PUBLICATIONS DE L'INSTITUT MATHEMATIQUE
Nouvelle série, tome 6(20), 1966.p. 155—163.

ON THE BOUNDARY PROBLEM FOR THE NON-LINEAR NAVIER-STOKES EQUATIONS ON A RIEMANNIAN MANIFOLD

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(Communicated Feb. 2, 1966)

This paper represents a continuation of the paper [2], and is dedicated to questions of existence and uniqueness of solutions of non-linear, in the sense of presence of inertial terms, classical Navier-Stokes equations on a n-dimensional Riemannian manifold $\mathfrak{R}$. The problem for consideration is formulated in Chapter 1. In Chapter 2 principally are considered the linearized Navier-Stokes equations on $\mathfrak{R}$. Namely, it is shown a way for the introduction of potentials of distributions. In the same chapter is given also the construction of Green's tensor. Chapter 3 is dedicated to the consideration of the non-linear Navier-Stokes equations on $\mathfrak{R}$, using the Green's tensor.

In this paper for the time being the generalized solutions of quoted equations will not be considered.

Contents

1. Introduction

Let exist on a connected domain $\mathfrak{D}$ with the boundary $\mathfrak{B}$ of an orientable, n-dimensional C^∞ Riemannian manifold $\mathfrak{R}$ with the metric $ds^2 = g_{ij} dx^i dx^j$ and on the complement $\mathfrak{D}_R$ of $\mathfrak{D}$ to a sphere S of the infinitely large radius R, the following partial equations, given in the system of local coordinates

$$v^k v^i_{,k} = F^i - \frac{1}{\rho} p_{,j} g^{ij} + \nu \Delta v^i, \tag{1.1}$$

$$v^i_{,i} = 0, \tag{1.2}$$

with the conditions

$$(v^i)_{\mathfrak{B}} = f^i, \tag{1.3}$$

$$(v^i)_{\mathfrak{B}_R} \to 0, \quad \text{when} \quad R \to \infty \tag{1.4}$$

and a condition that there is no streaming across $\mathfrak{B}$, i. e.

(1.5) $$\int_{\mathfrak{B}} f^i d\mathfrak{B}_i = 0$$

It is the purpose of this paper to find unique solutions of the equations (1.1)—(1.2), $v^i(x)$ and $p(x)$ in the class of infinitely differentiable functions, for the conditions (1.3)—(1.5), considering that $F^i(x)$ and $f^i(x)$ belong to the same class.

All notations have been taken over from my previous paper [2].

2. The linearized Navier-Stokes equations

Since, we have already studied in details in the paper [2] questions of the existence and of the uniqueness of solutions of the linearized Navier-Stokes equations, therefore, we shall here principally consider only the way for introduction of potentials of distributions. Let u^i, v^i and t^{ij}, $\overset{*}{t}{}^{ij}$ be continuous C^2 respectively C^1 tensor functions on the closure $\mathfrak{G} \cup \mathfrak{F}$ and let u^i and $\overset{*}{t}{}^{ij}$ have compact supports, and let us form the identity.

(2.1) $$\int_{\mathfrak{G}} [\nabla_k (t^{ik} u_i) - \nabla_k (\overset{*}{t}{}^{ik} v_i)]\, d\mathfrak{G} = \int_{\mathfrak{F}} (t^{ik} u_i - \overset{*}{t}{}^{ik} v_i)\, d\mathfrak{F}_k,$$

where $$t^{ij} = -p g^{ij} + 2\mu D^{ij}, \qquad D^{ij} = \frac{1}{2}(v^{i,j} + v^{j,i}),$$

(2.2) $$\overset{*}{t}{}^{ij} = q g^{ij} + 2\mu \overset{*}{D}{}^{ij}, \qquad \overset{*}{D}{}^{ij} = \frac{1}{2}(u^{i,j} + u^{j,i}).$$

We assume that $t^{ij} = t^{ij}(v)$ and $\overset{*}{t}{}^{ij} = \overset{*}{t}{}^{ij}(u)$, and instead of u^i and q we choose the fundamental solutions v^{ik} and p^k, then

(2.3) $$\overset{*}{t}{}^{ik}(v^j) = -p^j g^{ik} + \mu (v^{ji,k} + v^{jk,i}),$$

is a fundamental tensor satisfying the equation

(2.4) $$\overset{*}{t}{}^{ik}(v^j)_{,k} = 0,$$

respectively

(2.5) $$-p^k_{,j} g^{ij} + \mu \Delta v^{ki} = 0,$$

to which we need to associate also the equation

(2.6) $$v^{ki}_{,i} = 0.$$

It is evidently that the relations (3.1.4)—(3.1.8) from the paper [2] are valid.

