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ON MHD FLOW OF A VISCOUS FLUID PAST AN INFINITE PLATE WITH TIME-DEPENDENT SUCTION (I)

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Abstract

An analysis of a two-dimensional unsteady flow of an incompressible, viscous, electrically conducting fluid along an infinite flat plate with time-dependent suction, is presented. The effects of K (the magnetic field parameter), λ (frequency parameter), P (Prandtl number) on the functions affecting the mean velocity, mean wall shear stress and the mean temperature in the boundary layer are discussed. It is observed that the term of $0(\delta^2)$, where δ is the non-dimensional amplitude of the suction velocity assumed $\ll 1$, affects the mean shear stress which was observed to be absent in non-magnetic case.

1. Introduction

Lighthill [1] has studied the effect of fluctuations in free stream velocity on the skin-friction and heat transfer in a flow past two-dimensional bodies. Stuart [2] later on discussed the oscillatory flow over an infinite flat plate with constant suction and no heat transfer between the fluid and the plate. Stuart's problem was studied under magnetic field by Suryaprakasrao [3] and under slip flow boundary conditions by Reddy [4]. Recently Stuart's problem was extended to variable suction by Messiha [5] whereas Soundalgekar [6], following Messiha, studied Reddy's problem under variable suction. The mhd aspect of refs. [4, 6] was also discussed recently by Soundalgekar [7, 8] and that of ref. [5] was discussed by Pop [9] Soundalgekar [10] independently. Stuart's and Messiha's problems were also studied in case of elastico-viscous fluids by Kaloni [11] and Soundalgekar and Puri [12] respectively. In all these refs. [2—12] the attention was stressed on the unsteady flow superimposed on the mean flow. But, how the mean flow is affected by the superposition of oscillatory flow and variable suction, is also quite interesting. This problem was discussed in an interesting manner by Kelly [13] under time-dependent suction. The effects of slip-boundary conditions on Kelly's problem was recently studied by Soundalgekar [14].

It is interesting to study the effects of the transverse magnetic field on the mean flow superimposed by the oscillatory flow, past an infinite plate with time-dependent suction. This indeed is the motivation of the present investigation. The heat transfer aspect of the mean flow has not been discussed by Kelly. Hence in the present investigation, this aspect is also studied and during the course of discussion both non-magnetic and magnetic cases are compared.

In Section 2, the problem is posed mathematically. The velocity and temperature in the boundary layer are expressed as a Fourier expansion consisting of steady and unsteady parts. Two separate cases viz. (1) constant free stream velocity and (2) periodic free stream velocity are discussed and solutions are derived for velocity and temperature in the mean flow to $0(\delta^2)$, by perturbation method, where δ is the non-dimensional amplitude of the suction velocity assumed to be $\ll 1$. It is observed that, unlike the non-magnetic case, the mean value of the wall shear stress is affected, by the term of $0(\delta^2)$, by the oscillatory flow under transverse magnetic field and with time-dependent suction.

2. Mathematical analysis

(a) Velocity field: Here two-dimensional unsteady flow of an electrically conducting fluid along an infinite flat plate is assumed. The x-axis is chosen along the plate in the direction of flow and y-axis taken normal to it. u and v are the velocity components in the x and y directions respectively. A magnetic field of constant strength B_0 is supposed to be applied parallel to y-direction. As the plate is infinite in extent, the physical variables depend upon y and t only. Hence the equation of continuity is:

(1) $$\frac{\partial v}{\partial y}=0.$$

On neglecting the induced magnetic field and the electric field, under the assumption of small magnetic Reynolds number, the momentum equation governing the flow is:

(2) $$\frac{\partial u}{\partial t}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B_0^2 u}{\rho}$$

(3) $$\frac{\partial v}{\partial t}=-\frac{1}{\rho}\frac{\partial p}{\partial y}.$$

Eqn. (1) shows that the suction velocity v is a function of time only and hence following Kelly [13], we assume it to be given as:

(4) $$v(t)=v_0[1+\delta(e^{i\omega t}+e^{-i\omega t})]$$

where δ is the amplitude of the unsteady suction velocity. Also from (2), we have for the main-stream velocity U

(5) $$\frac{\partial U}{\partial t}=-\frac{1}{\rho}\frac{\partial p}{\partial x}-\frac{\sigma B_0^2 U}{\rho}.$$

Substituting for v from (4), and for $-\frac{1}{\rho}\frac{\partial p}{\partial x}$ from (5) in eqn. (2), we obtain

(6) $$\frac{\partial u}{\partial t}+v_0[1+\delta(e^{i\omega t}+e^{-i\omega t})]\frac{\partial u}{\partial y}=\frac{dU}{dt}+ \\ +\nu\frac{\partial^2 u}{\partial y^2}+\frac{\sigma B_0^2}{\rho}(U-u).$$

The boundary conditions are

(7) $$u(0, t)=0,\ u(\infty, t)=U_\infty(t).$$

We assume here that both $v(t)$ and $u(t)$ are taken to be slowly varying functions thus enabling the flow to remain laminar. As we are interested in understanding the nature of the mean flow in the boundary layer under the influence of the time-dependent suction and the magnetic field when the free stream is either constant or fluctuating, we assume the velocity field to be represented by

$$u(y,\ t)=u_0(y)+\sum_{n=1}^{\infty} u_n(y)\, e^{in\omega t}+\sum_{n=1}^{\infty}\overline{u_n(y)}\, e^{-in\omega t} \tag{8}$$

where $\overline{u_n(y)}$ is the complex conjugate of $u_n(y)$. We now consider the free-stream velocity to be (*i*) constant and (*ii*) varying.

