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MATRIX TRANSFORMS INTO THE SPEED-MADDOX SPACES OVER ULTRAMETRIC FIELDS II

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Abstract. Let K be a complete, non-trivially valued, ultrametric (or non-archimedean) field. We recall the notions of boundedness and convergence with speed and speed-Maddox spaces over K, where the speed is defined by a sequence $\mu = \{\mu_n\}$ in K with the property $0 < |\mu_n| \nearrow \infty$, $n \to \infty$, and define a new notion of absolute convergence with speed μ over K. Let λ be another speed in K. The necessary and sufficient conditions for a matrix A over K would transform all sequences that are λ-convergent or absolutely λ-convergent over K into the speed-Maddox spaces over K, where the speed is defined by μ.

1. Introduction

Let K be a complete, non-trivially valued, ultrametric (or non-archimedean) field. Also sequences, infinite series and infinite matrices have entries in K, and indices and summation indices run from 0 to ∞, unless otherwise stated. Given an infinite matrix $A = (a_{nk})$, and a sequence $x = \{x_k\}$, by the A-transform of x, we mean the sequence $A(x) = \{(Ax)_n\}$, where $(Ax)_n = \sum_{k=0}^{\infty} a_{nk}x_k$, $n = 0, 1, 2, \ldots$, assuming that the series on the right-hand side converges. If $(Ax)_n \to s$, $n \to \infty$, we say that x is A-summable or summable A to s.

If X, Y are sequence spaces, we write $A = (a_{nk}) \in (X, Y)$ if $\{(Ax)_n\} \in Y$, whenever $x = \{x_k\} \in X$. In the sequel, m, c, c_0 respectively denote the ultrametric Banach spaces of bounded, convergent and null sequences under the ultrametric norm $\|x\| = \sup_{k\geqslant 0} |x_k|$, $x = \{x_k\} \in m, c, c_0$.

Let $l = \{x = (x_n) : \sum_{k=0}^{n} |x_n| < \infty\}$ be the Banach space under the norm $\|x\| = \sum_{k=0}^{n} |x_n|$.

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In 1969, Kangro [**4**,**5**] introduced the concepts of convergence, zero-convergence and boundedness with speed over the field of complex numbers (see also [**1**]). Later Natarajan [**12**–**15**] defined the analogues of these concepts in the ultrametric set up. We recall these concepts. Let now, and the sequel, $\lambda = \{\lambda_n\}$ be a sequence in K, such that $0 < |\lambda_n| \nearrow \infty$, $n \to \infty$. A convergent sequence $\{x_n\}$ in K with the limit $\lim_{n\to\infty} x_n = s$ is said to be

a) λ-bounded if $\{\lambda_n(x_n - s)\} \in m$,
b) λ-convergent if $\{\lambda_n(x_n - s)\} \in c$,
c) λ-convergent to zero if $\{\lambda_n(x_n - s)\} \in c_0$.

In [**2**] we defined the concept of absolute convergence with speed over the field of complex numbers. Now we extend this concept to ultrametric case.

Definition 1.1. We say that a convergent sequence $\{x_n\}$ in K with the limit $\lim_{n\to\infty} x_n = s$ is absolutely convergent with speed λ or absolutely λ-convergent if $\{\lambda_n(x_n - s)\} \in l$.

By m^λ, c^λ and l^λ we denote correspondingly the sets of all λ-bounded, all λ-convergent, all absolutely λ-convergent sequences, and by c_0^λ the set of all sequences λ-convergent to zero. It is not difficult to see that $l^\lambda \subset c_0^\lambda \subset c^\lambda \subset m^\lambda \subset c$.

Let $p = \{p_n\}$ be a sequence of strictly positive real numbers, and let

$$c_0(p) = \{x = \{x_n\} : \lim_n |x_n|^{p_n} = 0\},$$
$$c(p) = \{x = \{x_n\} : \lim_n |x_n - l|^{p_n} = 0 \text{ for some } l \in K\},$$
$$m(p) = \{x = \{x_n\} : |x_n|^{p_n} = O(1)\}.$$

Earlier the sets $c_0(p)$, $c(p)$ and $m(p)$ were defined over the field of complex numbers, and called as Maddox spaces (see, for example, [**6**, **7**, **19**]). In that case, for a bounded sequence p these spaces are also linear paranormed spaces. The reader can refer to textbooks [**3**] and [**9**] on Maddox spaces and their various expansions or contractions, and related topics.

