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THERMAL BOUNDARY LAYER ON SLENDER BODIES OF REVOLUTION

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Notations

$x, y, r = r_0(x) + y$ — usual coordinates of the boundary layer,
$r_0(x)$ — radius of the body cross-section,
b — radius of the circular cylinder,
L — characteristic dimension of the body,
u, v — velocity projections along the directions x and y, respectively,
$U(x)$ — free stream velocity,
p — pressure,
T — temperature,
$T_1(x)$ — free stream temperature,
$\delta(x)$ — boundary layer thickness,
g — acceleration due to the gravity,
$\rho, \nu, c_p, \lambda, a = \lambda/\rho g c_p$ — usual denotations for the well known properties of the fluid,
$q = -\lambda (T_y)_{y=0}$ — heat flux,
$P = \nu/a$ — Prandtl number,
$R_x = U_\infty x/\nu$ — local Reynolds number,
$N_x = qx/\lambda (T_w - T_\infty)$ — local Nusselt number,
$E_c = U_\infty^2 / g c_p (T_w - T_\infty)$ — Eckert number,
$\Gamma(x)$ — gamma function,
$\psi(x)$ — logarithmic derivative of the gamma function,
$\gamma = -\psi(1) = 0{,}5772\ldots\ldots$ — Euler constant and
$\Phi(i, j; x), G(i, j; x)$ — confluent hypergeometric functions of the first and second kinds.

The meaning of remaining notations will be given later on in the paper.

1. Introduction

This paper considers the problem of the thermal boundary layer on bodies of revolution in which the ratio of the boundary layer thickness and the radius of the body cross-section is not a negligible quantity, but is approximately equal to unity, or is even greater than unity, so that the transverse curvature effect must be taken into account. Here, the laminar forced convection of an incompressible fluid is dealt with. The temperature field within the boundary layer is then given by the following equation.

$$(1)\qquad uT_x+vT_y=a\left(T_{yy}+\underline{\frac{1}{r_0+y}T_y}\right)+\frac{\nu}{gc_p}u_y^2+\frac{1}{\rho\, gc_p}up'(x).$$

If the transverse curvature effect were to be neglected, the underlined term in this equation would be ommitted. Such an equation has been solved already [1, 2] by Görtler's method [3], but the purpose of the present paper is to investigate the influence of that underlined term. Some purely qualititative conclusions, however, can be reached without first integrating the Equation (1). For instance, in the case of the thermometer problem, and in view of the fact that the body surface is thermally insulated, the underlined term in the immediate vicinity of the body will be extremely small, and therefore the transverse curvature effect for that region will be negligible. On the very surface of the body under consideration, the adiabatic wall temperature will be observed. In the case of the cooling problem, and in view of the fact that the heats due to friction and compression are taken into account, that is the last two terms of (1), the temperature gradient could change its sign in the boundary layer, and thus it is impossible to estimate in advance the influence of the underlined term in (1).

The velocity field in that case is given [4] by the dimension less stream function $F(\xi,\eta)$ and $W(\xi,\varphi)$ in the following manner: $F(\xi,\eta)=F_0(\xi,\eta)+F_1(\xi,\eta)\,\Delta(\xi)+F_2(\xi,\eta)\,\Delta^2(\xi)+\cdots$

for $\Delta(\xi)<1$, and:

$$W(\xi,\varphi)=\varphi+\frac{W_1(\xi,\varphi)}{\ln\Delta(\xi)}+\frac{W_2(\xi,\varphi)}{\ln^2\Delta(\xi)}+\cdots$$

for $\Delta(\xi)>1$. The coefficients of these series can be expanded into Görtler's series [3] where $F_0(\xi,\eta)$, $F_1(\xi,\eta)$, $F_2(\xi,\eta)$, ... can be calculated only numerically, but $W_1(\xi,\varphi)$, $W_2(\xi,\varphi)$, ... being obtainable in a closed form [4]. Here $\Delta(\xi)=2\nu L\sqrt{2\xi}/Ur_0^2$ is the so-called characteristic parameter [4], which is proportional to the ratio $\delta(x)/r_0(x)$, while ξ, η, and φ are the independent variables:

$$\xi=\frac{1}{\nu L^2}\int_0^x Ur_0^2\,dx,\quad \eta=\frac{Ur_0\,y}{\nu L\sqrt{2\xi}}\left(1+\frac{y}{2r_0}\right),\quad \varphi=\frac{1}{\Delta^2}\frac{r^2}{r_0^2}$$

