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ON NONLINEAR EFFECTS IN SLENDER BODY THEORY

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1 — Basic potential equations

The aerodynamical properties of high speed airplanes and missiles, having low aspect ratio wings and long tails, and travelling at subsonic or supersonic speeds, have been in recent years studied with considerable success by means of the slender body theory [1—5].

The development of this theory for an inviscid compressible flow about slender wing and body combinations of arbitrary cross sections has been based most frequently on the linearized potential equation for fluid flow.

The basic partial differential equation satisfied by the velocity potential in three-dimensional fluid flow is derived from Euler's momentum equation, the continuity condition and the relation for the speed of pressure propagation.

Eliminating the pressure and the density from these equations, and introducing a perturbation velocity potential φ, we obtain the differential potential equation for the flow field in the most general form [6]

$$\begin{aligned}(1)\qquad &(1-M_\infty^2)\,\varphi_{xx}+\varphi_{yy}+\varphi_{zz}=\\ &=\frac{\varphi_{xx}}{a_\infty^2}\left[(k+1)\,V_\infty\,\varphi_x+\frac{k+1}{2}\,\varphi_x{}^2+\frac{k-1}{2}\,(\varphi_y{}^2+\varphi_z{}^2)\right]+\frac{2}{a_\infty^2}\,\varphi_{xy}\,\varphi_y\,(V_\infty+\varphi_x)+\\ &+\frac{\varphi_{yy}}{a_\infty^2}\left[(k-1)\,V_\infty\,\varphi_x+\frac{k-1}{2}\,(\varphi_x{}^2+\varphi_z{}^2)+\frac{k+1}{2}\,\varphi_y{}^2\right]+\frac{2}{a_\infty^2}\,\varphi_{xz}\,\varphi_z\,(V_\infty+\varphi_x)+\\ &+\frac{\varphi_{zz}}{a_\infty^2}\left[(k-1)\,V_\infty\,\varphi_x+\frac{k-1}{2}\,(\varphi_x{}^2+\varphi_y{}^2)+\frac{k+1}{2}\,\varphi_z{}^2\right]+\frac{2}{a_\infty^2}\,\varphi_{yz}\,\varphi_y\,\varphi_z\end{aligned}$$

where M_∞ designates the free stream Mach number, equal to the ratio of the free stream velocity V_∞ and the speed of sound a_∞, k being the ratio of the specific heats.

Once the perturbation velocity potential φ is determined, the aerodynamical properties of slender bodies can be calculated.

Eq. (1) is the basic potential equation necessary for a full treatment of the aerodynamics of slender bodies. Its solution is, in general, a cumbersome mathematical problem.

— In most cases, the aerodynamical properties of slender bodies can be approximated by means of the *first order theory.* The body is assumed to disturb the free stream only slightly. Eq. (1) is then reduced to the familiar Prandtl-Glauert equation

$$(1 - M_\infty^2)\,\varphi_{xx} + \varphi_{yy} + \varphi_{zz} = 0 \tag{2}$$

which is valid for both subsonic and supersonic flows.

This is the basic potential equation of the linearized compressible flow, steady state theory.

Eq. (2) illustrates the change of mathematical character of the gas flow in the three speed regimes, subsonic, transonic and supersonic. This equation is elliptic, parabolic or hyperbolic when the first term is positive, zero or negative. It is a result of the small perturbation hypothesis that the character of the equation depends only on the Mach number of the undisturbed stream.

A general solution of Eq. (2) can be written in the following form [7]

$$\varphi(x, y, z) = \varphi_2(x; y, z) + g(x) \tag{3}$$

The first term on the right hand side is the general solution to Laplace's equation in two dimensions

$$\varphi_{yy} + \varphi_{zz} = 0 \tag{4}$$

for the specified boundary conditions.

The function $g(x)$ contains the entire dependence on the Mach number. It has been determined by Heaslet and Lomax [6], who used Fourier transforms, for subsonic flow ($M_\infty < 1$), and by Ward [8], who used Laplace transforms, for supersonic flow ($M_\infty > 1$). Actually, the entire difference between the subsonic and supersonic cases enters through a term which contributes to the pressure coefficient but not to the aerodynamical loading on the slender body surface.