Let $\mathfrak{G}$ be open set with the regular closed boundary $\mathfrak{F}$ such that $\overset{*}{t}{}^{ik}(v^j(x,\xi))$ is defined for arbitrary $x, \xi \in (\mathfrak{G} \cup \mathfrak{F})$. We describe a small geodesic sphere $S(\varepsilon)$ with the surface $\mathfrak{A}(\varepsilon)$ and with the radius ε around the point $x \in \mathfrak{G}$ and apply the identity (2.1) to the domain

$$\mathfrak{G}_\varepsilon = \mathfrak{G} - S(\varepsilon),$$

and afterwards we examine the behaviour of integrals at $\varepsilon \to 0$, in the same way as in the section 2.3 of the paper [2], taking (3.1) [2] and (2.4) and get

$$(2.7)\quad v^i(x) = \rho \int_{\mathfrak{G}} v^{ij}(x,\xi) F_j(\xi)\, d\mathfrak{G} + \int_{\mathfrak{F}} \overset{*}{t}{}^{jk}(v^i(x,\xi))\, v_j(\xi)\, d\mathfrak{F}_k - \int_{\mathfrak{F}} \overset{*}{t}{}^{jk}(v(\xi))\, v_j^i(x,\xi)\, d\mathfrak{F}_k$$

Applying the operator Δ to (2.7) and taking (2.3.28) and (3.2) from [2], (2.5) and that

$$\Delta_x \overset{*}{t}{}^{ij}(v^k(x.\xi))_\xi = -2p^i(x,\xi)^{,jk},$$

we obtain

$$(2.8)\quad p(x) = \int_{\mathfrak{G}} p_j(x,\xi) F^j(\xi)\, d\mathfrak{G} - 2\mu \int_{\mathfrak{F}} \nabla_j p^i(x,\xi)\, v^j(\xi)\, d\mathfrak{F}_i - \int_{\mathfrak{F}} t^{jk}(v(\xi))\, p_j(x.\xi)\, d\mathfrak{F}_k$$

Thus we can conclude that at solving our problem it is necessary to introduce potentials of following distributions in $\mathfrak{G}$ and on $\mathfrak{F}$, namely of

a) volume distributions

$$(2.9)\quad \begin{aligned} V_v^i(x,f) &= \int_{\mathfrak{G}} v^{ij}(x,\xi) f_j(\xi)\, d\mathfrak{G}, \\ V_p(x,f) &= \int_{\mathfrak{G}} p^i(x,\xi) f_i(\xi)\, d\mathfrak{G}; \end{aligned}$$

b) single distributions

$$(2.10)\quad \begin{aligned} \overline{\Pi}{}_v^i(x.\psi) &= \int_{\mathfrak{F}} v^{ik}(x,\xi)\, \psi_k(\xi)\, d\mathfrak{F}, \\ \overline{\Pi}_p(x,\psi) &= \int_{\mathfrak{F}} p^k(x,\xi)\, \psi_k(\xi)\, d\mathfrak{F}; \end{aligned}$$

c) double distributions

$$(2.11)\quad \begin{aligned} \overline{\overline{\Pi}}{}_v^i(x,\varphi) &= \int_{\mathfrak{F}} K^{ij}(x,\xi)\, \varphi_j(\xi)\, d\mathfrak{F}, \\ \overline{\overline{\Pi}}_p(x,\varphi) &= \int_{\mathfrak{F}} K^i(x,\xi)\, \varphi_i(\xi)\, d\mathfrak{F}, \end{aligned}$$

where

$$K^{ij}(x,\xi) = \overset{*}{t}{}^{ij}(v^k(x,\xi))\, n_k, \qquad K^i(x,\xi) = \nabla_j p^i(x,\xi)\, n^j,$$