2. Thermometer problem

To solve the thermometer problem when $\Delta(\xi)<1$, a dimension-less temperature $R(\xi,\eta)$ is introduced in the usual way [1]:

$$(2)\qquad T=T_t+\frac{U^2}{2gc_p}[R(\xi,\eta)-1]$$

where $T_t=T_1(x)+U^2(x)/2gc_p=\text{const}$, i.e., the so-called total temperature. Hence,

$$(3)\qquad \frac{1}{P}(1+\eta\Delta)\,R_{\eta\eta}+\left(F+2\xi\,F_\xi+\frac{\Delta}{P}\right)R_\eta-2\,\xi\,F_\eta\,R_\xi-2\,\beta(\xi)\,F_\eta\,R=$$

$$=-2(1+\eta\Delta)\,F_{\eta\eta}^2$$

with the following boundary conditions:

$$R_\eta=0 \quad \text{when} \quad \eta=0$$
$$R\to 0 \quad \text{when} \quad \eta\to\infty\,.$$

In this equation $\beta(\xi)=2\,\xi\nu L^2 U'/U^2 r_0^2$ is the well known [5] principal function. If it is assumed that the solution of (3) is in the form of:

$$R(\xi,\eta)=R_0(\xi,\eta)+R_1(\xi,\eta)\,\Delta(\xi)+\cdots$$

then:

$$\frac{1}{P}R_{0\eta\eta}+(F_0+2\,\xi F_{0\xi})\,R_{0\eta}-2\,\xi F_{0\eta}R_{0\xi}-2\,\beta(\xi)\,F_{0\eta}R_0=-2\,F_{0\eta\eta}^2$$

$$\frac{1}{P}R_{1\eta\eta}+(F_0+2\,\xi F_{0\xi})\,R_{1\eta}-2\,\xi F_{0\eta}R_{1\xi}-[2\,\beta(\xi)+\gamma(\xi)]F_{0\eta}R_1=$$
$$=-4\,F_{0\eta\eta}F_{1\eta\eta}-2\,\eta F_{0\eta\eta}^2-\frac{1}{P}\,\eta\,R_{0\eta\eta}-\left\{\frac{1}{P}+[1+\gamma(\xi)]\,F_1+\right.$$
$$\left.+2\,\xi F_{1\xi}\right\}R_{0\eta}+2\,\xi F_{1\eta}R_{0\xi}+2\,\beta(\xi)\,F_{1\eta}R_0$$

with the following boundary conditions:

$$R_{0\eta}=R_{1\eta}=0 \qquad \text{when} \quad \eta=0$$
$$R_0\to 0,\ R_1\to 0 \qquad \text{when} \quad \eta\to\infty\,.$$

In addition to the principal function $\beta(\xi)$ in the last equation there also appears the well known [4] so-called new principal function $\gamma(\xi)=2\,\xi\Delta'(\xi)/\Delta(\xi)$.

It can be shown that for $P=1$:

$$R_0(\xi,\eta)=1-F_{0\eta}^2(\xi,\eta)\,.$$

The function $R_0(\xi,\eta)$ itself represents a solution to the problem when the body transverse curvature effect is neglected, that is, that function represents a solution to the two-dimensional problem. It is interesting to note that Wrage [1] failed to see this simple relationship existing between the velocity and temperature fields, so that a considerable part of his paper is devoted to the evaluation of a system of universal functions for $R_0(\xi,\eta)$. Also, it can be shown that

$$R_1(\xi,\eta)=-2\,F_{0\eta}(\xi,\eta)\,F_{1\eta}(\xi,\eta)$$

and finally:

$$R(\xi,\eta)=1-F_\eta^2(\xi,\eta)\,.$$

If $\Delta(\xi)>1$, the dimensionless temperature $R(\xi,\varphi)$ is introduced in the same manner (2), so that the following equation is obtained:

$$\frac{1}{P}\varphi\,R_{\varphi\varphi}+\left\{\frac{1}{P}+[1+\gamma(\xi)]\;W+2\,\xi\,W_\xi\right\}R_\varphi-2\,\xi\,W_\varphi R_\xi-2\,\beta(\xi)\;W_\varphi R=$$
$$=-2\,\varphi\,W_{\varphi\varphi}^2$$

with the following boundary conditions:

$$R_\varphi=0 \qquad \text{when} \quad \varphi=1/\Delta^2$$
$$R\to 0 \qquad \text{when} \quad \varphi\to\infty$$

the solution of this equation being $P=1$:

$$R(\xi, \varphi)=1-W_{\varphi}^{2}(\xi, \varphi).$$

For the majority of gases, $P=1$ and, hence, the solutions obtained are of practical value. If $P\neq 1$, these solutions are to be sought numerically for each individual P, since it is known [1] that the energy equation (1) cannot be set free of the influence of the Prandtl number.

In a special case, when $U(x)=cx^{m}$, and $r_0(x)=ax^{n}$, the principal functions $\beta(\xi)$, and $\gamma(\xi)$, are reduced to constants, so that for $\Delta(\xi)>1$ the following is obtained:

$$R(\xi, \varphi)=\frac{\Gamma(2m+1)\,e^{-\zeta}\,G(2m+1,1;\zeta)}{\ln \Delta}+0\left(\frac{1}{\ln^{2}\Delta}\right)$$

where: $\zeta=2\varphi/(m+2n+1)$.

In Fig. 1, the temperature profiles are given for various values of the characteristic parameter $\Delta(\xi)$ provided $m=n=0$ and $P=1$. It can be seen that due to the body transverse curvature effect, the temperature in the immediate vicinity of the body decreases, while at some distance from the body, it increases. The temperatures in the vicinity of the body, when $\Delta(\xi)>1$, should be taken with a certain amount of reserve, because it is known [4] that the convergence of the asymptotic series for W_{φ} is slow, since the internal boundary condition for W_{φ} is satisfied when all the terms of the series are taken into account.

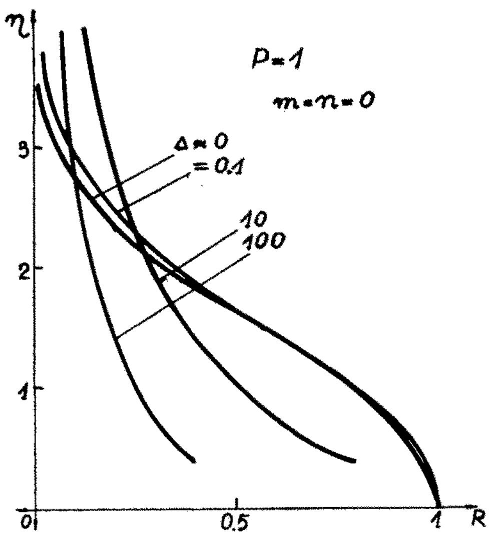

Fig. 1.

3. Cooling problem

The cooling problem is solved by introducing the so-called additional function (Zusatzfunktion) $H(\xi, \eta)$ in the following way:

$$(4)\quad T=T_1(x)+\frac{U^{2}(x)}{2gc_p}R(\xi, \eta)+H(\xi, \eta)$$

when $\Delta(\xi)<1$, this function will satisfy the equation:

$$(5)\qquad \frac{1}{P}(1+\eta\Delta)\,H_{\eta\eta}+\left(F+2\xi F_{\xi}+\frac{\Delta}{P}\right)H_{\eta}-2\xi F_{\eta} H_{\xi}=0$$

the boundary conditions for which are as follows:

$$H=H_w(\xi) \qquad \text{when} \quad \eta=0$$

$$H\to 0 \qquad \text{when} \quad \eta\to\infty$$

where:

$$H_w(\xi)=T_w(\xi)-T_1-\frac{U^{2}}{2gc_p}R(\xi, 0)$$

and $T_w(\xi)$ — for the time being an arbitrary distribution of the body temperature. It is readily seen that $H_w(\xi)=T_w(\xi)-T_t$ for $P=1$.