Further we assume that p is bounded. Then it is easy to prove [**8**, Corollary 2.11] that $c_0(p) \subset c_0$, $c(p) \subset c$, $m \subset m(p)$. If $p_n \equiv 1$, then $c_0(p) = c_0$, $c(p) = c$, $m(p) = m$. We note that in the ultrametric set up the Maddox spaces are studied by Natarajan [**10**]. Let

$$(c_0(p))^\lambda = \{x = \{x_n\} : \lim_{n\to\infty} x_n = s \,(\text{say}) \text{ and } \{\lambda_n(x_n - s)\} \in c_0(p)\},$$
$$(c(p))^\lambda = \{x = \{x_n\} : \lim_{n\to\infty} x_n = s \,(\text{say}) \text{ and } \{\lambda_n(x_n - s)\} \in c(p)\},$$
$$(m(p))^\lambda = \{x = \{x_n\} : \lim_{n\to\infty} x_n = s \,(\text{say}) \text{ and } \{\lambda_n(x_n - s)\} \in m(p)\}.$$

We call the sets $(c_0(p))^\lambda$, $(c(p))^\lambda$ and $(m(p))^\lambda$ speed-Maddox spaces. These spaces over the field of complex numbers are studied in [**16**, **17**]. The notions of paranormed zero-convergence, paranormed convergence and paranormed boundedness with speed λ over the field of complex numbers are also defined in [**16**, **17**]. Using these definitions we can say that for a bounded sequence p, in the classical case $(c_0(p))^\lambda$ consists of all paranormally zero-convergent sequences with speed λ,

$(c(p))^\lambda$ consists of all paranormally λ-convergent sequences, and $(m(p))^\lambda$ consists of all paranormally λ-bounded sequences.

Let μ be another speed. Necessary and sufficient conditions in ultrametric setup for a matrix A would transform c_0^λ into $(c_0(p))^\mu, (c(p))^\mu$ or $(m(p))^\mu$ are proved in [**18**]. Here we continue the studies started therein. We give the characterization of matrix classes $(c^\lambda, (c_0(p))^\mu)$, $(c^\lambda, (c(p))^\mu)$, $(c^\lambda, (m(p))^\mu)$, $(l^\lambda, (c_0(p))^\mu)$, $(l^\lambda, (c(p))^\mu)$ and $(l^\lambda, (m(p))^\mu)$ over the ultrametric field K.

2. Auxiliary results

We recall some auxiliary results, which are necessary in the sequel. First we present the Kojima–Schur theorem in ultrametric setup (see [**11**, Theorem 4.1])

LEMMA 2.1. *A matrix* $A \in (c, c)$ *if and only if*

$$\sup_{n,k} |a_{nk}| < \infty, \tag{2.1}$$

$$\lim_{n\to\infty} a_{nk} = a_k \text{ (say)},\ k = 0, 1, 2, \dots \tag{2.2}$$

$$\lim_{n\to\infty} \sum_{k=0}^{\infty} a_{nk} = a \text{ (say)}. \tag{2.3}$$

In such a case, $\lim_{n\to\infty}(Ax)_n = \sum_{k=0}^{\infty} a_k(x_k - s) + sa$, *where* $x = \{x_k\} \in c$ *with* $\lim_{k\to\infty} x_k = s$.

LEMMA 2.2. *A matrix* $A \in (l, c)$ *if and only if conditions* (2.1) *and* (2.2) *hold. Moreover, in this case* $\lim_{n\to\infty}(Ax)_n = \sum_{k=0}^{\infty} a_k x_k$.

The proof is similar to the proof of Lemma 2.1 from [**18**], therefore we omit it.

Using Theorems 5 and 7 of [**10**], we immediately get the following result.

LEMMA 2.3. *A matrix* $A \in (c_0, c_0(p)) = (l, c_0(p))$ *if and only if condition* (2.1) *holds, and*

$$\lim_{n\to\infty} |a_{nk}|^{p_n} = 0,\quad k = 0, 1, 2, \dots,$$

$$\lim_{M\to\infty} \limsup_{n\to\infty} \Big(M^{-1} \sup_{k\geqslant 0} |a_{nk}|\Big)^{p_n} = 0.$$

Using Theorems 8 and 10 of [**10**], we immediately obtain the following result.