Case I: Constant free stream velocity

$$U(t)=U_0. \tag{9}$$

We now substitute eqns. (8) and (9) in (6) and (7), equate like terms on both sides and obtain the following sets of equations:

$$\begin{cases} v_0\dfrac{du_0}{dy}+v_0\,\delta\left(\dfrac{du_1}{dy}+\dfrac{d\tilde{u}_1}{dy}\right)=\nu\dfrac{d^2u_0}{dy^2}+\dfrac{\sigma B_0^2}{\rho}(U_0-u_0)\\ u_0(0)=0,\ u_0(\infty)=U_0 \end{cases} \tag{10}$$

$$\begin{cases} i\omega u_1+v_0\dfrac{du_1}{dy}+v_0\,\delta\left(\dfrac{du_0}{dy}+\dfrac{du_2}{dy}\right)=\nu\dfrac{d^2u_1}{dy^2}-\dfrac{\sigma B_0^2u_1}{\rho}\\ u_1(0)=0,\ u_1(\infty)=0 \end{cases} \tag{11}$$

$$\begin{cases} in\omega u_n+v_0\dfrac{du_n}{dy}+v_0\,\delta\left(\dfrac{du_{n-1}}{dy}+\dfrac{du_{n+1}}{dy}\right)=\nu\dfrac{d^2u_n}{dy^2}-\dfrac{\sigma B_0^2}{\rho}u_n\\ u_n(0)=0,\ u_n(\infty)=0 \end{cases} \tag{12}$$

and similar equations for u_1 and $\tilde{u}_n$.

It can be observed from (10) that the mean velocity is affected by the oscillations in u_1 and $\tilde{u}_1$.

Introducing the following non-dimensional quantities

$$\eta=\frac{|v_0|}{\nu}y,\ \Phi_n=u_n/U_0,\ \lambda=\nu\omega/|v_0|^2,\ K=\frac{\sigma B_0^2\nu}{\rho|v_0|^2} \tag{13}$$

eqns. (10) and (12) reduce to the following:—

$$\begin{cases} \dfrac{v_0}{|v_0|}\Phi_0'+\dfrac{v_0\delta}{|v_0|}(\Phi_1'+\tilde{\Phi}_1')=\Phi_0''+K(1-\Phi_0)\\ \Phi_0(0)=0,\ \Phi_0(\infty)=1 \end{cases} \tag{14}$$

$$\begin{cases} in\lambda\,\Phi_n+\dfrac{v_0}{|v_0|}\Phi_n'+\dfrac{v_0\delta}{|v_0|}(\Phi_{n-1}'+\Phi_{n+1}')=\Phi_n''-K\Phi_n\\ \Phi_n(0)=0,\ \Phi_n(\infty)=0 \end{cases} \tag{15}$$

where primes henceforth denote differentiation with respect to η. In order to solve these coupled equations, we expand Φ_n's in powers of δ where $\delta \ll 1$. Hence substituting for Φ_n's the following:

$$\Phi_n(\eta) = \sum_{j=0}^{\infty} \Phi_{nj}(\eta)\,\delta^i \tag{16}$$

we obtain from (14) and (16), the eqn. for $\Phi_{00}(\eta)$ as follows:

$$\begin{cases} \Phi_{00}'' - \dfrac{v_0}{|v_0|}\,\Phi_{00}' - K\Phi_{00} = -K \\ \Phi_{00}(0) = 0,\ \Phi_{00}(\infty) = 1. \end{cases} \tag{17}$$

The solution to (17) exists only when $v_0 = -|v_0|$ i.e. for the case of suction. Hence its solution is

$$\Phi_{00}(\eta) = 1 - e^{-n\eta} \tag{18}$$

where

$$n = \frac{1}{2}\,[1 + (1 + 4K)^{1/2}].$$

The eqn. for $\Phi_{10}(\eta)$ is

$$\begin{cases} \Phi_{10}'' + \Phi_{10}' - (K + i\lambda)\,\Phi_{10} = 0 \\ \Phi_{10}(0) = 0,\ \Phi_{10}(\infty) = 0. \end{cases} \tag{19}$$

This leads to $\Phi_{10}(\eta) = 0$. Hence, we can also show that $\Phi_{n0}(\eta) \equiv 0$ for $n \geqslant 1$. Also $\Phi_{10}(\eta) = 0$ implies that $\Phi_{01}(\eta) = 0$.

The equation for $\Phi_{11}(\eta)$ is:

$$\begin{cases} \Phi_{11}'' + \Phi_{11}' - (K + i\lambda)\,\Phi_{11} = -n\,e^{-n\eta} \\ \Phi_{11}(0) = 0,\ \Phi_{11}(\infty) = 0. \end{cases} \tag{20}$$

This system leads to the solution for $\Phi_{11}(\eta)$ as:

$$\Phi_{11}(\eta) = \frac{n}{K + i\lambda}\,(e^{-n\eta} - e^{-h\eta}) \tag{21}$$

where

$$h = h_r + ih_i = \frac{1}{2}\,[1 + (1 + 4K + 4i\lambda)^{1/2}] \tag{22}$$

h_r and h_i are tabulated for different values of λ and K in Suryaprakasrao [3]. The equation for $\Phi_{02}(\eta)$ is

$$\begin{cases} \Phi_{02}'' + \Phi_{02}' - K\Phi_{02} = \dfrac{n}{K^2 + \lambda^2}\,[-nKe^{-n\eta} + \\ \qquad + (\lambda h_1 + Kh_r)\,e^{-h_r\eta}\cos h_1\eta + \\ \qquad + (Kh_i - \lambda h_r)\,e^{-h_r\eta}\sin h_i\eta] \\ \Phi_{02}(0) = 0,\ \Phi_{02}(\infty) = 0 \end{cases} \tag{23}$$

