underset{2}{\overline{\theta}}{}^{*n}_{\alpha\delta}}{L} (\partial_{[\gamma} \overline{\Gamma}^{*\delta}_{\beta]} - \partial \dot{u}^\varepsilon \overline{\Gamma}^{*\delta}_{[\beta} \overline{\Gamma}^{*\varepsilon}_{\gamma]}) \Big] du^\beta \delta u^\gamma +
\end{aligned} \tag{2.29}$$

$$+\left(L\frac{\partial \overset{1}{\overline{\theta}}{}^{*n}_{\alpha\beta}}{\partial \dot{u}^{\gamma}}+\underset{2}{\overline{\theta}}{}^{*n}_{\alpha\delta}\frac{\partial \overline{\Gamma}{}^{*\delta}_{\varepsilon\beta}}{\partial \dot{u}^{\gamma}}\dot{u}^{\varepsilon}-\underset{2}{\overline{\theta}}{}^{*n}_{\alpha\gamma|\beta}-\underset{1}{\overline{\theta}}{}^{*n}_{\delta\beta}A^{\delta}_{\alpha\gamma}\right)[du^{\beta},\overline{D}l^{\gamma}]+$$

$$+\left(A^{\delta}_{\alpha\beta}\underset{2}{\overline{\theta}}{}^{*n}_{\delta\gamma}-A^{\delta}_{\alpha\gamma}\underset{2}{\overline{\theta}}{}^{*n}_{\delta\beta}+L\frac{\partial \overset{2}{\overline{\theta}}{}^{*n}_{\alpha\beta}}{\partial \dot{u}^{\gamma}}-L\frac{\partial \overset{2}{\overline{\theta}}{}^{*n}_{\alpha\gamma}}{\partial \dot{u}^{\beta}}\right)\overline{D}l^{\beta}\overline{\Delta}l^{\gamma}.$$

In view of (2.7), (2.8), (2.9) and because of

$$\overline{R}{}^{n}_{\alpha\beta\gamma}=-\overline{R}{}^{n}_{\alpha\gamma\beta},\quad \overline{S}{}^{n}_{\alpha\beta\gamma}=-\overline{S}{}^{n}_{\alpha\gamma\beta} \tag{2.30}$$

(2.29) takes the form:

$$B^{n}_{\alpha}=\frac{1}{2}\overline{R}{}^{n}_{\alpha\beta\gamma}[du^{\beta},du^{\gamma}]+\left(\overline{P}{}^{n}_{\alpha\beta\gamma}-\underset{1}{\overline{\theta}}{}^{*n}_{\delta\beta}A^{\delta}_{\alpha\gamma}\right)[du^{\beta},\overline{D}l^{\gamma}]+ \tag{2.31}$$
$$+\frac{1}{2}\overline{S}{}^{n}_{\alpha\beta\gamma}[\overline{D}l^{\beta},\overline{D}l^{\gamma}].$$

Since the bivectors $[du^{\beta}, du^{\gamma}]$, $[du^{\beta}, \overline{D}l^{\gamma}]$, $[\overline{D}l^{\beta}, \overline{D}l^{\gamma}]$ are chosen arbitrarily, from (2.26) and (2.31) follow (2.20), (2.21) and (2.22) which proves the theorem.

§ 3. Generalisations of Gauss-Codazzi equations in a Finsler hypersurface.

If we take X^{i}_{α} as a vector of F_n, then

$$(\Delta D-D\Delta)X^{i}_{\alpha}=\frac{1}{2}R^{i}_{jkh}X^{j}_{\alpha}[dx^{k},dx^{h}]+P^{i}_{jkh}X^{j}_{\alpha}[dx^{k},Dl^{h}]+ \tag{3.1}$$
$$+\frac{1}{2}S^{i}_{jkh}X^{j}_{\alpha}[Dl^{k},Dl^{h}].$$