LEMMA 2.4. *A matrix* $A \in (c_0, c(p)) = (l, c(p))$ *if and only if condition* (2.1) *holds, and*

$$\lim_{n\to\infty} |a_{nk} - a_k|^{p_n} = 0,\quad k = 0, 1, 2, \dots,$$

$$\lim_{M\to\infty} \limsup_{n\to\infty} \Big(M^{-1} \sup_{k\geqslant 0} |a_{nk} - a_k|\Big)^{p_n} = 0.$$

Using Theorems 6 and 9 of [**10**], we immediately have the following result

LEMMA 2.5. *A matrix* $A \in (c_0, m(p)) = (l, m(p))$ *if and only if*

$$\sup_{n\geqslant 0} \Big(M^{-1} \sup_{k\geqslant 0} |a_{nk}|\Big)^{p_n} < \infty \text{ for some } M > 1.$$

3. Main results

We proceed to prove the main results of the paper. Let further $\lambda = \{\lambda_n\}$, $\mu = \{\mu_n\}$ be speeds of convergence over K, $p = \{p_n\}$, a bounded sequence of strictly positive real numbers, and $B = (b_{nk})$ the matrix, defined by

$$b_{nk} := \frac{\mu_n(a_{nk} - a_k)}{\lambda_k}, \quad n, k = 0, 1, 2, \dots,$$

provided that condition (2.2) holds. Also we will need the special sequences

$$e_k = \{0, \dots, 0, 1, 0, \dots\} \text{ where 1 is in the } k\text{-th position only } (k = 0, 1, 2, \dots),$$

$$e := (1, 1, \dots, 1, \dots), \quad e^\lambda = \left\{ \frac{1}{\lambda_0}, \frac{1}{\lambda_1}, \frac{1}{\lambda_2}, \dots \right\}.$$

We note that $e_k, e, e^\lambda \in c^\lambda$, and $e_k, e \in l^\lambda$.

THEOREM 3.1. *A matrix* $A = (a_{nk}) \in (c^\lambda, (c(p))^\mu)$ *if and only if*

$$A(e), A(e_k), A(e^\lambda) \in (c(p))^\mu, \quad k = 0, 1, 2, \dots, \tag{3.1}$$

$$\sup_{n,k} \left| \frac{a_{nk}}{\lambda_k} \right| < \infty, \tag{3.2}$$

$$\sup_{n,k} |b_{nk}| < \infty, \tag{3.3}$$

$$\lim_{n\to\infty} |b_{nk} - b_k|^{p_n} < \infty, \quad k = 0, 1, 2, \dots, \tag{3.4}$$

$$\lim_{M\to\infty} \limsup_{n\to\infty} \left(M^{-1} \sup_k |b_{nk} - b_k| \right)^{p_n} = 0, \tag{3.5}$$

where $\lim_{n\to\infty} b_{nk} = b_k$ *(say),* $k = 0, 1, 2, \dots$.

PROOF. *Necessity.* Let $A = (a_{nk}) \in (c^\lambda, (c_0(p))^\mu)$. Then condition (3.1) holds, since $e, e_k, e^\lambda \in c^\lambda$. As in this case also $A(e), A(e_k), A(e^\lambda) \in c$, then conditions (2.2), (2.3) hold, and

$$\text{there exists } \lim_{n\to\infty} \sum_{k=0}^{\infty} \frac{a_{nk}}{\lambda_k} = a^\lambda \text{ (say)}. \tag{3.6}$$

Let, now, $x = \{x_k\} \in c^\lambda$. Hence there exists the limit $\lim_{k\to\infty} x_k = s$ (say), since $x \in c$, and $\{v_k\} \in c$, where

$$v_k = \lambda_k(x_k - s), \quad k = 0, 1, 2, \dots. \tag{3.7}$$

As from (3.7) we obtain

$$x_k = \frac{v_k}{\lambda_k} + s, \quad k = 0, 1, 2, \dots,$$

then we can write

$$(Ax)_n = \sum_{k=0}^{\infty} a_{nk} x_k = \sum_{k=0}^{\infty} \frac{a_{nk}}{\lambda_k} v_k + s \sum_{k=0}^{\infty} a_{nk}. \tag{3.8}$$