The solution for $\Phi_{02}(\eta)$ is

$$\Phi_{02}(\eta)=\frac{n}{K^2+\lambda^2}\Bigg[\left\{\frac{(Kh_i-\lambda h_r)\,Y-(\lambda h_i+Kh_r)\,X}{X^2+Y^2}-\frac{nK\eta}{1-2n}\right\}e^{-n\eta}+ \tag{24}$$
$$+\frac{\{(\lambda h_i+Kh_r)(X\cos h_i\eta+Y\sin h_i\eta)+(Kh_i-\lambda h_r)(X\sin h_i\eta-Y\cos h_i\eta)\}\,e^{-h_r\eta}}{X^2+Y^2}\Bigg]$$

where

$$X=h_r^2-h_i^2-h_r-K,\quad Y=h_i(1-2h_r)$$
$$h_r=\frac{1}{2}+\frac{1}{2}\left[\frac{1}{2}\left\{(1+4K)^2+64\lambda^2\right\}^{1/2}+\frac{1+4K}{2}\right]^{1/2}$$
$$h_i=\frac{1}{2}\left[\frac{1}{2}\left\{(1+4K)^2+64\lambda^2\right\}-\frac{1+4K}{2}\right]^{1/2}.$$

From (24), we have,

$$\Phi_{02}'(0)=\frac{n}{K^2+\lambda^2}\Bigg[n\,\frac{X(\lambda h_i-Kh_r)-Y(Kh_i-\lambda h_r)}{X^2+Y^2}-\frac{nK}{1+2n}+ \tag{25}$$
$$+\frac{(\lambda h_i+Kh_r)(Xh_i-Xh_r)+(Kh_i-\lambda h_r)(Xh_i+Yh_r)}{X^2+Y^2}\Bigg].$$

Hence the mean profile to $0(\delta^2)$ can now be expressed as

$$\Phi_0(\lambda,\ \eta,\ K)=1-e^{-n\eta}+\delta^2\,\Phi_{02}(\eta). \tag{26}$$

In the absence of the magnetic field, Kelly (13) has observed that $\Phi_{02}'(0)=0$ i.e. the mean value of the wall shearing stress is not affected by terms to $0(\delta^2)$. But this is not true in the present case for $\Phi_{02}'(0)\neq 0$ from (24). Hence un-

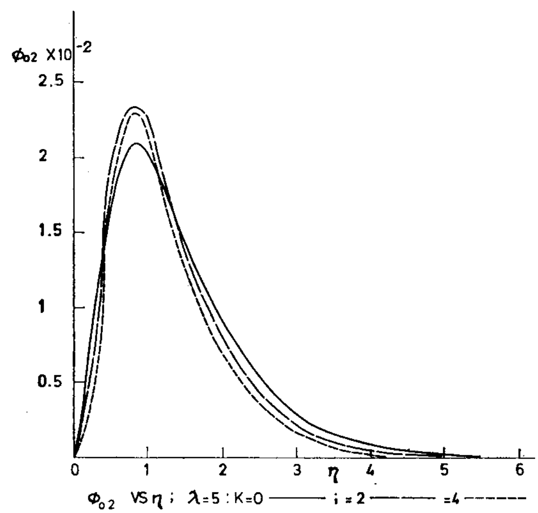

Fig. 1

der the transverse magnetic field, the mean value of the wall shearing stress is affected by λ and K. The function $\Phi_{02}(\eta)$ is shown in Figs. 1 and 2 for different values of K and λ, whereas the numerical values of $\Phi'_{02}(0)$ are entered in Table I.

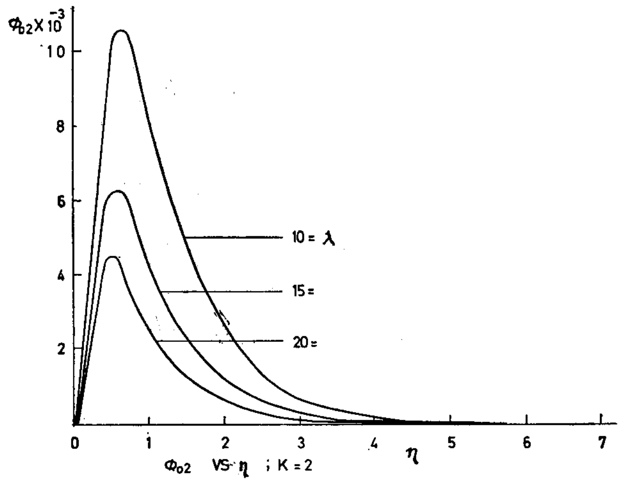

Fig. 2

TABLE I

$\Phi'_{02}(0)$

K\λ	0	5	10	15	20
0	0	0	0	0	0
2	2.1547	−0,0179	−0.0069	−0.0038	−0.0025
4	1.1708	−0.0333	−0,0135	−0.0076	−0.0051
6	1.6203	−0.4501	−0.0195	−0.0112	−0.0075
			$\Phi'_{01,1}(0)$		
0	0	2	2	2	2
2	0	1.8423	1.8864	1.9066	1.9189
4	0	1.7403	1.8105	1.8437	1.8639
6	0	1.6626	1.7509	1.7936	1.8199
			$\Phi'_{01,2}(0)$		
0	0	0	0	0	0
2	0	−0.1065	−0.0862	−0.0745	−0.0667
4	0	−0.1559	−0.1325	−0.1168	−0.1058
6	0	−0.1837	−0.1629	−0.1461	−0.1336

