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ON INVERSION IN THE CASE OF THE FUNDAMENTALLY FINITE INTEGRABLE VEKUA COMPLEX DIFFERENTIAL EQUATION

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Dedicated to Professor Hari Mohan Srivastava on the occasion of his 80th birthday

Abstract. The class of so called fundamentally finite integrable Vekua CDE is defined using the fixed point of the inversion and where one solution is equal to the coefficient of the equation. Then the different manifestations of inversion in relation to the general solution, an arbitrary analytical function inside and the core of the coefficient are examined. It shows that all the major problems of the Vekua equation theories, including boundary value problems can be interpreted and solved using the principle of inversion. The main significance of the fundamentally finite integrable Vekua equation is that the real and imaginary part of the solution can be separated, which in many mechanical and technique problems have certain physical meanings.

1. Introduction

Inversion is a ruling principle in nature, science and human society; in all phenomena and processes around us and inside us. Mathematics has studied and adopted inversion in the best way possible, which is the reason why it is present in all its areas. We can also find the affirmation of this fact in the theory of the Vekua complex differential equations (CDE).

In his well known monograph [**1**], Vekua thoroughly analyzed the elliptic system of partial equations

$$\begin{aligned} u'_x - v'_y &= a(x,y)u + b(x,y)v + c(x,y) \\ u'_y + v'_x &= b(x,y)u - a(x,y)v + d(x,y) \end{aligned} \tag{1.1}$$

where $a(x,y)$, $b(x,y)$, $c(x,y)$ and $d(x,y)$ are the given continuous functions in some simply connected domain T. This system has a big theoretical and practical meaning, as well as many applications in the various areas of mechanics. If the

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second equation in (1.1) multiples with i and adds to the first, we get a canonical form of the Vekua CDE

$$w'_{\bar{z}} = A\bar{w} + B$$

$$\Big(A = \frac{a+ib}{2}; \quad B = \frac{c+id}{2}; \quad w\big(z, \bar{z} = u(x,y) + iv(x,y)\big)\Big) \tag{1.2}$$

In many cases it is possible to determine a single particular solution to the w_0 equation, using various procedures (1.2). Substituting $w = w_0 + V$ where V is a new unknown function, equation (1.2) transforms into a homogenous equation

$$V'_{\bar{z}} = A\bar{V} \tag{1.3}$$

The general solution to (1.2) or (1.3) given by Vekua [**1**] cannot be used in practice, because besides infinite series and recurrent relations it also contains double singular integrals Cauchy-type, which are very difficult to solve.

For a very long time, there has not been a single example of equation (1.3) which has a solution in the finite and explicit form, and this question is of a great importance. Indeed, in many mechanical and technical problems, the real and imaginary parts of the solution have a certain physical meaning. If the general solution to Vekua equation (1.3) could be mapped into an "attractive", finite, closed and explicit form

$$w = w\big(z, \bar{z}, Q(z), Q(\bar{z})\big)$$

where $Q(z)$ is an arbitrary analytic function, then it would be possible to separate real and imaginary parts of the solution and assign it an adequate physical meaning.

Čanak (see [**2**], 2003) has determined a single class of these Vekua equations for the first time. These equations are

$$w'_{\bar{z}} = -\frac{\varphi'(z)}{\varphi + \bar{\varphi}}\bar{w} \tag{1.4}$$

where $\varphi(z)$ is a given analytic function. Their general solution has the following form

$$w(z, \bar{z}) = Q'(z) - (Q + \bar{Q})\frac{\varphi'(z)}{\varphi + \bar{\varphi}} \tag{1.5}$$

where $A(z)$ is an arbitrary analytic function of the class $\mathcal{A}$. (These are the analytic functions where Taylor series has only real coefficients, equally to $\overline{Q(z)} = Q(\bar{z})$, that is $\overline{Q'(z)} = Q'_{\bar{z}}(\bar{z})$, (conjugation term by term)).

What is particularly interesting is the fact that this result can be achieved using the principle of inversion. In this paper, different aspects of inversion of the Vekua equation are analyzed, pointing out their importance.