Since $\{(Ax)_n\} \in c$ and condition (2.3) holds, then the matrix $A_\lambda := (a_{nk}/\lambda_k)$ transforms this sequence $\{v_k\} \in c$ into c. But, for every sequence $\{v_k\} \in c$, the sequence $\{v_k/\lambda_k\} \in c_0$, and for $\{v_k/\lambda_k\} \in c_0$ there exists a convergent sequence $x = \{x_k\}$ with $\lim_{k\to\infty} x_k = s$, such that $v_k/\lambda_k = x_k - s$. Thus, we have proved that, for every sequence $\{v_k\} \in c$ there exists a sequence $\{x_k\} \in c^\lambda$ such that (3.7) holds. Hence, $A_\lambda \in (c, c)$. Therefore, by Lemma 2.1, condition (3.2) holds, and

$$\eta := \lim_{n\to\infty} (Ax)_n = \sum_{k=0}^{\infty} \frac{a_k}{\lambda_k}(v_k - v) + va^\lambda + sa \tag{3.9}$$

for every $x = \{x_k\} \in c^\lambda$, where $v := \lim_{k\to\infty} v_k$ in view of (3.8) and (3.9), we have

$$(Ax)_n - \eta = \sum_{k=0}^{\infty} \frac{(a_{nk} - a_k)}{\lambda_k}(v_k - v) + v\biggl(\sum_{k=0}^{\infty} \frac{a_{nk}}{\lambda_k} - a^\lambda\biggr) + s\biggl(\sum_{k=0}^{\infty} a_{nk} - a\biggr),$$

and so

$$\mu_n((Ax)_n - \eta) = \sum_{k=0}^{\infty} b_{nk}(v_k - v) + v\mu_n\biggl(\sum_{k=0}^{\infty} \frac{a_{nk}}{\lambda_k} - a^\lambda\biggr) + s\mu_n\biggl(\sum_{k=0}^{\infty} a_{nk} - a\biggr) \tag{3.10}$$

for every $x \in c^\lambda$. By assumption, $\{(Ax)_n\} \in (c(p))^\mu$ for every $x \in c^\lambda$, hence

$$\{\mu_n((Ax)_n - \eta)\} \in c(p) \tag{3.11}$$

for every $x \in c^\lambda$. Moreover, since $A(e^\lambda), A(e) \in (c(p))^\mu$, then correspondingly

$$\biggl\{\mu_n\biggl(\sum_{k=0}^{\infty} \frac{a_{nk}}{\lambda_k} - a^\lambda\biggr)\biggr\} \in c(p), \tag{3.12}$$

$$\biggl\{\mu_n\biggl(\sum_{k=0}^{\infty} a_{nk} - a\biggr)\biggr\} \in c(p). \tag{3.13}$$

Consequently, from (3.10) we can conclude that $B = (b_{nk}) \in (c_0, c(p))$. Therefore conditions (3.3)–(3.5) are satisfied by Lemma 4, completing the proof of the necessity part.

Sufficiency. Let conditions (3.1)–(3.5) be fulfilled. Then conditions (2.2), (2.3), (3.6) are satisfied by (3.1), and relation (3.8) holds for every $x \in c_0^\lambda$, where $\lim_{k\to\infty} x_k = s$ and v_k is defined by (3.7). We note that

$$\lim_{n\to\infty} \frac{a_{nk}}{\lambda_k} = \frac{a_k}{\lambda_k},\ k = 0, 1, 2, \dots$$

by (2.2). Hence, using Lemma 2.1, we obtain by (3.2) that $A_\lambda \in (c, c)$ and relation (3.9) holds for every $x \in c^\lambda$. Then also (3.10) holds for every $x \in c^\lambda$ by (3.8) and (3.9). Now, relations (3.12) and (3.13) hold by (3.1), and, using Lemma 2.4, we can conclude that $B \in (c_0, c(p))$ by (3.3)–(3.5). Therefore, it follows from (3.10) that (3.11) is also satisfied for every $x \in c^\lambda$, completing the proof of the theorem. □

The next two results can be proved in a similar fashion, therefore we omit their proofs. We only note that for the proof of Theorem 3.2, instead of Lemma 2.4 we use Lemma 2.3, and for the proof of Theorem 3.3, instead of Lemma 2.2 we use Lemma 2.5.

