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# A NOTE ON A THEOREM OF BOAS

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**1.** Let $\lambda=(\lambda_{n,k})$ be a triangular matrix of real numbers, then $(\lambda)$ mean of a sequence $\{s_n\}$ is given by

$$t_n=\sum_{k=0}^{n}\lambda_{n,k}\,s_k.$$

If we take

$$\lambda_{n,k}=\frac{A_{n-k}^{\alpha-1}}{A_n^{\alpha}},\quad \alpha>-1,\quad A_n^{\alpha}=\binom{n+\alpha}{\alpha},$$

then the sequence $t_n$ becomes $\sigma_n^{\alpha}$, the $n$-th Cesàro mean of order $\alpha$ of the sequence $\{s_n\}$.

**2.** Generalizing a result of Tomić [2], Boas [1] has recently obtained the following theorem.

Theorem *A*. If the series $\Sigma\, b_n \sin nx$ has $b_n\geqslant-\lambda_n$, where $\lambda_n$ are the Fourier-Stieltjes sine coefficients of a function of bounded variation, and if

$$\int_0^{\varepsilon}|\,\sigma_n^1(x)\,|\,dx=O(1), \tag{2.1}$$

for some $\varepsilon>0$, then the series $\Sigma\, b_n/n<\infty$.

The object of this note is to show that this theorem remains true even if we replace $\sigma_n^1$ in (2.1) by the $(\lambda)$ mean of the sequence of partial sums of $\Sigma\, b_n \sin nx$, where $\lambda_{n,k}$ satisfies certain conditions.

**3.** Let

$$\lambda_{n,k}>0,\quad \sum_{k=0}^{n}\lambda_{n,k}<C\quad\text{and}\quad \sum_{[n/2]}^{n}\lambda_{n,k}\geqslant\delta>0\quad\text{for sufficiently large } n. \tag{3.1}$$

Following the lines of proof of Boas [1] we have

$$\left|\sum_{k=1}^{n}\Big(\sum_{\mu=k}^{n}\lambda_{n,\mu}\Big)\,b_k\,(k^{-1}+O(k^{-2}))\right|<C,$$

where $C$ is a constant.

Let $b_n \geqslant 0$. If $m$ is so large that the term $O(k^{-2})$ is less than $k^{-1}/2$ for $k \geqslant m$, then for $n > 2m$ we have

$$\frac{\delta}{2} \sum_{k=m}^{\left[\frac{n}{2}\right]} b_k/k \leqslant \sum_{k=m}^{\left[\frac{n}{2}\right]} \Big( \sum_{\mu=k}^{n} \lambda_{n,\mu} \Big) b_k \left(k^{-1} + O(k^{-2})\right).$$
$$< C.$$

Hence $\Sigma\, b_k/k < \infty$.

The proof for the case $b_n \geqslant -\lambda_n$ when $\lambda_n \downarrow O$ and $\sum \frac{\lambda n}{n} < \infty$, is similar to that of Boas.

Thus it follows that the theorem of Tomić remains true if we replace (2.1) by

$$\int_0^{\varepsilon} |\, t_n^{*}(x) \,|\, dx = O(1),$$

where $\lambda_{n,k}$ satisifies the condition (3.1) and $t_n^{*}(x)$ is the corresponding mean of $\Sigma\, b_n \sin nx$.

**4.** If we take $\lambda_{n,k} = \frac{A_{n-k}^{\alpha-1}}{A_n^{\alpha}}$, $\alpha > 0$ we find that condition (3.1) is satisfied. Thus we deduce [1, p. 383], that in Theorem $A$ condition (2.1) can be replaced by

$$\int_0^{\varepsilon} |\, \sigma_n^{\alpha}(x) \,|\, dx = O(1), \quad \alpha > 1, \tag{4.1}$$

which, is a lighter condition.

## REFERENCES

[1] R. P. Boas, *On sine series with positive coefficients,* Math. Zeits. 80 (1963), 382—383.

[2] M. Tomić, *Über trigonometrische Sinus-Reihen mit positiven Koeffizienten,* Math. Zeits. 75 (1961), 53—56.