It is thus apparent, from Eq. (3), that the three-dimensional velocity field induced by slender airplanes or missiles, flying at either subsonic or supersonic speeds, is approximated in the vicinity of the airplane or missile by a velocity field that satisfies the two-dimensional Laplace's equation and the boundary conditions in transverse planes, plus a longitudinal velocity field that depends on the longitudinal rate of change of the cross-sectional area and is independent of the cross flow coordinates y and z.

— Further approximation to the slender body theory has been obtained by giving attention to the fact that there are certain flows about elongated bodies in which the conditions near the body can be approximated by neglecting the first term $(1 - M_\infty^2)\varphi_{xx}$ in comparison with φ_{yy} and φ_{zz} in the potential equation (2), the x direction being the direction of the elongation of the body.

This was originally used by Munk [1] in studying the aerodynamics of slender airships. R. T. Jones [2] and J. R. Spreiter [3] extended this method to the study of low aspect ratio pointed wings and of slender wing and body combinations.

In this assumption that for a very long slender body whose cross section varies slowly with x, Eq. (2) is reduced to a simple parabolic differential equation (4) in three dimensions. Consequently, for this class of problems, the calculation of the flow is independent of the Mach number.

The simplified form (4) of potential equation permits the analysis to be undertaken as a two-dimensional potential flow problem. Each cross flow plane, therefore, may be treated independently of the adjacent planes in the determination of the velocity potential. Thus, the potential is determined for any arbitrary x plane normal to the body axis.

This *simple slender body theory* of Munk and Jones has been applied with considerable success to the prediction of pressure distributions, loadings and forces on inclined elongated bodies, highly swept back wings, slender wing and body combinations [4, 9, 10], and the like. There is also considerable experimental verification.

It should be remembered that these results give only rough estimates. Accurate results require more detailed investigation.

2 — Nonlinear potential equations

It has been known for some time that the linearized slender body theory, based on the solutions of homogeneous differential equations (2) or (4) for steady state fluid flow, has serious limitations. This theory gives only a first approximation and in many cases needs important corrections. It has been verified that in these cases, even the study of small perturbation flows requires a solution of the nonlinear equation (1).

— The various developed methods [11] which give higher approximations are, in general, methods of successive approximation taking into account the terms on the right hand side of the basic potential equation (1).

As an example of the results of these methods, a certain insight into the complexity of the determination of nonlinear effects has been obtained by Van Dyke [12] in his *second order theory*. Using an iteration procedure and taking as the initial step the first order solution of the homogeneous differential equation (2), a second order solution of Eq. (1) is obtained. This technique for numerical solution is rather complicated.

The nonlinear slender body theories are closer to the theoretically exact adiabatic flow and constitute methods for overcoming the shortcomings of the linearized theories. These methods are tedious but are, in general, capable of yielding any desired accuracy.

It can be concluded, however, that precise determination of nonlinear effects associated with the potential equation (1) is too complicated a problem for really useful results to be expected in practice.

— Nevertheless, the treatment of the general equation of motion (1) can be considerably simplified in two special cases, corresponding to transonic and hypersonic speed ranges, in which the linearization of Eq. (1), even for small disturbances, is not justified [6, 7, 13].

It has been verified that for thin wings and elongated bodies at small angles of attack, an approximate analysis based on the linearized potential equation (2) is valid for flight Mach numbers sufficiently removed from unity and not of excessive magnitude. The treatment of these two latter regimes, that is, transonic and hypersonic, forces us to abandon the simplification inherent in the linearized theory and requires a solution of the nonlinear Eq. (1).

The assumptions that the free stream velocity V_∞ is of the same order of magnitude as the sound velocity a_∞, or that it is large in comparison with the sound velocity, leads to a simplification of the general equation (1).