Because of $dx^{k}=X^{k}_{\beta}du^{\beta}$ and

$$Dl^{i}=\overline{D}l^{\varepsilon}X^{i}_{\varepsilon}+\left(\underset{1}{\overline{\theta}}{}^{*n}_{\varepsilon\beta}du^{\beta}+\underset{2}{\overline{\theta}}{}^{*n}_{\varepsilon\beta}\overline{D}l^{\beta}\right)l^{\varepsilon}\underset{n}{N^{i}}=\overline{D}l^{\varepsilon}X^{i}_{\varepsilon}+O^{n}_{\beta}du^{\beta}\underset{n}{N^{i}}$$

we get:

$$[dx^{k},dx^{h}]=X^{kh}_{\beta\gamma}[du^{\beta},du^{\gamma}], \tag{3.2}$$

$$[dx^{k},Dl^{h}]=X^{k}_{\beta}[du^{\beta},\overline{D}l^{\gamma}X^{h}_{\gamma}+O^{n}_{\gamma}du^{\gamma}\underset{n}{N^{h}}] \tag{3.3}$$

$$[Dl^{k},Dl^{k}]=[\overline{D}l^{\beta}X^{k}_{\beta}+O^{n}_{\beta}du^{\beta}\underset{n}{N^{k}},\ \overline{D}l^{\gamma}X^{h}_{\gamma}+O^{n}_{\gamma}du^{\gamma}\underset{n}{N^{h}}]. \tag{3.4}$$

Substituting (3.2), (3.3) and (3.4) into (3.1) we obtain

$$\begin{aligned}(\Delta D-\Delta D)X^{i}_{\alpha}=&\frac{1}{2}\Big[R^{i}_{jkh}X^{jkh}_{\alpha\beta\gamma}+P^{i}_{jkh}X^{j}_{\alpha}\big(X^{k}_{\beta}O^{n}_{\gamma}\underset{n}{N^{h}}-X^{k}_{\gamma}O^{n}_{\beta}\underset{n}{N^{h}}\big)+\\&+S^{i}_{jkh}X^{j}_{\alpha}O^{n}_{\beta}\underset{n}{N^{k}}O^{n}_{\gamma}\underset{n}{N^{h}}\Big][du^{\beta},du^{\gamma}]+\\&+\big(P^{i}_{jkh}X^{jkh}_{\alpha\beta\gamma}+S^{i}_{jkh}X^{jh}_{\alpha\gamma}O^{n}_{\beta}\underset{n}{N^{k}}\big)[du^{\beta},\overline{D}l^{\gamma}]+\\&+\frac{1}{2}S^{i}_{jkh}X^{jkh}_{\alpha\beta\gamma}[\overline{D}l^{\beta},\overline{D}l^{\gamma}].\end{aligned} \tag{3.5}$$

Substituting the value of $(\Delta D - D\Delta) X^i_\alpha$, A^ε_α and B^n_α respectively from (3.5), (2.19) and (2.31) into (2.1) and by equating the corresponding coefficients of bivectors $[du^\beta, du^\gamma]$, $[du^\beta, \overline{D}l^\gamma]$, $[\overline{D}l^\beta, \overline{D}l^\gamma]$ we get the equations:

$$R^i_{jkh} X^{jkh}_{\alpha\beta\gamma} + P^i_{jkh} X^j_\alpha \left(X^k_\beta \underset{n}{O^n_\gamma} N^h - X^k_\gamma \underset{n}{O^n_\beta} N^h\right) + S^i_{jkh} X^j_\alpha \underset{n}{O^n_\beta} N^k \underset{n}{O^n_\gamma} N^h =$$

$$(3.6) \qquad = [\overline{R}^\varepsilon_{\alpha\beta\gamma} - (\underset{1}{\overline{\theta}^{*n}_{\alpha\beta}} g^{\varepsilon\delta} \underset{1}{\overline{\theta}^{*n}_{\delta\gamma}} - \underset{1}{\overline{\theta}^{*n}_{\alpha\gamma}} g^{\varepsilon\delta} \underset{1}{\overline{\theta}^{*n}_{\delta\beta}})] X^i_\varepsilon + \overline{R}^n_{\alpha\beta\gamma} \underset{n}{N^i},$$

$$P^i_{jkh} X^{jkh}_{\alpha\beta\gamma} + S^i_{jkh} X^{jh}_{\alpha\gamma} \underset{n}{O^n_\beta} N^k =$$