Case II: Periodic free stream

We now study the case of free stream velocity oscillating with the same frequency as that of the suction-velocity given in (4), but with an arbitrary phase angle α. Thus let

$$U_\infty = U_0 [1 + 2\varepsilon \cos(\omega t + \alpha)] \\ = U_0 [1 + \varepsilon_1 e^{i\omega t} + \tilde{\varepsilon}_1 e^{-i\omega t}] \tag{27}$$

where

$$\varepsilon_1 = \varepsilon e^{i\alpha}. \tag{28}$$

We still assume the velocity field to be given by (8). Then substituting (8), (27), (28) in (6) and (7), equating like terms on both sides, we have in view of (14), the equation for $\Phi_1(\eta)$ as

$$\begin{cases} i\lambda \Phi_1 - \Phi_1' - \delta(\Phi_0' + \Phi_2') = i\lambda\varepsilon_1 + \Phi_1'' - K\Phi_1 \\ \Phi_1(0) = 0, \ \Phi_1(\infty) = \varepsilon_1 \end{cases} \tag{29}$$

with a corresponding set for $\tilde{\Phi}_1(\eta)$. Hence the equation for $\Phi_{10}(\eta)$ for suction, is now given by

$$\begin{cases} \Phi_{10}'' \Phi_{10}' - (K + i\lambda)\Phi_{10} = i\lambda\varepsilon_1 \\ \Phi_{10}(0) = 0, \ \Phi_{10}(\infty) = \varepsilon_1. \end{cases} \tag{30}$$

The solution to this system in (30) can be obtained as:

$$\Phi_{10}(\eta) = \varepsilon_1 (1 - e^{-h\eta}) \tag{31}$$

where

$$h = \frac{1}{2}(1 + (1 + 4K + 4i\lambda)^{1/2}).$$

As $e^{2i\omega t}$ is absent in (27), we conclude that $\Phi_{20}(\eta) = 0$. Hence $\Phi_{11}(\eta)$, $\Phi_{02}(\eta)$ remain the same. The non-zero value of $\Phi_{10}(\eta)$ shows that $\Phi_{01}(\eta)$ also exists which is determined by

$$\begin{cases} \Phi_{01}'' + \Phi_{01}' - K\Phi_{01} = \dfrac{d}{d\eta}(\Phi_{10} + \tilde{\Phi}_{10}) \\ \Phi_{01}(0) = 0, \ \Phi_{01}(\infty) = 0 \end{cases} \tag{32}$$

If we assume

$$\begin{cases} \varepsilon_1 = \varepsilon_{ir} + i\varepsilon_{1i} \\ \Phi_{01}(\eta) = \varepsilon_{1r}\Phi_{01,1}(\eta) + \varepsilon_{1i}\Phi_{01,2}(\eta) \end{cases} \quad \text{and} \tag{33}$$

then in view of (31), (32) and (33), the equations for $\Phi_{01,1}(\eta)$ and $\Phi_{01,2}(\eta)$ are

$$\begin{cases} \Phi_{01,1}'' \Phi_{01,1}' - K\Phi_{01,1} = 2(h_r \cos h_i\eta + h_i \sin h_i\eta)\, e^{-h_r\eta} \\ \Phi_{01,1}(0) = 0, \ \Phi_{01,1}(\infty) = 0 \end{cases} \tag{34}$$

and

$$\begin{cases} \Phi_{01,2}'' + \Phi_{01,2}' - K\Phi_{01,2} = 2(h_i \cos h_i\eta - h_r \sin h_i\eta)\, e^{-h_r\eta} \\ \Phi_{01,2}(0) = 0 \quad \Phi_{01,2}(\infty) = 0 \end{cases} \tag{35}$$

The solutions to systems (34) and (35) are

$$\Phi_{01,1}(\eta) = 2\,e^{-n\eta}\left[\frac{h_r X - h_i Y}{X^2+Y^2}\right] - \tag{36}$$
$$-2\left[\frac{(h_r X - h_i Y)\cos h_i \eta + (h_r Y + h_i X)\sin h_i \eta}{X^2+Y^2}\right] e^{-h_r \eta}$$

$$\Phi_{01,2}(\eta) = -2\,e^{-n\eta}\left[\frac{h_i X + h_r Y}{X^2+Y^2}\right] + \tag{37}$$
$$+2\left[\frac{(h_i X + h_r Y)\cos h_i \eta + (h_i Y - h_r X)\sin h_i \eta}{X^2+Y^2}\right] e^{-h_r \eta}.$$

In non-magnetic case, it has been observed by Kelly [13] that $\Phi_{01,2}(\eta)$ is multiple of $\Phi_{02}(\eta)$ which is not the case under the action of the transverse magnetic field. Thus $\varepsilon_r \Phi_{01,1}(\eta)$ is the solution when the free stream velocity is either in phase or directly out of phase with the suction velocity, whereas $\varepsilon_{1i}\Phi_{01,2}(\eta)$ is the solution when the free stream velocity is 90° out of phase with the suction velocity. Hence the solution for the mean velocity profile is now given, to $0(\delta^2)$, as:

$$\Phi_0(\eta) = 1 - e^{-n\eta} + \delta(\varepsilon_{1r}\Phi_{01,1} + \varepsilon_{1i}\Phi_{01,2}) + \delta^2 \Phi_{02}. \tag{38}$$

As the functions affecting the mean-velocity are independent of each other, the wall shearing stress is also affected by them. It is given by

$$\tau_w = \mu \frac{du_0}{dy}\bigg|_{y=0} = \frac{\mu U_0 |v_0|}{\nu}\left[n + \delta\left(\varepsilon_{1r}\Phi'_{01,1} + \varepsilon_{1i}\Phi'_{01,2}\right) + \delta^2 \Phi'_{02}\right]. \tag{39}$$

Hence from (36) and (37), we get

$$\Phi'_{01,1}(0) = \frac{2}{X^2+Y^2}\left[(h_r - n)(h_r X - h_i Y) - h_i(h_i X + h_r Y)\right] \tag{40}$$

$$\Phi'_{01,2}(0) = -\frac{2}{X^2+Y^2}\left[(h_r - n)(h_i X + h_r Y) + h_i(h_r X - h_i Y)\right]. \tag{41}$$

The functions $\Phi_{01,1}(\eta)$ and $\Phi_{01,2}(\eta)$ are shown on Figs. 3—6 and the numerical values of $\Phi'_{01,1}(0)$ and $\Phi'_{01,2}(0)$ are entered in Table I.

b) **The energy equation:** The equation of energy, including viscous dissipation and neglecting Joule dissipation, is given in the present case as

$$\frac{\partial T}{\partial t} + v\frac{\partial T}{\partial y} = \frac{K}{\rho c_p}\frac{\partial^2 T}{\partial y^2} + \frac{\nu}{c_p}\left(\frac{\partial u}{\partial y}\right)^2. \tag{42}$$

The term representing the Joule dissipation can be neglected on the assumption that the induced magnetic field is negligible.