In above equations $f^i \in C^\infty(\mathfrak{G})$, $\psi^i \in C^\infty(\mathfrak{F})$ and $\varphi^i \in C^\infty(\mathfrak{F})$ are densities of given potentials. We assume them as continuous functions. The potential of a single distribution $\overline{\Pi}{}_v^i$ is continuous almost everywhere in $\mathfrak{G}$ including also $\mathfrak{F}$, as it becomes on $\mathfrak{F}$ equal to zero. We have already spoken about the potential of a double distribution in the paper [2], and therefore, for the determination of the density field φ^i we can immediately write down the integral equations

$$(2.12)\qquad \begin{aligned} {}^{+}\overline{\overline{\Pi}}{}_v^i(\xi) &= \frac{1}{2}\varphi^i(\xi) + \int_{\mathfrak{F}} K^{ij}(\xi,\eta)\,\varphi_j(\eta)\,d\mathfrak{F}, \\ {}^{-}\overline{\overline{\Pi}}{}_v^i(\xi) &= -\frac{1}{2}\varphi^i(\xi) + \int_{\mathfrak{F}} K^{ij}(\xi,\eta)\,\varphi_j(\eta)\,d\mathfrak{F}, \end{aligned}$$

for every $\xi \in \mathfrak{F}$. ${}^{+}\overline{\overline{\Pi}}{}_v^i$ and ${}^{-}\overline{\overline{\Pi}}{}_v^i$ represent inside and outside boundary values on $\mathfrak{F}$ and are continuous C^∞ functions of $\xi \in \mathfrak{F}$.

We can also consider the second boundary problem, together with the first one, at which on $\mathfrak{F}$ is imposed a field

$$(2.13)\qquad (t^{ij}(v)\,\eta_j)_{\mathfrak{F}} = \rho^i, \qquad i = 2, \ldots, n.$$

To determine the density field ψ^i, we write the stress tensor t^{ij} for the potential $\overline{\Pi}{}_v^i$. From (2.2) and (2.10) it follows

$$(2.14)\qquad t^{ij}(\overline{\Pi}_v) = \int_{\mathfrak{F}} \overset{*}{t}{}^{ij}(v^k)\,\psi_k\,d\mathfrak{F}.$$

From here it is obvious that $t^{ij}(\overline{\Pi}_v)$ has the same form, and therefore, also qualities as $\overline{\overline{\Pi}}{}_v^i$. Hence, it is easy to write the integral equations for the determination of ψ_i, namely

$$(2.15)\qquad \begin{aligned} t^{ij}(\overline{\Pi}_v)^{+}n_j &= \frac{1}{2}\psi^i(\xi) + \int_{\mathfrak{F}} t^{ij}(v^k(\xi,\eta))\,\psi_k(\eta)\,d\mathfrak{F}_j, \\ t^{ij}(\overline{\Pi}_v)^{-}n_j &= -\frac{1}{2}\psi^i(\xi) + \int_{\mathfrak{F}} \overset{*}{t}{}^{ij}(v^k(\xi,\eta))\,\psi_k(\eta)\,d\mathfrak{F}_j, \end{aligned}$$

for every $\xi \in \mathfrak{F}$.

If f^i is a vector function from the class $\mathfrak{L}^2(\mathfrak{G})$ (see [2]) then we can give the following estimations for potentials of volume distributions for

$$|V_v(x)| \leqslant K_1 \cdot \|f\|_{L^2(\mathfrak{G})},$$

$$|V_p(x)| \leqslant K_2 \cdot \|f\|_{L^2(\mathfrak{G})},$$

bounded $\mathfrak{G}$. But, in order that the integrals (2.9) converge absolutely and represent continuous functions, when the region $\mathfrak{G}$ is unbounded, it is necessary also a condition of vanishing of the field f^i at infinity, namely the requirement that f^i has a compact support. For details about properties of these integrals see: Calderon-Zygmund [1], Mikhlin [2] and Sobolev [3], therefore, we shall not stop on them.

2.1 *The Green's tensor*

In this section we shall establish the existence of the Green's tensor for the linearized Navier-Stokes equations on a set $\mathfrak{G}$, namely the existence of a tensor

$$G^{ij}(x,\xi), \qquad (x,\xi\in\mathfrak{G}), \tag{2.1.1}$$

with following properties:

i) It belongs to the class $C^{\infty}(\mathfrak{G})$.

ii) For a given field $b^i(x,\xi)\in C^{\infty}(\mathfrak{G})$ it satisfies the equations

$$\begin{aligned} &\mu\,\Delta\, G^{ij}-b^{i,j}=0,\\ &G^{ij}{}_{,j}=0, \end{aligned} \tag{2.1.2}$$

on $\mathfrak{G}$.