$$(3.7) \qquad = [\overline{P}^\varepsilon_{\alpha\beta\gamma} - (\underset{1}{\overline{\theta}^{*n}_{\alpha\beta}} g^{\varepsilon\delta} \underset{2}{\overline{\theta}^{*n}_{\delta\gamma}} - \underset{1}{\overline{\theta}^{*n}_{\delta\beta}} g^{\varepsilon\delta} \underset{2}{\overline{\theta}^{*n}_{\alpha\gamma}})] X^i_\varepsilon + (\overline{P}^n_{\alpha\beta\gamma} - \underset{1}{\overline{\theta}^{*n}_{\delta\beta}} A^\delta_{\alpha\gamma}) \underset{n}{N^i},$$

$$(3.8) \qquad S^i_{jkh} X^{jkh}_{\alpha\beta\gamma} = [\overline{S}^\varepsilon_{\alpha\beta\gamma} - (\underset{2}{\overline{\theta}^{*n}_{\alpha\beta}} g^{\varepsilon\delta} \underset{1}{\overline{\theta}^{*n}_{\delta\gamma}} - \underset{2}{\overline{\theta}^{*n}_{\alpha\gamma}} g^{\varepsilon\delta} \underset{2}{\overline{\theta}^{*n}_{\delta\beta}})] X^i_\varepsilon + \overline{S}^n_{\alpha\beta\gamma} \underset{n}{N^i}.$$

We next *prove the formulas of Gauss for hypersurface of Finsler space, giving relations between tensors* $\overline{R}^\varepsilon_{\alpha\beta\gamma}$, $\overline{P}^\varepsilon_{\alpha\beta\gamma}$ *and* $\overline{S}^\varepsilon_{\alpha\beta\gamma}$ *defined by* (2.7), (2.8), (2.9) *and corresponding tensors R, P and S (indices suppressed) of the imbedding space* F_n. *Namely we prove.*

Theorem 3:

$$(3.9) \qquad \overline{R}_{\alpha\delta\beta\gamma} - (\underset{1}{\overline{\theta}^{*n}_{\alpha\beta}} \underset{1}{\overline{\theta}^{*n}_{\delta\gamma}} - \underset{1}{\overline{\theta}^{*n}_{\alpha\gamma}} \underset{1}{\overline{\theta}^{*n}_{\delta\beta}}) = R_{jrkh} X^{jrkh}_{\alpha\delta\beta\gamma} + P_{rjkh} X^{jr}_{\alpha\delta} (X^k_\beta O^n_\gamma - X^k_\gamma O^n_\beta) \underset{n}{N^h} + S_{jrkh} X^{jr}_{\alpha\delta} O^n_\beta \underset{n}{N^k} O^n_\gamma \underset{n}{N^h},$$

$$(3.10) \qquad \overline{P}_{\alpha\delta\beta\gamma} - (\underset{1}{\overline{\theta}^{*n}_{\alpha\beta}} \underset{2}{\overline{\theta}^{*n}_{\delta\gamma}} - \underset{2}{\overline{\theta}^{*n}_{\alpha\gamma}} \underset{1}{\overline{\theta}^{*n}_{\delta\beta}}) = P_{jrkh} X^{jrkh}_{\alpha\delta\beta\gamma} + S_{jrkh} X^{jrh}_{\alpha\delta\gamma} O^n_\beta \underset{n}{N^k},$$

$$(3.11) \qquad \overline{S}_{\alpha\delta\beta\gamma} - (\underset{2}{\overline{\theta}^{*n}_{\alpha\beta}} \underset{2}{\overline{\theta}^{*n}_{\delta\gamma}} - \underset{2}{\overline{\theta}^{*n}_{\alpha\gamma}} \underset{2}{\overline{\theta}^{*n}_{\delta\beta}}) = S_{jrkh} X^{jrkh}_{\alpha\delta\beta\gamma}.$$

Proof. The above equations follow directly from (3.6), (3.7) and (3.8) if they are multiplied by $g_{ir} X^r_\delta$.