2. The inversion of the Vekua equation (1.3) in relation to the coefficient $A(z, \bar{z})$

Let observe the Vekua equation

$$w'_{\bar{z}} = A(z, \bar{z})\bar{w} \tag{2.1}$$

where $A = A(z, \bar{z})$ is the given differentiable complex function. We can observe this equation as "mapping", where the given equation $A(z, \bar{z})$ represents an entrance, and the solutions to equation (2.1) represents an exit. If we want to engage into inverse mapping, then we are familiar with the importance of the fixed points of this mapping. And this is the reason for the following question: Is it possible for a solution to equation (2.1) to be equal to the coefficient A, that is, can the exit be equal to the entrance, and the image to the original?

If we substitute $w = A$, $\bar{w} = \bar{A}$, $w'_{\bar{z}} = A'_{\bar{z}}$ in (2.1) we will get a following condition

$$A'_{\bar{z}} = A \cdot \bar{A} \tag{2.2}$$

for the function A. But now we can make an inversion, and also consider that condition as one new CDE which we want to solve.

The right-hand part of equation (2.2) is real, so the left-hand side has to be too. Fempl [**3**] has shown that expression $A'_{\bar{z}}$ is real only when the function A has the following form

$$A = \psi'_x - i\psi'_y$$

where $\psi = \psi(x, y)$ is an arbitrary real double continuous differentiable function. Then it is also valid

$$A'_{\bar{z}} = \tfrac{1}{2}\psi''_{xx} + \tfrac{1}{2}\psi''_{yy}$$

and equation (2.2) becomes

$$\psi''_{xx} + \psi''_{yy} = 2\big(\psi_x^{'2} + \psi_y^{'2}\big), \quad \text{i.e.,} \quad \Delta\psi = 2|\nabla u|^2. \tag{2.3}$$

Let us find a solution to (2.3) in a form of $\psi = f(h)$ where $h = h(x, y)$ is an arbitrary harmonic function, and $f(h)$ a wanted differentiable function. Then

$$\psi'_x = f'_h \cdot h'_x; \quad \psi''_{xx} = f'' \cdot h_x^{'2} + f' \cdot h''_{xx}$$
$$\psi'_y = f'_h \cdot h'_y; \quad \psi''_{yy} = f'' \cdot h_y^{'2} + f' \cdot h''_{yy}$$

Substituting these values in (2.3), after some simplifying, we get a nonlinear DE which is DE of the first order (2.3)

$$f'' = 2f^{'2} \tag{2.4}$$

One solution to (2.4) is $f = -\frac{1}{2}\ln h$ and the general solution to the partial DE is

$$\psi(x, y) = -\tfrac{1}{2}\ln[h(x, y)]$$

where $h(x, y)$ is an arbitrary strictly positive harmonic function.

Furthermore, it can be written in a complex form $h(x, y) = \varphi(z) + \overline{\varphi(z)}$ where $\varphi(z)$ is a new arbitrary analytic function of the complex variable, the real part of which is strictly positive. Then the general solution to equation (2.3) is

$$\psi = -\tfrac{1}{2}\ln\big(\varphi(z) + \overline{\varphi(z)}\big)$$

and the general solution to auxiliary equation (2.2)

$$A = -\frac{\varphi'(z)}{\varphi + \bar{\varphi}} \tag{2.5}$$

Where, through an immediate check, we can see that $\varphi = \varphi(z)$ can be an arbitrary analytic function, where $\operatorname{Re}\varphi$ has no zero.

Now we have met with the first inversion of the Vekua CDE (2.1) in a relation to the coefficient A. The result is that w and coefficient A have changed their roles, so now we are looking for that coefficient which can meet a certain condition. That condition is that the coefficient has to be a fixed point of inverse mapping.

For solution (2.5) the Vakua equation becomes equation (1.4) the general solution of which is given in (1.5). But if we look for the solution which is equal to the coefficient in that formula, then that condition will be met for $Q = \frac{1}{2}$.

From all above we can obtain the following theorem.