Thus, for transonic flow fields, only the first term on the right hand side of Eq. (1) is retained, so that the potential equation

$$(1-M_\infty^2)\varphi_{xx}+\varphi_{yy}+\varphi_{zz}-(k+1)\frac{M_\infty^2}{V_\infty}\varphi_x\varphi_{xx}=0 \tag{5}$$

is used.

On the other hand, for two-dimensional hypersonic flow, the potential equation takes the foım

$$M_\infty^2\,\varphi_{zz}-\left[1-(k-1)\frac{M_\infty}{a_\infty}\varphi_x-\frac{k+1}{2}\frac{\varphi_z{}^2}{a_\infty^2}\right]\varphi_{zz}+2\frac{M_\infty}{a_\infty}\varphi_z\varphi_{xz}=0. \tag{6}$$

In [14], the difference between the results of the linearized theory and the actual conditions in the transonic and hypersonic speed ranges is shown by comparing the computed values of the coefficient of the pressure acting on an inclined plane surface in a two-dimensional flow.

3 — Nonlinear pressure-velocity relation

Another, very important source of nonlinearity in the slender body theory are the nonliear terms in the pressure-velocity relation, used to formulate the pressure coefficient.

Usually, one utilises the pressure coefficient C_p expressed with respect to the perturbation velocity components u, v, w along the body axes x, y, z, in terms of the angle of attack α and the angle of sideslip β, in the form

$$C_p=\frac{p-p_\infty}{q_\infty}\approx-\frac{2}{V_\infty}(u-\beta v+\alpha w)-\frac{v^2+w^2}{V_\infty^2}-\dots \tag{7}$$

where p designates the pressure on the body surface, and p_∞, q_∞, V_∞ are the pressure, the dynamic pressure and the velocity of the free stream.

The loading coefficient or the difference between the pressure coefficients at corresponding points on the lower and upper surface, is defined by

$$\Delta C_p=C_p^+-C_p^- \tag{8}$$

where the superscript (+) refers to the impact pressures of the lower surface, while (—) refers to the suction pressures of the upper surface.

In the classical slender body theory of Munk and Jones, and sometimes also in the first order theory, the quadratic terms $(v^2+w^2)/V_\infty^2$ in Eq. (7) are neglected.

It has been known for some time that the use of such an approximate expression for the pressure coefficient, in which due account is not taken of the nonlinear terms, yields considerable disagreement with the actual, measured values, and also both incorrect pressure and loading distributions along the surface of slender bodies [7,15].

On the other hand it has been observed that the more precise expression (7) for the pressure coefficient C_p including quadratic terms in the perturbation velocity components affords an improvement in the approximation to C_p on the surface of a slender body in cases where a comparison can be made with more exact solutions or with experimental values.

It is noted that the inclusion of a quadratic term is not obviously consistent with the approximations already made in deriving the potential differential equation (2).

— To illustrate the order of magnitude of this disagreement or error, more precise expressions for the pressure coefficient C_p and the loading coefficient ΔC_p on a slender cruciform wing and body combination [7,15,16] are evaluated by means of Eq. (7).

As shown in Fig. 1, the combination is formed of a circular body and flat highly swept back, low aspect ratio wings, inclined at small angles of pitch α and yaw β.

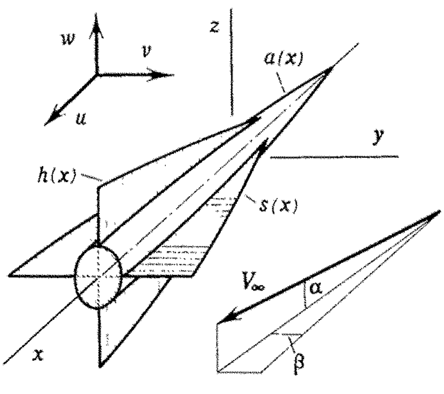

Fig 1

In this general case, it is easily established that the total potential function for the perturbation velocities is given by the sum

(9) $$\varphi = \varphi_0 + \varphi_\alpha + \varphi_\beta$$

where φ_0 is the perturbation velocity potential due to the component of velocity along the body axis, while φ_α and φ_β are the perturbation potentials due to the angles of attack α and yaw β.