If the solution of (5) is assumed to be in the form:

$$H(\xi,\eta)=H_0(\xi,\eta)+H_1(\xi,\eta)\bar{\Delta}(\xi)+\cdots$$

then it is obtained

$$\frac{1}{P}H_{0\eta\eta}+(F_0+2\xi F_{0\xi})H_{0\eta}-2\xi F_{0\eta}H_{0\xi}=0 \tag{6}$$

$$\frac{1}{P}H_{1\eta\eta}+(F_0+2\xi F_{0\xi})H_{1\eta}-2\xi F_{0\eta}H_{1\xi}-\gamma(\xi)F_{0\eta}H_1=-\frac{\eta}{P}H_{0\eta\eta}-$$
$$-\left[F_1+2\xi F_{1\xi}+\gamma(\xi)F_1+\frac{1}{P}\right]H_{0\eta}+2\xi F_{1\eta}H_{0\xi}$$

with the following boundary conditions

$$H_0=H_w(\xi),\ H_1=0 \qquad \text{when} \quad \eta=0$$
$$H_0\to 0,\ H_1\to 0 \qquad \text{when} \quad \eta\to\infty$$

As in the case of the thermometer problem, the function $H_0(\xi,\eta)$ itself represents a solution to the two-dimensional problem. In order to make (6) possible for the solution, it is necessary [1] to assume function $H_w(\xi)$ in the following form:

$$H_w(\xi)=\sum_{i=0}^{\infty}\alpha_{\frac{i}{2}}\,\xi^{\frac{i}{2}}$$

where $\alpha_{i/2}$ are the constants. This expression represents a certain restriction on possible distributions of the body temperatures $T_w(\xi)$, but this restriction, as shown by Wrage [1], is of no great importance. The universal functions for $H_0(\xi,\eta)$ have already been derived [1, 2]. The universal function for $H_1(\xi,\eta)$ can also be derived in exactly the same way, but this will not be undertaken here because of lack of space.

For $\Delta(\xi)>1$, the additional function $H(\xi,\varphi)$ will satisfy the equation:

$$\frac{1}{P}\varphi H_{\varphi\varphi}+\left\{\frac{1}{P}+[1+\gamma(\xi)]W+2\xi W_\xi\right\}H_\varphi-2\xi W_\varphi H_\xi=0 \tag{7}$$

the boundary conditions of which are

$$H=H_w(\xi) \qquad \text{when} \quad \varphi=1/\bar{\Delta}^2$$
$$H\to 0 \qquad \text{when} \quad \varphi\to\infty\,.$$

If the solution of this equation is assumed to be in the form of an asymptotic series:

$$H(\xi,\varphi)=H_0(\xi,\varphi)+\frac{H_1(\xi,\varphi)}{\ln\Delta(\xi)}+\frac{H_2(\xi,\varphi)}{\ln^2\bar{\Delta}(\xi)}+\cdots \tag{8}$$

then a recursive system of differential equations will be obtained for the determination of unknown functions $H_0(\xi,\varphi)$, $H_1(\xi,\varphi),\ldots$, each of which can be represented in the form of:

$$H_i(\xi,\varphi)=\sum_{j=0}^{\infty}H_{i\frac{j}{2}}(\varphi)\,\xi^{\frac{j}{2}},\quad i=0,1,2,\ldots \tag{9}$$

It is well known [4] that in this case of flow, the behaviour of the velocity profile in the immediate vicinity of the body is of a logarithmic nature. In view of the fact that the velocity profiles have a similar feature, it is quite natural to assume that the temperature field in the immediate vicinity of the body behaves logarithmically, i.e., that for small φ:

$$H_{i\frac{j}{2}}(\varphi)\sim K_{i\frac{j}{2}}+L_{i\frac{j}{2}}\ln\varphi\quad(i,j=0,1,2,\ldots)$$

where $K_{i\frac{j}{2}}$ and $L_{i\frac{j}{2}}$ are constants. On the body surface itself, there shall be:

$$H_{i\frac{j}{2}}\left(\frac{1}{\Delta^2}\right)=K_{i\frac{j}{2}}-2L_{i\frac{j}{2}}\ln\Delta\,.$$