THEOREM 2.1. *The Vekua CDE* (1.4) *will have a solution which is equal to the coefficient of the equation, if that coefficient is a member of function class* (2.5)*.*

Because of its great significance, as well as the connection with the inversion and finite integrability, we call function (1.4) fundamentally, finite-integrable Vekua CDE. Analytic function $\varphi(z)$ is the kernel of function (1.4).

3. The inversion of Vekua fundamental equation (1.4) in relation to the kernel

In the Vekua CDE

$$w'_{\bar{z}} = -\frac{\varphi'(z)}{\varphi + \varphi'}\bar{w} \tag{3.1}$$

the first question could be how to determine the solution if the analytic function $\varphi(z)$ is given, (the kernel of the function). The inverse question would be if we assumed that the function $w(z, \bar{z})$ is given, and that we are looking for the unknown analytic function $\varphi(z)$ which identically satisfies equation (3.1).

We can write this equation as

$$\varphi'(z) + E(\varphi + \bar{\varphi}) = 0; \quad \Big(E = \frac{w'_{\bar{z}}}{\bar{w}}\Big) \tag{3.2}$$

where $E = E(z, \bar{z})$ is the given complex equation. Bearing in mind the fixed point of the first inversion, let us write the coefficient E of equation (3.2) in that same form, that is, $E = \frac{-\zeta'(z)}{\zeta+\zeta}$ where $\zeta(z)$ is the new analytic function (if it is even possible to be presented like that). Then equation (3.2) becomes

$$\varphi'_z - \frac{\zeta'(z)}{\zeta + \bar{\zeta}}(\varphi + \bar{\varphi}) = 0$$

and its solution is

$$\varphi(z) = \zeta(z)$$

NOTE 3.1. Inverse equation (3.2) is not the Vekua-type equation, but it can be reduced to it. Indeed, let us label with $\mathcal{A}$ the class of all analytic functions where the Taylor series contains only the real coefficients, that is

$$\varphi(z) = c_0 + c_1 z + c_2 z^2 + \cdots + c_n z^n + \cdots \quad (c_n \in R,\ n = 1, 2, \ldots)$$

Then $\overline{\varphi(z)} = \varphi(\bar{z})$ and $\overline{\varphi'(z)} = \varphi'_{\bar{z}}(\bar{z})$. Then equation (3.2) conjugates into

$$\varphi'_{\bar{z}} + \bar{E}(\varphi + \varphi') = 0$$

that is, into the Vekua equation.

From all above we can obtain the following theorem.

THEOREM 3.1. *A sufficient condition for solving inverse CDE* (3.2) *is that the coefficient E is a member of the class function which has the form*

$$E(z, \bar{z}) = -\frac{\zeta'(z)}{\zeta + \bar{\zeta}} \tag{3.3}$$

where $\zeta(z)$ is an arbitrary analytic function. Then the solution to equation (3.2) *is $\varphi(z) = \zeta(z)$. Equation* (3.2) *is not Vekua-type equation, but it can become through conjugation.*

EXAMPLE 3.1. To find the analytic solution to CDE

$$\varphi'_z - \frac{3z^2}{z^3 + \bar{z}^3}(\varphi + \bar{\varphi}) = 0 \tag{3.4}$$

Solution. Equation (3.4) is the equation of the type (3.2) and the coefficient has the form (3.3). According to the Theorem 3.1 the result of equation (3.4) is $\varphi(z) = z^3$ which can be immediately checked.

4. The inversion of the Vekua fundamental equation (1.4) in a relation to an arbitrary analytic function $Q(z)$ in the general solution

The general solution to the Vekua CDE contains one arbitrary analytic function $Q(z)$. Under the term inversion in a broader sense, we will consider all those problems (the number of which is not a small one) which, unlike the main problem, are directed to the function $Q(z)$ and question their mutual relation. The two most important inverse problems for us are:

a) Using the given general solution to eliminate arbitrary analytic function and form adequate Vekua CDE;
b) To consider boundary problems for the Vekua CDE and determine the function $Q(z)$ based on the given boundary conditions.

In this chapter we will explain these two aspects of inversion of the Vekua equation in the simplest way possible.