If we now assume that in the boundary layer, the mean temperature $T_0(y)$ is being superimposed by the unsteady temperature, then the temperature can be represented as follows:

$$T = T_0(y) + \sum_{n=1}^{\infty} T_n(y)\, e^{in\omega t} \sum_{n=1}^{\infty} \widetilde{T_n(y)}\, e^{-in\omega t} \tag{43}$$

where „——" denotes the complex conjugate. We also assume that the plate is thermally insulated and the free stream temperature is zero. Then the boundary conditions for the present case are

$$\frac{dT}{dy}=0 \text{ at } y=0 \text{ and } T\to 0 \text{ as } y\to\infty \tag{44}$$

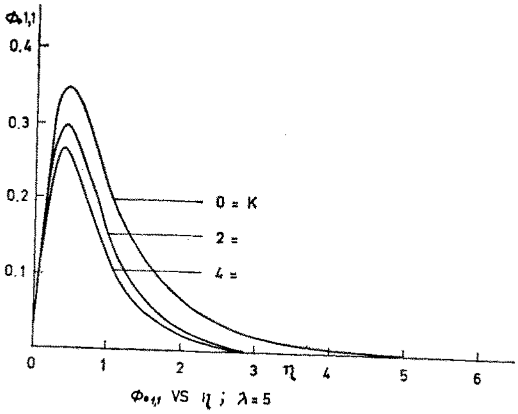

Fig. 3

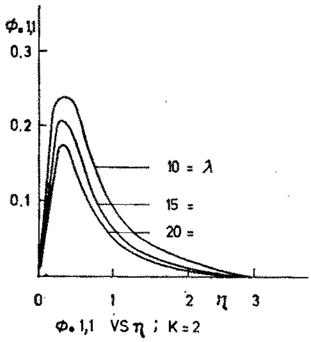

Fig. 4

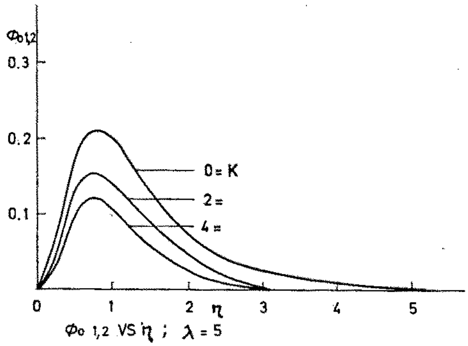

Fig. 5.

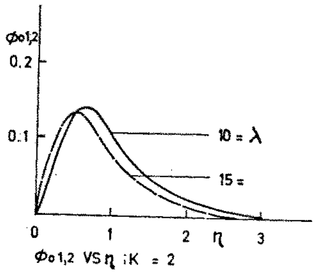

Fig. 6

Substituting (8), (43) in (42) and (44), on equating the like terms, we have the following:

$$\begin{cases} v_0\dfrac{dT_0}{dy}=\dfrac{K}{\rho\, c_p}\dfrac{d^2 T}{dy^2}+\dfrac{\nu}{c_p}\left[\left(\dfrac{du_0}{dy}\right)^2+2\dfrac{du_1}{dy}\dfrac{d\bar{u}_1}{dy}\right] \\ \dfrac{dT_0}{dy}\bigg|_{y=0}=0 \text{ and } T_0(\infty)=0 \end{cases} \tag{45}$$

and

$$(46)\qquad \begin{cases} \text{in}\,\omega\, T_n + v_0 \dfrac{dT_n}{dy} + v_0 \delta \left(\dfrac{dT_{n-1}}{dy} + \dfrac{dT_{n+1}}{dy} \right) = \\ \qquad = \dfrac{K}{\rho c_p} \dfrac{d^2 T_n}{dy^2} + \dfrac{2\nu}{c_p} \left[\dfrac{du_0}{dy} \dfrac{du_n}{dy} + \dfrac{d\bar{u}_1}{dy} \dfrac{du_{n+1}}{dy} \right] \\ \left.\dfrac{dT_n}{dy}\right|_{y=1} = 0 \text{ and } T_n(\infty) = 0. \end{cases}$$

In addition to the non-dimensional quantities defined in (13), we now introduce the following non-dimensional quantities

$\theta_n = T_n/T^*$, $P = \mu c_p/K$, Prandtl Number

$E = U_0^2/c_p T^*$ Eckert number

in (45) and (46) which reduce to the following equations for the case of suction:

$$(47)\qquad \begin{cases} \theta_0'' + P\theta_0' = -PE[\Phi_0'^2 + 2\Phi_1' \overline{\Phi}_1'] \\ \theta_0'(0) = 0, \ \theta_0(\infty) = 0 \end{cases}$$

$$(48)\qquad \begin{cases} \text{in}\,\lambda\,\theta_n - \theta_n' - \delta(\theta_{n-1}' + \theta_{n+1}') = \dfrac{1}{P}\theta_n'' + 2E[\Phi_0' \Phi_n' + \overline{\Phi}_n' \Phi_{n+1}'] \\ \theta_n'(0) = 0), \ \theta_n(\infty) = 0. \end{cases}$$

where primes again denote differentation with respect to η.