If the imbedding space is reduced to an n dimensional Riemann space, and the hypersurface of Finsler space to an $n-1$ dimensional Riemann space, then $P_{jrkh}=0$, $S_{jrkh}=0$, $\underset{2}{\overline{\theta}^{*n}_{\alpha\beta}}=0$ and equations (3.9), (3.10) and (3.11) reduce to the form:

$$(3.12) \qquad \overline{R}_{\alpha\delta\beta\gamma} - (\underset{1}{\overline{\theta}^{*n}_{\alpha\beta}} \underset{1}{\overline{\theta}^{*n}_{\delta\gamma}} - \underset{1}{\overline{\theta}^{*n}_{\alpha\gamma}} \underset{1}{\overline{\theta}^{*n}_{\delta\beta}}) = R_{jrkh} X^{jrkh}_{\alpha\delta\beta\gamma}$$

$$(3.13) \qquad \overline{P}_{\alpha\delta\beta\gamma} = 0$$

$$(3.14) \qquad \overline{S}_{\alpha\delta\beta\gamma} = 0$$

Equation (3.12) is that of [8] p. 242 (formula (4.5)) which is the Gauss equation for hypersurface of Riemann space, since $\overset{*}{\underset{1}{\bar\theta}}{}^{n}_{\alpha\beta}$ corresponds to h_{ab} from [8].

The equations of Codazzi for hypersurface of Finsler space give the relations between tensors $\bar R^{n}_{\alpha\beta\gamma}$, $\bar P^{n}_{\alpha\beta\gamma}$, $\bar S^{n}_{\alpha\beta\gamma}$ defined by (2.23), (2.24), (2.25) *and corresponding tensors R, P, S of imbedding space F_n and have the form, as it is shown in the following.*

Teorem 4:

$$\bar R^{n}_{\alpha\beta\gamma}=R_{jrkh}X^{jkh}_{\alpha\beta\gamma}\underset{n}{N^r}+P_{jrkh}X^{j}_{\alpha}\underset{n}{N^r}\left(X^{k}_{\beta}\underset{n}{O^{n}_{\gamma}}N^h-X^{k}_{\gamma}\underset{n}{O^{n}_{\beta}}N^h\right)+S_{jrkh}X^{j}_{\alpha}\underset{n}{N^r}\underset{n}{O^{n}_{\beta}N^k}\underset{n}{O^{n}_{\gamma}N^h} \tag{3.15}$$

$$\bar P^{n}_{\alpha\beta\gamma}-\overset{*}{\underset{1}{\bar\theta}}{}^{n}_{\delta\beta}A^{\delta}_{\alpha\gamma}=P_{jrkh}X^{jkh}_{\alpha\beta\gamma}\underset{n}{N^r}+S_{jrkh}X^{j}_{\alpha}\underset{n}{N^r}\underset{n}{O^{n}_{\beta}N^k}X^{h}_{\gamma} \tag{3.16}$$

$$\bar S^{n}_{\alpha\beta\gamma}=S_{jrkh}X^{jkh}_{\alpha\beta\gamma}\underset{n}{N^r} \tag{3.17}$$

Proof. The above equations follow directly from (3.6), (3.7) and (3.8) if they are multiplied by $g_{ir}\underset{n}{N^r}$.

If the imbedding space F_n is reduced to an n dimensional Riemann space in which $P_{ijkh}=0$, $S_{ijkh}=0$, and hypersurface of F_n is reduced to an $n-1$ dimension Riemann space in which $\overset{*}{\underset{2}{\bar\theta}}{}^{n}_{\alpha\beta}=0$, $A^{\gamma}_{\alpha\beta}=0$, then equations (3.15) (3.16) and (3.17) are reduced to the form:

$$\bar R^{n}_{\alpha\beta\gamma}=R_{jrkh}X^{jkh}_{\alpha\beta\gamma}\underset{n}{N^r} \tag{3.18}$$

$$\bar P^{n}_{\alpha\beta\gamma}=0 \tag{3.19}$$

$$\bar S^{n}_{\alpha\beta\gamma}=0 \tag{3.20}$$

Equation (3.18) is equivalent to Codazzi's formula (4.6) in [8] (p. 242) $(2\,'\nabla_d h_{cb}=B^{\nu\mu\lambda}_{dcb}K_{\nu\mu\lambda\varkappa}n^{\varkappa})$ for hypersurface od Riemann space, since $\overset{*}{\underset{1}{\bar\theta}}{}^{n}_{\alpha\beta}$ corresponds to tensor h_{cb} and in Riemann space $\overset{*}{\underset{2}{\bar\theta}}{}^{n}_{\alpha\beta}=0$, $\partial\dot u^{\delta}\overset{*}{\underset{1}{\bar\theta}}{}^{n}_{\alpha\beta}=0$, so that $\bar R^{n}_{\alpha\beta\gamma}$ defined by (2.23) corresponds to $2\,'\nabla_{[d}h_{c]b}$.

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