Now, in order to satisfy the internal boundary condition of (7), there must be

$$\sum_{i,j=0}^{\infty}\frac{\left(K_{i\frac{j}{2}}-2L_{i\frac{i}{2}}\ln\Delta\right)\xi^{\frac{j}{2}}}{\ln^i\Delta}=\sum_{i=0}^{\infty}\alpha_{\frac{i}{2}}\,\xi^{\frac{i}{2}}$$

or, after equating the coefficients adjacent to the terms of the same degree in terms of $\ln\Delta$:

$$-2L_{00}-2L_{0\frac{1}{2}}\xi^{\frac{1}{2}}-L_{01}\xi-\cdots=0$$

$$(K_{00}-2L_{10})+\left(K_{0\frac{1}{2}}-2L_{1\frac{1}{2}}\right)\xi^{\frac{1}{2}}+(K_{01}-2L_{11})\xi+\cdots=$$

$$=\alpha_0+\alpha_{\frac{1}{2}}\xi^{\frac{1}{2}}+\alpha_1\xi+\cdots$$

$$(K_{10}-2L_{20})+\left(K_{1\frac{1}{2}}-2L_{2\frac{1}{2}}\right)\xi^{\frac{1}{2}}+(K_{11}-2L_{21})\xi+\cdots=0$$

. . . .

Since all these latest equations must be satisfied for any ξ, it is obvious that the following must be true:

$$\left.\begin{aligned}&L_{0\frac{j}{2}}=0\\&K_{0\frac{j}{2}}-2L_{1\frac{j}{2}}=\alpha_{\frac{j}{2}}\\&K_{i\frac{j}{2}}=2L_{i+1,\frac{j}{2}}\end{aligned}\right\}\quad\begin{aligned}&j=0,1,2,\ldots\\&i=1,2,3,\ldots\end{aligned}\tag{10}$$

Differential equations for the first two terms of the series (8) will be:

$$\frac{1}{P}\varphi H_{0\varphi\varphi}+\left\{\frac{1}{P}+[1+\gamma(\xi)]\varphi\right\}H_{0\varphi}-2\xi H_{0\xi}=0 \tag{11}$$

$$\frac{1}{P}\varphi H_{1\varphi\varphi}+\left\{\frac{1}{P}+[1+\gamma(\xi)]\varphi\right\}H_{1\varphi}-2\xi H_{1\xi}=$$

$$=-\{[1+\gamma(\xi)]W_1+2\xi W_{1\xi}\}H_{0\varphi}+2\xi W_{1\varphi}H_{0\xi} \tag{12}$$

where the external boundary condition is:

$$\varphi\to\infty:\quad H_0\to 0,\ H_1\to 0\,.$$

The internal boundary condition, however, will be satisfied by the fulfilment of the equations (10).

If the solution of (11) is assumed to be in the form of Görtler's series (9), the following will be obtained for the first term

$$\frac{1}{P}\varphi H_{00}'' + \left[\frac{1}{P}+(1+\gamma_0)\varphi\right]H_{00}'=0 \tag{13}$$

with the following boundary conditions

$$H_{00}\sim K_{00}\qquad \text{when}\quad \varphi\to 0$$

$$H_{00}\to 0\qquad \text{when}\quad \varphi\to\infty$$

where γ_0 is the first term in the expansion of the principal function $\gamma(\xi)$ [4] (for solid bodies of revolution with the forward stagnation point $\gamma_0=-1/2$).

If the following substitutions of the variables are introduced:

$$(1+\gamma_0)P\varphi=\zeta,\quad H_{00}(\varphi)=e^{-\zeta}\Phi(\zeta) \tag{14}$$

(13) is reduced to a confluent hypergeometric equation:

$$\zeta\Phi''+(1-\zeta)\Phi'-\Phi=0$$

the general solution of which is as follows:

$$\Phi(\zeta)=M\Phi(1,1;\zeta)+NG(1,1;\zeta).$$

If the external boundary condition is applied, it follows that there must be $M=0$; if, however, the internal boundary condition is applied, it leads to the following relation:

$$-N(\ln\zeta+\gamma_0)\sim K_{00}$$

which can be satisfied only if simultaneously $N=0, K_{00}=0$, and therefore; the equations (13), together with the above given boundary conditions, will have only their trivial solutions. It is possible to show in exactly the same way that equations for the remaining coefficients of the Görtler series for the function $H_0(\xi,\varphi)$ will have only their trivial solutions, i.e.,