THEOREM 3.2. *A matrix* $A = (a_{nk}) \in (c^{\lambda}, (c_0(p))^{\mu})$ *if and only if conditions* (3.2), (3.3) *hold, and*

$$A(e), A(e_k), A(e^{\lambda}) \in (c_0(p))^{\mu}, \quad k = 0, 1, 2, \dots, \tag{3.14}$$

$$\lim_{n\to\infty} |b_{nk}|^{p_n} = 0, \quad k = 0, 1, 2, \dots, \tag{3.15}$$

$$\lim_{M\to\infty} \limsup_{n\to\infty} \Big(M^{-1} \sup_k |b_{nk}|\Big)^{p_n} = 0. \tag{3.16}$$

THEOREM 3.3. *A matrix* $A = (a_{nk}) \in (c^{\lambda}, (m(p))^{\mu})$ *if and only if conditions* (3.2), (3.3) *hold, and*

$$A(e), A(e_k), A(e^{\lambda}) \in (m(p))^{\mu}, \quad k = 0, 1, 2, \dots, \tag{3.17}$$

$$\sup_n \Big(M^{-1} \sup_k |b_{nk}|\Big)^{p_n} < \infty \ \textit{ for some } M > 1. \tag{3.18}$$

Further we characterize the matrix class $(l^{\lambda}, (c(p))^{\mu})$.

THEOREM 3.4. *A matrix* $A = (a_{nk}) \in (l^{\lambda}, (c(p))^{\mu})$ *if and only if conditions* (3.2)–(3.5) *hold, and*

$$A(e), A(e_k) \in (c(p))^{\mu}, \quad k = 0, 1, 2, \dots. \tag{3.19}$$

PROOF. *Necessity.* Let $A \in (l^{\lambda}, (c(p))^{\mu})$. Then condition (3.19) holds, since $e, e_k \in l^{\lambda}$. As in this case also $e_k, e \in l$, then conditions (2.2), (2.3) hold.

As in the proof of Theorem 3.1, we define $\{v_k\} \in l$ for every $x = \{x_k\} \in l^{\lambda}$ by (2.2), where $\lim_{k\to\infty} x_k = s$. Then relation (3.8) holds for every $x \in l^{\lambda}$. Since $\{(Ax)_n\} \in c$ and condition (2.3) holds, then, similarly to the proof of $A_{\lambda} \in (c, c)$ in Theorem 3.1, it is possible to prove that in this case $A_{\lambda} \in (l, c)$. This implies that condition (3.2) is satisfied and

$$\lim_{n\to\infty} \sum_{k=0}^{\infty} \frac{a_{nk}}{\lambda_k} v_k = \sum_{k=0}^{\infty} \frac{a_k}{\lambda_k} v_k.$$

for every $\{v_k\} \in l$ by Lemma 2.2. Hence by (2.3) we obtain from (3.8) that

$$\eta = \lim_{n\to\infty} (Ax)_n = \sum_{k=0}^{\infty} \frac{a_k}{\lambda_k} v_k + sa \tag{3.20}$$

for every $x \in l^{\lambda}$. Now, using (3.8) and (3.20), we can write

$$(Ax)_n - \eta = \sum_{k=0}^{\infty} \frac{a_{nk} - a_k}{\lambda_k} v_k + s\bigg(\sum_{k=0}^{\infty} a_{nk} - a\bigg),$$

and so

$$\mu_n((Ax)_n - \eta) = \sum_{k=0}^{\infty} b_{nk} v_k + s\mu_n\bigg(\sum_{k=0}^{\infty} a_{nk} - a\bigg).$$

for every $x \in l^{\lambda}$. By assumption, $\{(Ax)_n\} \in (c(p))^{\mu}$ for every $x \in l^{\lambda}$, hence condition (3.11) holds for every $x \in l^{\lambda}$. In addition, since $A(e) \in (c(p))^{\mu}$, then condition (3.13) is also satisfied. Consequently, from (3.21) we conclude that $B \in$

$(l, c(p))$. Therefore conditions (3.3)–(3.5) hold by Lemma 2.4, completing the proof of the necessity part.

Sufficiency. We suppose that conditions (3.2)–(3.5) and (3.19) are fulfilled. Let us briefly go through the proof. Using this assumption and relation (3.8) (which holds), we note that $A_\lambda \in (c, c)$ and $\{(Ax)_n\} \in c$ for every $x \in l^\lambda$. Using again $\eta = \lim_{n\to\infty}(Ax)_n$, we also in this case obtain that (3.20) holds for every $x \in l^\lambda$, from which we easily can prove that (3.11) holds for every $x \in l^\lambda$, i.e., $\{(Ax)_n\} \in (c(p))^\mu$ for every $x \in l^\lambda$. □

In a similar fashion, it is possible to prove the following results; hence we omit their proofs. We only note that for the proof of Theorem 3.5, instead of Lemma 2.4 it is necessary to use Lemma 2.3, and for the proof of Theorem 3.6, instead of Lemma 2.2 it is necessary to use Lemma 2.5.