To solve these coupled equations for $\theta_0's$, we now assume, in addition to (16), for θ_n the following expansion:

$$(49)\qquad \theta_n = \sum_{j=0}^{\infty} \theta_{nj} \delta^j.$$

a) **Constant free stream velocity:** Substituting (16) and (49) in (47), we have the following equation for $\theta_{00}(\eta)$:

$$(50)\qquad \begin{cases} \theta_{00}'' + P\theta_{00}' = -PE\Phi_{00}'^2 \\ \theta_{00}'(0) = 0, \ \theta_{00}(\infty) = 0. \end{cases}$$

Substituting for $\Phi_{00}(\eta)$ from (18) and solving (50), we get

$$(51)\qquad \theta_{00}(\eta) = \frac{E_n}{2n - P} \left[ne^{-P\eta} - \frac{P}{2} e^{-2n\eta} \right].$$

The equation for $\theta_{10}(\eta)$ is

$$(52)\qquad \begin{cases} \theta_{10}'' + P\theta_{10}' - i\lambda\theta_{10} = 0 \\ \theta_{10}'(\eta) = 0, \ \theta_{10}(\infty) = 0 \end{cases}$$

which, when solved, gives

$$\theta_{10}(\eta)=0. \tag{53}$$

We can also show similarly that $\theta_{n0}=0$, $n \geqslant 1$. (53) also implies that $\theta_{01}(\eta)=0$. From this we conclude that the terms of $0(\delta)$ do not affect the mean temperature.

The equation for $\theta_{02}(\eta)$ is

$$\begin{cases} \theta''_{02}+P\theta'_{02}=-2PE\left(\Phi'_{00}\Phi'_{02}+\Phi'_{11}\tilde{\Phi}'_{11}\right) \\ \theta'_{02}(0)=0, \ \ \theta_{02}(\infty)=0. \end{cases} \tag{54}$$

Substituting for Φ_{00}, Φ_{02}, Φ_{11} and its conjugate from (18), (24), (21) respectively in (54), simplifying and solving, we can obtain the solution for Φ_{02}. Hence the solution for the mean temperature profile to $0(\delta^2)$ may be expressed as

$$\theta_0(\eta)=\frac{2\eta E}{2n-P}\left\{ne^{-P\eta}-\frac{P}{2}e^{-2n\eta}\right\}+\delta^2\theta_{02}(\eta). \tag{55}$$

The functions $\theta_{00}(\eta)$, $\theta_{02}(\eta)$ which influence the mean temperature are shown on figs 7—9.

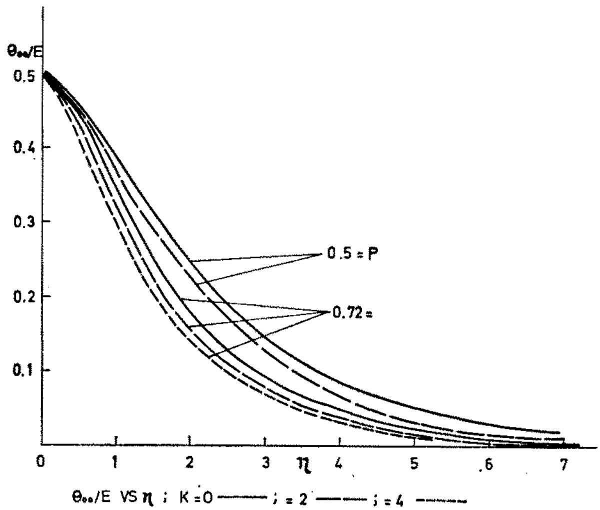

Fig. 7

b) **Periodic free stream:** In this case, as $\Phi_{10}(\eta)\neq 0$, $\theta_{10}(\theta)$ can be found. However, this is not an important function affecting the mean temperature. The only function which is significant in this case is $\theta_{01}(\eta)$, the equation for which is

$$\begin{cases} \theta''_{01}+P\theta'_{01}=-2PE\left(\Phi'_{00}\Phi'_{01}+\Phi'_{10}\overline{\Phi}'_{11}+\Phi'_{11}\overline{\Phi}'_{10}\right) \\ \theta'_{01}(0)=0, \ \ \theta_{01}(\infty)=0. \end{cases} \tag{56}$$

We now split $\theta_{01}(\eta)$ into two components, namely,

$$\theta_{01}(\eta) = \varepsilon_{1r}\,\theta_{01,1} + \theta_{1i}\,\theta_{01,2} \tag{57}$$

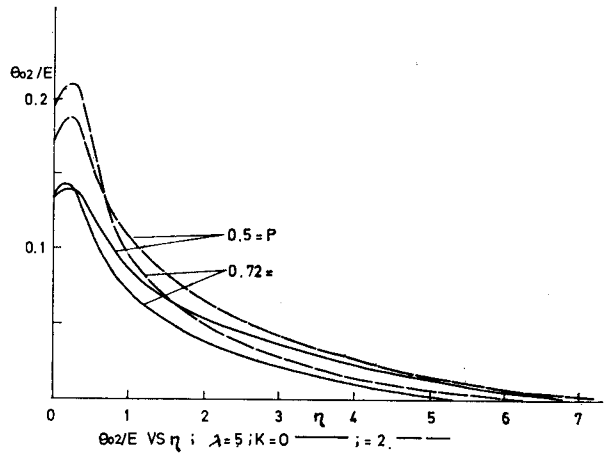

Fig. 8

Then on substituting (57) and (33) into (56) and again substituting for Φ_{00}, Φ_{10}, Φ_{11}, $\Phi_{01,1}$ and $\Phi_{01,2}$ from (18), (31), (21), (36), (37) respectively, the following two equations are obtained.