iii) On the boundary $\mathfrak{F}$ of a set $\mathfrak{G}$ it becomes equal to zero, i. e.

$$G^{ij}(x,\xi)=0, \tag{2.1.3}$$

for $x\in\mathfrak{F}$ and any point $\xi\in\mathfrak{G}$.

iv) If $\Gamma(x,\xi)\to 0$ then it exhibits the principal singularity characterized by the representation of the fundamental solution, namely

$$\Gamma^{-m}U+\log\Gamma\cdot V+W, \tag{2.1.4}$$

where U, V, and W are regular functions of x in a vicinity of ξ.

v) Inside $\mathfrak{G}$ it is non-negative.

vi) It is a symmetrical tensor.

The existence of the Green's tensor we shall establish by the following

Theorem. *The tensor function*

$$G^{ij}(x,\xi)=v^{ij}(x,\xi)-\int_{\mathfrak{F}}\nabla_k v^{is}(x,\zeta)\,\omega_s^{j}(\zeta,\xi)\,d\mathfrak{F}^k, \tag{2.1.5}$$

with

$$b^i(x,\xi)=p^i(x,\xi)-\int_{\mathfrak{F}}\nabla_k p^s(x,\zeta)\,\omega_s^{i}(\zeta,\xi)\,d\mathfrak{F}^k, \tag{2.1.6}$$

where $v^{ij}(x,\xi)$ and $p^i(x,\xi)$ are the fundamental solutions-fundamental tensors defined by (3.1.4)—(3.1.8) *from the paper* [2], *and*

$$\omega^{ik}(\zeta,\xi)=\overset{*}{v}{}^{ik}(\zeta,\xi)+\sum_{\nu=1}^{\infty}\lambda^{\nu}K^{ik}_{(\nu)}(\zeta,\xi), \tag{2.1.7}$$

$$K^{ik}_{(\nu)}(\zeta,\xi)=\int_{\mathfrak{F}}K^{ij}_{(\nu)}(\zeta,\eta)\,\overset{*}{v}{}^{k}_{j}(\eta,\xi)\,d\mathfrak{F},$$

$$K^{ik}_{(1)}=K^{ik}=\nabla_j v^{ik}n^j; \qquad K^{ij}_{(\nu)}(\zeta,\eta)=\int_{\mathfrak{F}}K^{ik}_{(1)}(\zeta,\xi)\,K^{j}_{(\nu-1)k}(\xi,\eta)\,d\mathfrak{F},$$

at which $\lambda=\pm 2$ and $\overset{*}{v}{}^{ij}=\pm 2v^{ij}$ in the case $\mathfrak{D}$ respectively $\mathfrak{D}_R$, satisfies all before mentioned conditions i)—vi), and therefore, it represents the Green's tensor.

Proof. To prove the above theorem we observe two fields

$$G^{ij}(x,\xi)=v^{ij}(x,\xi)-h^{ij}(x,\xi), \tag{2.1.8}$$

$$b^i(x,\xi)=p^i(x,\xi)-a^i(x,\xi), \tag{2.1.9}$$

where v^{ij} and p^i are the fundamental tensors fields, and h^{ij} and a^i are regular fields on $\mathfrak{G}$, which are found as solutions of the following system of partial equations

$$\mu\,\Delta\,h^{ij}-a^{i,j}=0, \tag{2.1.10}$$

$$h^{ij}{}_{,j}=0, \tag{2.1.11}$$

for the condition

$$(h^{ij}(x,\xi))_{x\in\mathfrak{F}}=(v^{ij}(x,\xi))_{x\in\mathfrak{F}}. \tag{2.1.12}$$

Thus, to find h^{ij} and a^i we need to solve the Dirichlet's problem in the same way as in the section 3.1 of the paper [2]. By introducing the solutions (2.1.10)–(2.1.12) into (2.1.8)–(2.1.9) we easily find relations (2.1.5) –(2.1.7). On the proof of the convergence of the series (2.1.7) as also on the question of the existence and the uniqueness of solutions, we shall not stop because it is completely related to the proof and the same questions are considered in the section 3.1 of [2]. Since, $v^{ij}(x,\xi)\in C^\infty(\mathfrak{G})$ then also $h^{ij}(x,\xi)$ and $G^{ij}(x,\xi)$ belong to the same class. From (2.1.12) it follows that $G^{ij}(x,\xi)=0$ at $x\in\mathfrak{F}$ and $\xi\in\mathfrak{G}$. The singularity of $G^{ij}(x,\xi)$ is given by singularity of a fundamental tensor, as $h^{ij}(x,\xi)$ is a regular function in $\mathfrak{G}$. Moreover, since $G^{ij}(x,\xi)\to+\infty$ when $\Gamma(x,\xi)\to 0$, and since the maximum principle shows that G^{ij} cannot have a negative minimum in the interior of $\mathfrak{G}$, its minimum must occur on the boundary $\mathfrak{F}$. Since on $\mathfrak{F}$ is fulfilled the condition (2.1.3) we have