THEOREM 3.5. *A matrix* $A = (a_{nk}) \in (l^\lambda, (c_0(p))^\mu)$ *if and only if conditions* (3.2)*,* (3.5)*,* (3.15)*,* (3.16) *hold, and* $A(e), A(e_k) \in (c_0(p))^\mu$*,* $k = 0, 1, 2, \ldots$.

THEOREM 3.6. *A matrix* $A = (a_{nk}) \in (l^\lambda, (m(p))^\mu)$ *if and only if conditions* (3.2)*,* (3.18) *hold, and* $A(e), A(e_k) \in (m(p))^\mu$*,* $k = 0, 1, 2, \ldots$.

References

1. A. Aasma, H. Dutta, P. N. Natarajan, *An Introductory Course in Summability Theory*, Wiley, Hoboken, 2017.
2. A. Aasma, P. N. Natarajan, *Absolute convergence with speed and matrix transforms*, TWMS J. Appl. Engin. Math. **14** (2024), 1109–1120.
3. F. Başar, *Summability Theory and its Applications*, 2^{nd} ed., CRC Press, Boca Raton, London, New York, 2022.
4. G. Kangro, *On the summability factors of the Bohr-Hardy type for a given speed I*, Eesti NSV Tead. Akad. Toim., Füüs.-Mat. **18** (1969), 137–146.
5. ______, *On the summability factors of the Bohr-Hardy type for a given speed II*, Eesti NSV Tead. Akad. Toim., Füüs.-Mat. **18** (1969), 387–395.
6. I. J. Maddox, *Continuous and Köthe-Toeplitz duals of certain sequence spaces*, Proc. Cambridge Philos. Soc. **65** (1969), 431–435.
7. I. J. Maddox, M. A. L. Willey, *Continuous operators on paranormed spaces and matrix transforms*, Pac. J. Math. **53** (1974), 217–228.
8. E. Malkowsky, F. Başar, *A survey on some paranormed sequence spaces*, Filomat **31** (2017), 1099-–1122.
9. M. Mursaleen, F. Başar, *Sequence Spaces: Topics in Modern Summability Theory*, Math. Appl.: Model. Engin. Soc. Sci., CRC Press, Boca Raton, FL, 2020
10. P. N. Natarajan, *Continuous duals of certain sequence spaces and the related matrix transformations over non-Archimedian fields*, Indian J. Pure Appl. Math. **21** (1990), 82–87.
11. ______, *An Introduction to Ultrametric Summability Theory*, Springer, New Delhi, 2014.
12. ______, *On convergence and summability with speed in ultrametric fields*, An. Ştiinţ. Univ. Al. I. Cuza Iaşi. Mat. (N.S.) **68** (2022), 195–202.
13. ______, *On boundedness with speed* λ *in ultrametric fields*, Facta Univ. Ser. Math. Inform. **37** (2022), 643—649.
14. ______, *Matrix Transformations between certain new sequence spaces over ultrametric fields*, Filomat **36** (2022), 6359-–6367.
15. ______, *Characterization of some new matrix classes over ultrametric fields*, J. Comb. Inf. Syst. Sci. **47** (2002), 9–26.

16. P. N. Natarajan, A. Aasma, *Some classes of matrix transforms connected with subspaces of Maddox spaces defined by speeds of convergence*, Filomat **38**(23) (2024), 8039–8046.
17. ______, *Matrix transforms into the subsets of Maddox spaces defined by speed*, WSEAS Trans. Math. **24** (2025), 82–89
18. ______, *Matrix transforms into the speed-Maddox spaces over ultrametric fields*, An. Ştiinţ. Univ. Al. I. Cuza Iaşi. Mat. (N.S.) **71**(1) (2025), 71–79.
19. S. Simons, *The sequence spaces $l(p_\nu)$ and $m(p_\nu)$*, Proc. Lond. Math. Soc. (3) **15** (1965), 422—436.

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