$$H_0(\xi,\varphi)\equiv 0,\ K_{0\frac{j}{2}}=0 \tag{15}$$

which changes to some extent the second of Eq. (10), in such a way that it will read as follows:

$$L_{1\frac{j}{2}} = -\frac{1}{2}\alpha_{\frac{j}{2}}$$

The function $H_0(\xi, \varphi)$ in itself represents a solution of the problem in the so-called first approximation. It is not difficult to see that in the first approximation, the temperature field is constant when $\Delta(\xi) > 1$, both in the thermometer problem and in the cooling problem, that is the temperature in the boundary layer at each point is equal to the temperature on its outer boundary. This result is in full agreement with the corresponding result obtained in investigating the problem of velocity boundary layer on slender bodies of revolution [4].

If (15) are taken into consideration and the solution of (12) is attempted in the form of the Görtler series (9); the first term is obtained in the form of

$$\frac{1}{P}\varphi H_{10}'' + \left[\frac{1}{P} + (1+\gamma_0)\varphi\right] H_{10}' = 0 \tag{16}$$

with the following boundary conditions:

$$H_{10} \sim K_{10} + L_{10}\ln\varphi \qquad \text{when} \quad \varphi \to 0$$

$$H_{10} \to 0 \qquad \text{when} \quad \varphi \to \infty .$$

The general solution of this equation is same as the one for the equation (13); the corresponding boundary conditions will lead to:

$$M = 0, \quad -N(\ln\zeta + \gamma) \sim K_{10} + L_{10}\ln\varphi$$

whence it follows:

$$N = -L_{10} = \alpha_0/2$$

$$K_{10} = \frac{\alpha_0}{2}[\gamma + \ln P + \ln(1+\gamma_0)].$$

Hence, the solution of (16) will be as follows:

$$H_{10}(\varphi) = \frac{\alpha_0}{2} e^{-\zeta} G(1, 1; \zeta).$$

The equation for the second term of the series for the function $H_1(\xi, \varphi)$ will be non-homogeneous:

$$\frac{1}{P}\varphi H_{1\frac{1}{2}}'' + \left[\frac{1}{P} + (1+\gamma_0)\varphi\right] H_{1\frac{1}{2}}' - H_{1\frac{1}{2}} = -\gamma_{\frac{1}{2}}\varphi H_{10}' \tag{17}$$

with the following boundary conditions:

$$H_{1\frac{1}{2}} \sim K_{1\frac{1}{2}} + L_{1\frac{1}{2}}\ln\varphi \quad \text{when} \quad \varphi \to 0$$

$$H_{10} \to 0 \qquad \text{when} \quad \varphi \to \infty .$$

The application of some recurrent relations which satisfy confluent hypergeometric functions of the second kind [6] will prove that:

$$\varphi H_{10}' = -\frac{\alpha_0}{2} e^{-\zeta}$$

thus, the general solution of (17) will be:

$$H_{1\frac{1}{2}} = Me^{-\zeta} \Phi\left(\frac{2+\gamma_0}{1+\gamma_0}, 1; \zeta\right) + Ne^{-\zeta} G\left(\frac{2+\gamma_0}{1+\gamma_0}, 1; \zeta\right) - \frac{\alpha_0 \gamma_{1/2}}{2(2+\gamma_0)} e^{-\zeta}$$

When the boundary conditions are used, the last equation will yield:

$$M=0, \ N = -\Gamma\left(\frac{2+\gamma_0}{1+\gamma_0}\right) L_{1\frac{1}{2}} = \frac{\alpha_{1/2}}{2} \Gamma\left(\frac{2+\gamma_0}{1+\gamma_0}\right)$$

$$K_{1\frac{1}{2}} = -\frac{\alpha_{1/2}}{2}\left[\psi\left(\frac{2+\gamma_0}{1+\gamma_0}\right) + 2\gamma + \ln P + \ln(1+\gamma_0)\right] - \frac{\alpha_0 \gamma_{1/2}}{2(2+\gamma_0)}$$

and the solution of (17) will finally be as follows:

$$H_{1\frac{1}{2}}(\varphi) = e^{-\zeta}\left[\frac{\alpha_{1/2}}{2} \Gamma\left(\frac{2+\gamma_0}{1+\gamma_0}\right) G\left(\frac{2+\gamma_0}{1+\gamma_0}, 1; \zeta\right) - \frac{\alpha_0 \gamma_{1/2}}{2(2+\gamma_0)}\right].$$

The remaining coefficients of the Görtler series for the function $H_1(\xi, \varphi)$ can be obtained in exactly the same way, but the working out of these coefficients quickly becomes extremely complicated, so it is probably much more economical to use a computing machine.