The potential at zero angles of attack and yaw, or the thickness potential φ_0, which contains the effects of the body thickness and the Mach number, is given as the real part of the complex potential

(10) $$F_0(\zeta) = V_\infty a \frac{da}{dx} \ln \zeta + g(x)$$

where $\zeta = y + iz$ is the complex variable in the crossflow plane Oyz, and a is the local radius of the body in the crossflow plane corresponding to x. The function $g(x)$ contains the effects of the Mach number and the longitudinal rate of change of the cross-sectional area.

The perturbation velocity potential φ_α, due to the transversal component of velocity $V_\infty \sin\alpha \approx V_\infty \alpha$, normal to the body axis, is independent of the Mach number. It is determined as the real part of the complex potential

(11) $$F_\alpha(\zeta) = -iV_\infty \alpha \left\{ \left[\left(\zeta + \frac{a^2}{\zeta}\right)^2 - \left(s + \frac{a^2}{s}\right)^2 \right]^{\frac{1}{2}} - \zeta \right\}$$

where s is the local semispan of the horizontal wing.

From a similar expression, the perturbation velocity potential φ_β, due to the transversal velocity component $V_\infty \sin\beta \approx V_\infty \beta$, is determined.

The perturbation velocity potentials φ_0, φ_α and φ_β produce velocity components which are linearly superposable.

Let u_0, v_0, w_0 be the perturbation velocity components associated with φ_0, while u_α, v_α, w_α and u_β, v_β, w_β designate those associated with φ_α and φ_β, respectively.

Then, from Eq. (7), the pressure coefficient, including quadratic terms in the perturbation velocity components, is given by

$$C_p = -\frac{2}{V_\infty}\left[(u_0+u_\alpha+u_\beta)-\beta(v_0+v_\alpha+v_\beta)+\alpha(w_0+w_\alpha+w_\beta)\right]-$$

(12) $$-\frac{1}{V_\infty^2}\left[(v_0+v_\alpha+v_\beta)^2+(w_0+w_\alpha+w_\beta)^2\right].$$

From Eqs (10) and (11), the perturbation velocity components on the upper surface of the horizontal wing are easily obtained as follows

$$u_0 = \frac{\partial\varphi_0}{\partial x} = 0$$

$$u_\alpha = \frac{\partial\varphi_\alpha}{\partial x} = -\frac{V_\infty\alpha}{A}\left[\left(1-\frac{a^4}{s^4}\right)\frac{ds}{dx}+2\frac{a^3}{s^3}\left(1-\frac{s^2}{y^2}\right)\frac{da}{dx}\right]$$

$$u_\beta = \frac{\partial\varphi_\beta}{\partial x} = -\frac{V_\infty\beta}{B}\left[\left(1-\frac{a^4}{h^4}\right)\frac{dh}{dx}+2\frac{a^3}{h^3}\left(1+\frac{h^2}{y^2}\right)\frac{da}{dx}\right]$$

$$v_0 = \frac{\partial\varphi_0}{\partial y} = V_\infty\frac{a}{y}\frac{da}{dx}$$

$$v_\alpha = \frac{\partial\varphi_\alpha}{\partial y} = \frac{V_\infty\alpha}{A}\frac{y}{s}\left(1-\frac{a^4}{y^4}\right)$$

$$v_\beta = \frac{\partial\varphi_\beta}{\partial y} = -\frac{V_\infty\beta}{B}\frac{y}{h}\left(1-\frac{a^4}{y^4}\right)+V_\infty\beta$$

$$w_0 = \frac{\partial\varphi_0}{\partial z} = 0$$

$$w_\alpha = \frac{\partial\varphi_\alpha}{\partial z} = -V_\infty\alpha$$

(13) $$w_\beta = \frac{\partial\varphi_\beta}{\partial z} = 0$$

where

$$A = \left[\left(1+\frac{a^4}{s^4}\right)-\frac{y^2}{s^2}\left(1+\frac{a^4}{y^4}\right)\right]^{\frac{1}{2}}$$

(14) $$B = \left[\left(1+\frac{a^4}{h^4}\right)+\frac{y^2}{h^2}\left(1+\frac{a^4}{y^4}\right)\right]^{\frac{1}{2}}$$

h being the local semispan of the vertical wing.