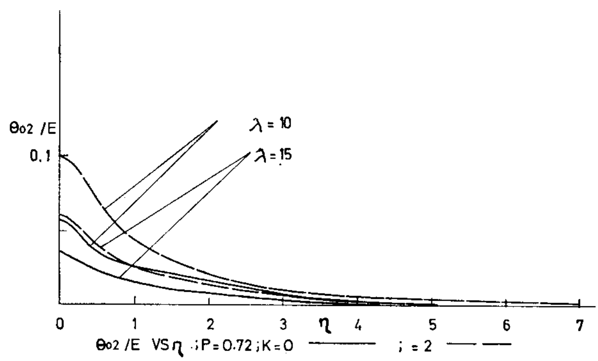

Fig. 9

$$(58)\quad \begin{cases} \theta''_{01,1} + P\,\theta'_{01,1} = -2\,PE\,[(C_5 - C_{10})\,e^{-(n+h_r)\eta}\cos h_i\eta + \\ \qquad + (C_6 - C_{11})\,e^{-(n+h_r)\eta}\sin h_i\eta + (C_9 + C_{12})\,e^{-2h_r\eta} - \\ \qquad - C_7\,e^{-2n\eta}\,] \\ \theta'_{01,1}(0) = 0,\quad \theta_{01,1}(\infty) = 0 \end{cases}$$

$$(59)\quad \begin{cases} \theta''_{01,2} + P\,\theta'_{01,2} = -2\,PE\,[C_{11} - C_6)\,e^{-(n+h_r)\eta}\cos h_i\eta + \\ \qquad + (C_5 - C_{10})\,e^{-(n+h_r)\eta}\sin h_i\eta - \\ \qquad - (C_{13} - C_{14})\,e^{-2h_r\eta} + C_8\,e^{-2n\eta}] \\ \theta'_{01,2}(0) = 0,\quad \theta_{01,2}(\infty) = 0 \end{cases}$$

where

$$C_5=\frac{2n\{h_r(h_r X-h_i Y)-h_i(h_i X+h_r Y)\}}{X^2+Y^2}, \qquad C_{15}=X^2+Y^2$$

$$C_6=\frac{2n\{h_i(h_r X-h_i Y)+h_r(h_i X+h_r Y)\}}{X^2+Y^2}, \qquad C_{16}=K^2+\lambda^2$$

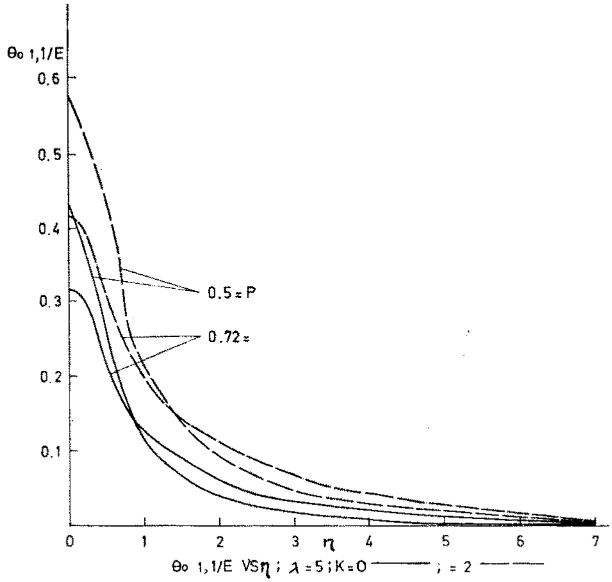

Fig. 10

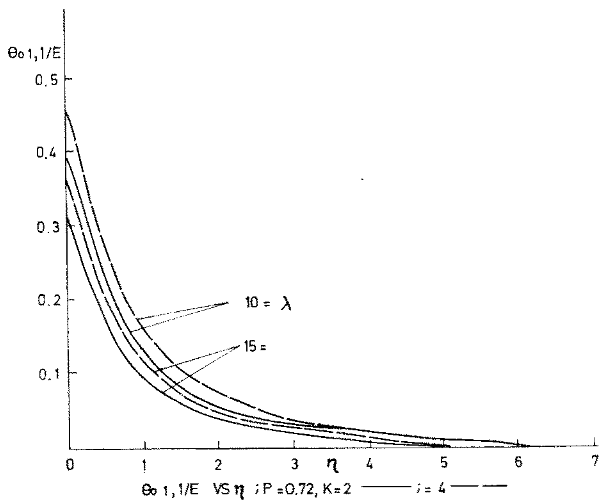

Fig. 11

$$C_7 = 2n^2(h_r X - h_i Y)/C_{15}, \quad C_{11} = 2n(Kh_i + \lambda h_r)/C_{16}$$

$$C_8 = 2n^2(h_i X + h_r Y)/C_{15}, \quad C_{12} = 2h_i(Kh_i + \lambda h_r)/C_{16}$$

$$C_9 = 2h_r(Kh_r - \lambda h_i)/C_{16}, \quad C_{13} = 2h_r(Kh_i + \lambda h_r)/C_{16}$$

$$C_{10} = 2n(Kh_r - \lambda h_i)/C_{16}, \quad C_{14} = 2h_i(Kh_r - \lambda h_i)/C_{16}$$