$$G^{ij}(x,\xi)>0,$$

inside $\mathfrak{G}$. The symmetry property follows from the symmetry of the fundamental solution. In this way the theorem is proved.

3. The non-linear Navier-Stokes equations

We shall use the Green's tensor that is established in the preceding section and put up the following

Theorem. *The first boundary problem for the equations* (1.1) *and* (1.2) *is uniquely solvable in the class C^∞ on domains $\mathfrak{D}$ and $\mathfrak{D}_R$ respectively, in dependence if the solutions of the linearized equations are unique or not, on the same domains. The completely continuous solutions are given in the following form*

$$v^i(x)=v^i_{(o)}(x)-\rho\int_{\mathfrak{G}} G^i_j(x,\xi)\,v^k(\xi)\,v^j_{;k}(\xi)\,d\mathfrak{G}, \tag{3.1}$$

$$v^i_{;j}(x)=v^i_{(o);j}(x)-\rho\int_{\mathfrak{G}} G^i_{s,j}(x,\xi)\,v^k(\xi)\,v^s_{;k}(\xi)\,d\mathfrak{G}, \tag{3.2}$$

$$p(x)=p_{(o)}(x)-\int_{\mathfrak{G}} b_j(x,\xi)\,v^k(\xi)\,v^j_{;k}(\xi)\,d\mathfrak{G}; \tag{3.3}$$

respectively

(3.4) $$v^i(x)=\sum_{p=o}^{\infty}(-\rho)^p v^i_{(p)}x, \qquad v^i_{,j}(x)=\sum_{p=o}^{\infty}(-\rho)^p v^i_{(p),j}(x),$$

(3.5) $$v^i_{(p+1)}(x)=\int_{\mathfrak{G}}\sum_{q=o}^{p}G^i_j(x,\xi)\,v^k_{(q)}(\xi)\,v^j_{(p-q),k}(\xi)\,d\mathfrak{G},$$
$$v^i_{(p+1),k}(x)=\int_{\mathfrak{G}}\sum_{q=0}^{p}G^i_{j,k}(x,\xi)\,v^s_{(q)}v^j_{(p-q),s}(\xi)\,d\mathfrak{G},$$

where $\overset{i}{v}_{(o)}(x)$ *and* $p_{(o)}^{(x)}$ *represent the solutions of the linearized Navier-Stokes equations for the conditions* (1.3)–(1.5), *and accordingly to the Riemannian metric,* $d\mathfrak{G}$ *represents the measure, namely* $d\mathfrak{G}=(g(x))^{1/2}\,dx^1\cdots dx^n$.

Proof. Let $\mathfrak{G}$ be a bounded region. Let us first prove that (3.1) and (3.3) satisfy the equations (1.1) and (1.2). If we apply the operator Δ to (3.1), and if (3.3) once covariant differenciate, all in the same way as in the section (2.3) of [2], and taking into account the symmetry properties of Green's tensor, then (2.1.2), and also that $\overset{i}{v}_{(o)}$ and $p_{(o)}$ satisfy the linearized equations

$$\mu\,\Delta\,v^i_{(o)}-p^{;i}_{(o)}=-\rho\,F^i,$$
$$v^i_{(o),i}=0,$$

it is easy to verify that equations (1.1) and (1.2) are satisfied.

Since, the right side of (3.1) contains unknown v^i and $v^{i}_{,j}$, then by covariant differentiation of it, we obtain the equation (3.2). These two equations represent $n(n+1)$ integral equations for the determination of unknown v^i, $v^{i}_{,j}\in C^\infty(\mathfrak{G})$. If, we suppose solutions of integral equations in the form of series (3.4), then introducing them into (3.1) and (3.2), we obtain recurrent formulae (3.5).