By means of these expressions and using Eq. (12), the pressure distribution on the upper surface of the horizontal wing, shown in Fig. 1, can be calculated [15].

The effect of nonlinear terms in the pressure-velocity relation (7) alters [16] the pressure coefficient by

$$(C_p)_2 = -\frac{1}{V_\infty^2}[(v_0+v_\alpha+v_\beta)^2+(w_0+w_\alpha+w_\beta)^2]=$$

(15)
$$= -\left[\frac{a}{y}\frac{da}{dx}+\left(1-\frac{a^4}{y^4}\right)\left(\frac{\alpha}{A}\frac{y}{s}-\frac{\beta}{B}\frac{y}{h}\right)+\beta\right]^2-\alpha^2 .$$

The symmetry properties of the wing yield that the perturbation velocity components u_0, u_β, v_0, v_β, w_α have the same sign on the upper and lower surfaces.

The loading coefficient for the horizontal wing is then given by

$$\Delta C_p = -\frac{4}{V_\infty}(u_\alpha-\beta v_\alpha)-\frac{4}{V_\infty^2}v_\alpha(v_0+v_\beta)=$$

$$=\frac{4\alpha}{A}\left\{\left(1-\frac{a^4}{s^4}\right)\frac{ds}{dx}+\frac{a}{s}\left[2\left(\frac{a^2}{s^2}-1\right)+\left(1-\frac{a^2}{y^2}\right)^2\right]\frac{da}{dx}+\right.$$

(16)
$$\left.+\frac{\beta}{B}\frac{y^2}{sh}\left(1-\frac{a^4}{y^4}\right)^2\right\}$$

wherein

$$(\Delta C_p)_2 = -\frac{4}{V_\infty^2}v_\alpha(v_0+v_\beta)=$$

$$=-\frac{4\alpha}{A}\frac{y}{s}\left(1-\frac{a^4}{y^4}\right)\left[\frac{a}{y}\frac{da}{dx}-\frac{\beta}{B}\frac{y}{h}\left(1-\frac{a^4}{y^4}\right)+\beta\right]$$

represents the effect of nonlinear terms in Eq. (7).

A similar analysis can be carried out for the body.

The perturbation velocity components obtained for the upper surface of the body are as follows

$$u_0=\frac{\partial\varphi_0}{\partial x}$$

$$u_\alpha=\frac{\partial\varphi_\alpha}{\partial x}=-\frac{V_\infty\alpha}{E}\left[\left(1-\frac{a^4}{s^4}\right)\frac{ds}{dx}+2\frac{a}{s}\left(1+\frac{a^2}{s^2}-2\frac{y^2}{a^2}\right)\frac{da}{dx}\right]$$

$$u_\beta=\frac{\partial\varphi_\beta}{\partial x}=-\frac{V_\infty\beta}{F}\left[\left(1-\frac{a^4}{h^4}\right)\frac{dh}{dx}-2\frac{a}{h}\left(1-\frac{a^2}{h^2}-2\frac{y^2}{a^2}\right)\frac{da}{dx}\right]$$

$$v_0=\frac{\partial\varphi_0}{\partial y}=V_\infty\frac{y}{a}\frac{da}{dx}$$

$$v_\alpha=\frac{\partial\varphi_\alpha}{\partial y}=\frac{4V_\infty\alpha}{E}\frac{y}{s}\left(1-\frac{y^2}{a^2}\right)$$

$$v_\beta=\frac{\partial\varphi_\beta}{\partial y}=-\frac{4V_\infty\beta}{F}\frac{y}{h}\left(1-\frac{y^2}{a^2}\right)+V_\infty\beta$$

