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AN EXAMPLE CONCERNING THE CATEGORY NUMBERS

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Let S be a topological space, $I(S)$ the set of all isolated points of S, and $D(S)=S\setminus I(S)$. A set $X\subset S$ is called nowhere dense in S if $\overline{C(\overline{X})}=S$, where $C\overline{X}=S\setminus\overline{X}$. Đ. Kurepa, in [1], has considered the concept of a category number of a topological space. Namely, the minimal cardinal number n such that there are n nowhere dense sets, the union of which coincides with $D(S)$ is called the category number of S, and is denoted by $\operatorname{ct}(S)$.

In [1], the following problem has been given: n is any cardinal number satisfying $\aleph_0\leqslant n<\operatorname{ct}(S)$, is there a subspace S' of S satisfying $\operatorname{ct}(S')=n$? The example which follows, answers the problem in negative.

1. *The space* S_1. Let S_1 be any set of cardinality $\aleph_3$, i.e. $\operatorname{card}(S_1)=\aleph_3$. Topologize S_1 in the following manner: $A\subset S_1$ is open iff $A=\Phi$ or card $(S_1\setminus A)\leqslant\aleph_2$. Then,

(i) Φ and S_1 are open;

(ii) If A_1 and A_2 are open then $A_1\cap A_2$ is open, for either $A_1\cap A_2=\Phi$ or

$$\operatorname{card}(S_1\setminus A_1\cap A_2)=\operatorname{card}(S_1\setminus A_1)\cup(S_1\setminus A_2)$$
$$\leqslant\operatorname{card}(S_1\setminus A_1)+\operatorname{card}(S_1\setminus A_2)\leqslant\aleph_2+\aleph_2=\aleph_2;$$

(iii) A union of open sets is obviously open.

Let us prove that $\operatorname{ct}(S_1)=\aleph_3$. First $D(S_1)=S_1$, for S_1 has no isolated point. Further, if $B\subset S_1$ is such that $\operatorname{card}(B)=\aleph_3$ then B can not be nowhere dense in S, for

$$\overline{B}=S_1,\quad C\overline{B}=\Phi\ \text{and}\ \overline{C\overline{B}}=\Phi\neq S_1.$$

Therefore, if B is nowhere dense in S_1, then $\operatorname{card}(B)\leqslant\aleph_2$. Now, S_1 is not a union of $\leqslant\aleph_2$ nowhere dense sets B_ξ in S_1, because we would have

$$\operatorname{card}(S_1)=\operatorname{card}U\{B_\xi:\xi\in Z\}\leqslant\aleph_2\cdot Z\leqslant\aleph_2\cdot\aleph_2=\aleph_2.$$

So, $\operatorname{ct}(S_1)=\aleph_3$.

Next, we prove that for each subspace S_1' of S_1, $\operatorname{ct}(S_1')=\aleph_3$ or 0. Let card $(S_1')=\aleph_3$. Then S_1' is homeomorphic to S_1 and so $\operatorname{ct}(S_1')=\aleph_3$. If card $(S_1')\leqslant\aleph_2$, for each $x\in S_1'$, the set $(S_1\setminus S_1')\cup\{x\}$ is open in S_1 and the set $S_1'\cap[(S_1\setminus S_1')\cup\{x\}]=\{x\}$ is open in S_1', what means that S_1' is discrete and so $\operatorname{ct}(S_1')=0$ $(D(S_1')=\Phi$ and $\operatorname{ct}(\Phi)=0)$.

2. *The space* S_2. Let S_2 be Euclidian line.

3. *The space* S. Let S be the topological sum of the spaces S_1 and S_2. Let $S' = S_1' \cup S_2'$ be a subspace of S, $S_1' \subset S_1$ and $S_2' \subset S_2$. When $\operatorname{card}(S_1') = \aleph_3$, then $\operatorname{ct}(S') = \aleph_3$. When $\operatorname{card}(S_1') \leqslant \aleph_2$, then $\operatorname{ct}(S') = \operatorname{ct}(S_2') \leqslant \aleph_1$ (excepting that $2^{\aleph_0} = \aleph_1$). Therefore, there in no subspace S' of the space S such that $\operatorname{ct}(S') = \aleph_2$.

REFERENCES

[1] Đ. Kurepa, *On the category numbers of topological spaces*, Publ. Inst. Math., t. 8 (22), 1968, pp. 149—152.

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