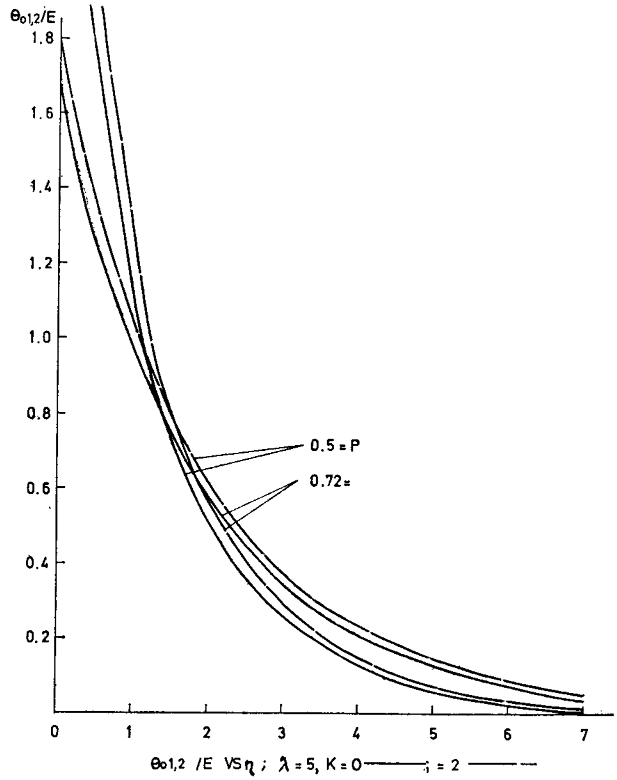

Fig. 12

Equations (58) and (59) can now be solved for $\theta_{01,1}(\eta)$ and $\theta_{01,2}(\eta)$. Hence the solution for $\theta_0(\eta)$ is given to $0(\delta^2)$ by

(60) $$\theta_0(\eta) = \theta_{00} + \delta(\varepsilon_{1r}\theta_{01,1} + \varepsilon_{1i}\theta_{01,2}) + \delta^2\theta_{02}.$$

The functions $\theta_{01,1}$ and $\theta_{01,2}$ are shown on figs. 10—13.

3. **Conclusions:** (1) An increase in the frequency λ leads to a decrease in $\Phi_{02}(\eta)$. (2) An increase in the magnetic field parameter K or λ leads to a decrease in $\Phi_{01,1}(\eta)$. Same is the case with $\Phi_{01,2}(\eta)$. (3) $\Phi_{00}(\eta)$ decreases with an increase in K or P, the Prandtl number. (4) $\theta_{02}(\eta)$ increases suddenly, as P increases, near the plate and then fades away. It is more in the presence of the magnetic field than in the absence of the magnetic field. An increase in λ leads to a decrease in $\theta_{02}(\eta)$. (5) The behaviour of the function $\theta_{01,1}(\eta)$ is the same as that of $\theta_{02}(\eta)$. (6) Compared to $\theta_{01,1}(\eta)$, $\theta_{01,2}(\eta)$ increases sharply

near the plate with increasing *K*. Under the magnetic field, it increases with increasing *P*. (7) An increase in λ leads to a decrease in $\theta_{01,2}(\eta)$.

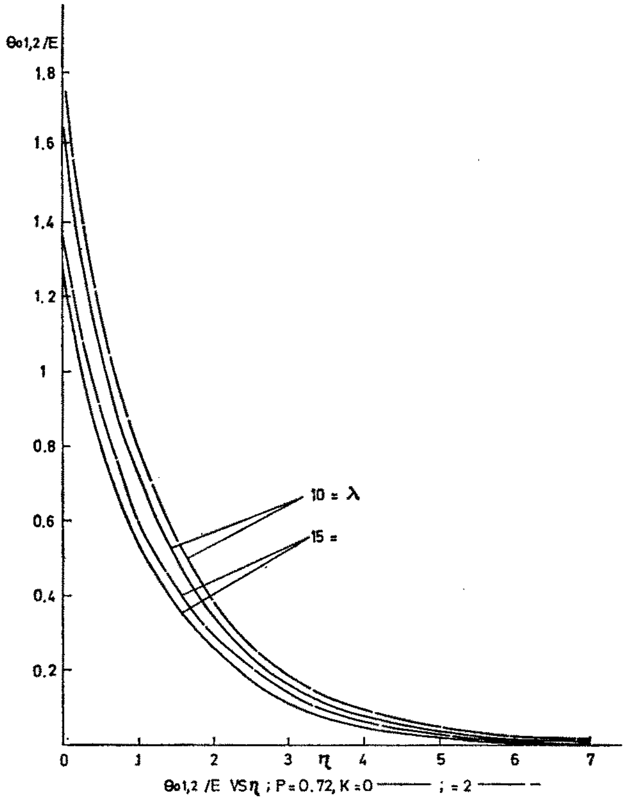

Fig. 13

Mean Shearing Stress:

(1) For $\lambda = 0$, which is the case of suction and free-stream velocity constant, the function $\Phi'_{02}(0)$ affecting the mean shearing stress is observed to be positive and decreasing with increasing the magnetic field strength. For $\lambda \neq 0$, i.e. time dependent suction and constant or periodic free stream velocity, the value of the function $\Phi'_{02}(0)$ is observed to be negative in the presence of the magnetic field. It decreases with increasing λ but increases with increasing *K*. Hence the tendency of separation may increase at small values of λ and large values of *K*. However for the same *K*, the tendency of separation decreases with increasing λ. Hence the effect of the term to $0(\delta^2)$ in oscillatory flow with time-dependent suction, under magnetic field, can be understood by the above description.

In the presence of an oscillatory main stream and time-dependent suction, the mean wall shear stress is affected by both the functions in the term to $0(\delta)$ when magnetic field is present. The first function viz. $\Phi'_{01,1}(0)$, which is due to the solution when the main stream is in phase or directly out of phase with the suction velocity, is observed to be always positive for non-zero values of K. Hence the effect of this term is to avoid the separation. An increase in λ or K leads to an increase of $\Phi'_{01,1}(0)$.

The second term viz. $\Phi'_{01,2}(0)$, which is the contribution when the free stream velocity is 90° out of phase with the suction velocity, was observed to be zero by Kelly [13] in non-magnetic case. It has been found to have negative values for non-zero values of λ or K. However, it decreases with increasing λ and increases with increasing K. Hence the contribution of this function towards the tendency of separation is more with increasing K whereas it is less with increasing λ.

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