$$w_0=\frac{\partial\varphi_0}{\partial z}=V_\infty\left(1-\frac{y^2}{a^2}\right)^{1/2}\frac{da}{dx}$$

$$w_a=\frac{\partial\varphi_a}{\partial z}=-\frac{4V_\infty\alpha}{E}\frac{y^2}{as}\left(1-\frac{y^2}{a^2}\right)^{1/2}-V_\infty\alpha$$

$$w_\beta=\frac{\partial\varphi_\beta}{\partial z}=\frac{4V_\infty\beta}{F}\frac{y^2}{ah}\left(1-\frac{y^2}{a^2}\right)^{1/2} \tag{18}$$

wherein

$$E=\left[\left(1+\frac{a^2}{s^2}\right)^2-4\frac{y^2}{s^2}\right]^{1/2}$$

$$F=\left[\left(1-\frac{a^2}{h^2}\right)^2+4\frac{y^2}{h^2}\right]^{1/2}. \tag{19}$$

From these expressions and using Eq. (12), the pression distribution on the upper surface of the circular body, shown in Fig. 1, can be calculated [15].

The effect of nonlinear terms in Eq. (7) is represented by

$$(C_p)_2=-\left[\frac{y}{a}\frac{da}{dx}+4\left(1-\frac{y^2}{a^2}\right)\left(\frac{\alpha}{E}\frac{y}{s}-\frac{\beta}{F}\frac{y}{h}\right)+\beta\right]^2-$$

$$-\left[\left(1-\frac{y^2}{a^2}\right)^{1/2}\frac{da}{dx}-4\frac{y}{a}\left(1-\frac{y^2}{a^2}\right)^{1/2}\left(\frac{\alpha}{E}\frac{y}{s}-\frac{\beta}{F}\frac{y}{h}\right)-\alpha\right]^2 \tag{20}$$

Taking account of the symmetry properties of the body, the loading coefficient for the body is obtained as

$$\Delta C_p=-\frac{4}{V_\infty}(u_\alpha-\beta v_\alpha+\alpha w_\beta)-\frac{4}{V_\infty^2}(v_\alpha v_\beta+w_\alpha w_\beta)=$$

$$=\frac{4\alpha}{E}\left[\left(1-\frac{a^4}{s^4}\right)\frac{ds}{dx}+2\frac{a}{s}\left(1+\frac{a^2}{s^2}-2\frac{y^2}{a^2}\right)\frac{da}{dx}\right]+\frac{64\alpha\beta}{EF}\frac{y^2}{sh}\left(1-\frac{y^2}{a^2}\right) \tag{21}$$

wherein the effect of nonlinear terms in Eq. (7) is given by

$$(\Delta C_p)_2=-\frac{4}{V_\infty^2}[v_\alpha(v_0+v_\beta)+w_\alpha(w_0+w_\beta)]=$$

$$=-\frac{16\alpha}{E}\frac{y}{s}\left(1-\frac{y^2}{a^2}\right)\left[\frac{y}{a}\frac{da}{dx}-\frac{4\beta}{F}\frac{y}{h}\left(1-\frac{y^2}{a^2}\right)+\beta\right]+$$

$$+4\left(1-\frac{y^2}{a^2}\right)^{1/2}\left[\frac{4\alpha}{E}\frac{y^2}{as}\left(1-\frac{y^2}{a^2}\right)^{1/2}+\alpha\right]\left(\frac{da}{dx}+\frac{4\beta}{F}\frac{y^2}{ah}\right). \tag{22}$$

It should be remembered that the loading is not influenced by thickness effects associated with the perturbation velocity potential φ_0.

From these results it appears that the nonlinear terms in Eq. (7) have considerable effect on both pressure and loading distribution over the surface of inclined slender wing and body combinations.

In the limiting case of a slender body of revolution, without wing, it results from Eqs (21) and (22) that the neglected quadratic terms yield an error of order of 100% in the loading distribution.

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