In the cooling problem, too, there exists a special case where the temperature field can be expressed directly by means of a velocity field. This is the case where $P=1$, $U(x) = \text{const.}$ and $T_w = \text{const.}$ Then,

$$H = (T_w - T_t)\left(1 - \frac{u}{U}\right).$$

4. *Example*

It seems that only the problem of a flow past a circular cylinder has been solved so far when $U(x) = U_\infty = \text{const.}$ and $T_w = \text{const.}$ This problem was solved by Seban and Bond [7] for the region in which $\delta(x)/r_0(x) < 1$ when $P = 0{,}72$, and by Bourne and Davies [8] for the region in which $\delta(x)/r_0(x) > 1$ when P takes any value, but they all neglected the heat due to viscosity and compressibility

For the case where $P=1$ we obtain the following values for the ratio of Nusselt's number and the square root of the local Reynolds number:

$$\frac{N_x}{\sqrt{R_x}} = \frac{1}{\sqrt{2}}\left(1 - \frac{1}{2} E_c\right)\left[F_0''(0) + 2\sqrt{\frac{2\nu x}{U_\infty b^2}} F_1''(0)\right], \quad \Delta(\xi) < 1$$

$$\frac{N_x}{\sqrt{R_x}} = \left(1 - \frac{1}{2} E_c\right) \frac{\sqrt{\dfrac{4\nu x}{U_\infty b^2}}}{\ln \dfrac{8\nu x}{U_\infty b^2}} \left(1 + \frac{\gamma + \ln 2}{\ln \dfrac{8\nu x}{U_\infty b^2}}\right), \quad \Delta(\xi) > 1.$$

If the heat due to the viscosity and the compressibility is neglected ,then it should be put formally that $E_c=0$. Bourne and Davies obtained for such a case the following results:

$$\frac{N_x}{\sqrt{R_x}}=\frac{\sqrt{\dfrac{4\,\nu x}{U_\infty\, b^2}}}{\ln\dfrac{4\,\nu x}{U_\infty\, b^2}}\left(1+\frac{\gamma}{\ln\dfrac{4\,\nu x}{U_\infty\, b^2}}\right).$$

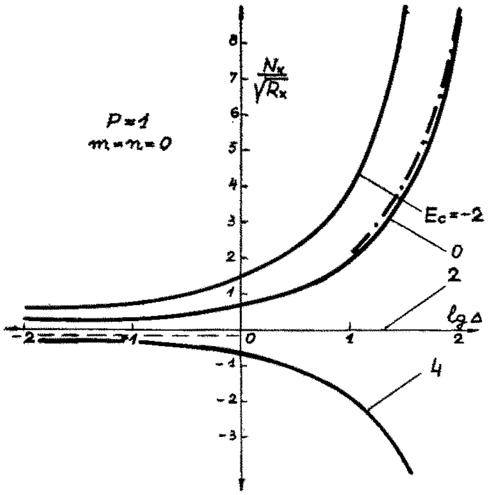

Fig. 2

The results are given graphically in Fig. 2. For $\Delta(\xi)=1$ the results are obtained by interpolation. It is easy to see that for $E_c=2$, the body involved behaves as it if were thermally insulated, but if $E_c>2$, the heat is convected from the fluid to the body regardless of the fact that $T_w>T_\infty$. The curve — · — · — represents the results obtained by Bourne and Davies, while the curve — — — represents the solution for the case where the transverse curvature effect is neglected for $E_c=2$. It is seen that this curve gives a good approximation only in the region in which $\Delta(\xi)\ll 1$. Also, it can be observed that in region in which $\Delta(\xi)>1$, a very intensive exchange of heat between the body and the fluid takes place.

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