Now, we shall prove the convergence of the series (3.4). Let us consider a space $C(\mathfrak{G})$ of real-valued continuous tensor functions $f(x)$ on the set $\mathfrak{G}$ normed by

$$\|f\|=\sup_{x\in\mathfrak{G}}|f(x)|.$$

Then from (3.4) it follows

(3.6) $$|v(x)|\leqslant\sum_{p=o}^{\infty}|\rho|^p H_p, \qquad |\nabla v(x)|\leqslant\sum_{p=o}^{\infty}|\rho|^p H_p,$$

where

$$\sup_{x\in\mathfrak{G}}|v_p(x)|\leqslant H_p, \qquad \sup_{x\in\mathfrak{G}}|\nabla v_p(x)|\leqslant H_p;$$

and are valid the recurrent relations

$$|v_{(p+1)}(x)|\leqslant G\sum_{q=o}^{p}H_q\,H_{(p-q)},$$

$$|\nabla v_{(p+1)}(x)|\leqslant G\sum_{q=o}^{p}H_q\,H_{(p-q)},$$

where

$$G = \sup_{x \in \mathfrak{G}} \int_{\mathfrak{G}} |G(x, \xi)| \, d\xi.$$

From (3.1) we readily obtain

$$|v(x)| \leqslant H_o + \rho \, G \, H^2,$$

respectively

(3.7) $$H \leqslant H_o + \rho \, G \, H^2$$

This inequality enables us to conclude that series (3.6) will be convergent on $C(\mathfrak{G})$ if is fulfilled the condition

(3.8) $$4 \rho \, G \, H_o < 1.$$

The convergence of (3.6) as a series of elements in the Banach space $C(\mathfrak{G})$ means that (3.4) are uniformly convergent on $\mathfrak{G}$; thus, the function v^i and its first covariant derivative are continuous tensor functions of x on $\mathfrak{G}$.

The condition (3.8) will impose a limitation upon the *Re*-number, namely

(3.9) $$R_e < \frac{1}{4 \mu \, A \, B},$$

where $G = AL$, $H_o = BV$, and A and B are characteristic length, for instance, a diameter of the region $\mathfrak{G}$, and any characteristic velocity of the linearized Navier-Stokes equations respectively.

For the uniqueness proof we shall use the maximum principle. Let in a bounded region $\mathfrak{G}$, v^i and w^i be solutions of the stated problem, which possess the same boundary values $(v^i)_{\mathfrak{B}} = (w^i)_{\mathfrak{B}} = f^i$, then their difference u^i on $\mathfrak{B}$ has the value $(u^i)_{\mathfrak{B}} = 0$. By using the maximum principle it may be seen that the function u inside $\mathfrak{G}$ cannot be either larger or less of the boundary value; thus, it is equal to zero, respectively $v^i = w^i$ everywhere in $\mathfrak{G}$. So, if we form a solution (3.1) by u^i, because $\overset{i}{v}_{(o)}$ as the solution of the linearized Navir-Stokes equations satisfies the boundary condition, then taking that $\overset{i}{u}_{(o)}$ on $\mathfrak{B}$ will have the zero value, we have

$$\| u_{(o)} \| = 0, \qquad \text{respectively} \qquad H_o = 0,$$

everywhere in $\mathfrak{G}$, which may be easily seen from the section 3.1 of [2]; thus

$$\| u \| = 0.$$

This proves the uniqueness $v^i = w^i$ of the solution of the non-linear Navier-Stokes equations in a bounded region $\mathfrak{G}$.

The case of an unbounded region may be similarly treated as well as for the linearized equations. Namely, if the number of dimensions is odd it is simple to verify, from the behaviour of G^{ij} and $\overset{i}{v}_{(o)}$ at infinity, that also v^i at infinity has the zero meaning. It is a simple matter to prove this, therefore, we omit to work about it. The theorem is proved.

REFERENCES

[1] A. P. Calderon and A. Zygmund, *On the existence of certain singular integrals. Acta Math.*, 88, № 1—2, 1952. 85—139.

[2] M. Đurić. *On the Dirichlet's problem for the Navier-Stokes equations on a Riemanian manifold* (*S*ee same number).

[3] С. Г. Михлин, *Многомерные сингулярные интегралы и интегральные уравнения*, Гос. Изд. Физ — Мат. лит., Москва, 1962.

[4] С. Л. Соболев, *Некоторые применения функционалного анализа в математической физике*, Изд. Сибир. Отд. АН СССР, Новосибирск, 1962.