4.1. Creating the Vekua CDE. It is well known that in many cases the general solution to the real DE of the first order $f(x, y, y') = 0$ can be expressed in the following form $y = g(x, C)$, that is, containing one arbitrary real constant C. If we determine the first derivative of this solution $y' = g'(x, C)$, then we can eliminate the constant C and form a differential equation.

There is a similar situation with the so-called degenerate Vekua CDE. Its general form is

$$w'_{\bar{z}} + A(z, \bar{z})w + B(z, \bar{z}) = 0 \tag{4.1}$$

This equation was solved by Fempl (see [**4**]). He has shown that its general solution has the following form

$$w(z,\bar{z}) = e^{-I(A)}[Q(z) - I(Be^{I(A)})] \tag{4.2}$$

where $Q(z)$ is an arbitrary analytic function, and I is an operator inverse to the differential operator $\partial/\partial\bar{z}$, which is often reduced to the common complex integration of z variable.

It is not too difficult to differentiate function (4.1), and then eliminate the analytic function $Q(z)$ and finally reach the starting degenerate Vekua CDE which general solution is given in advance. It can be seen in Example 4.1.

Example 4.1. Forming the degenerate Vekua CDE the general solution of which is

$$w(z,\bar{z}) = Q(z)\cdot e^{z\bar{z}^2} \tag{4.3}$$

Differentiating by $\bar{z}$, we get

$$w'_{\bar{z}} = 2z\bar{z}Q(z)e^{z\bar{z}^2}, \tag{4.4}$$

From (4.3) we get $Q(z) = w(z,\bar{z})/e^{z\bar{z}^2}$. Substituting this value in (4.4) we obtain

$$w'_{\bar{z}} = 2z\bar{z}w$$

which represents the starting CDE, the general solution of which is given with (4.3).

However, if we try to get the nondegenerate Vekua CDE $w'_{\bar{z}} = A\bar{w}$ the general solution of which is given with (4.3), we will see that it is not possible. The obvious problem is that the general solution to the nondegenerate Vekua equation besides an arbitrary analytic function $Q(z)$, also contains its conjugation $\overline{Q(z)}$ and its complex derivative $Q'(z)$.

Now a valid question can be posed "is it possible to make an example of the *nondegenerate* Vekua CDE, the general solution of which is given in advance". The answer to this question is given in Example 4.2.

Example 4.2. Forming the nondegenerate Vekua CDE the general solution of which is

$$w(z,\bar{z}) = Q'(z) - (Q+\bar{Q})\frac{\varphi'(z)}{\varphi+\bar{\varphi}} \tag{4.5}$$

where $\varphi(z)$ is the given analytic function, and $Q(z)$ an arbitrary analytic function.

From (4.5) we can directly obtain

$$\bar{w} = \bar{Q}'_{\bar{z}} - \frac{\bar{\varphi}'_{\bar{z}}(Q+\bar{Q})}{\varphi+\bar{\varphi}} \tag{4.6}$$

$$w'_{\bar{z}} = -\bar{Q}'_{\bar{z}}\frac{\varphi'(z)}{\varphi+\bar{\varphi}} + \frac{(Q+\bar{Q})}{(\varphi+\bar{\varphi})^2}\varphi'_z\cdot\bar{\varphi}'_{\bar{z}} \tag{4.7}$$

and further from (4.6)

$$\frac{\bar{\varphi}'_{\bar{z}}(Q+\bar{Q})}{\varphi+\bar{\varphi}} = \bar{Q}'_{\bar{z}} - \bar{w} \tag{4.8}$$

Substituting (4.8) into (4.7), and after simplifying, we get

$$w'_{\bar{z}} = -\frac{\varphi'_z}{\varphi + \bar{\varphi}} \cdot \bar{w} \tag{4.9}$$

which is the wanted Vekua CDE.

We can see that the answer to the question is affirmative, and that the direct inversion can be used on the fundamentally, finite-integrable Vekua CDE, that is, to eliminate the analytic function $Q(z)$. There is no visible way how to use this method with the general Vekua formula, where the analytic functions $Q(z)$ and $\overline{Q(z)}$ are under the double singular Cauchy-type integrals.

Therefore, the following theorem can be formulated.

THEOREM 4.1. *For all complex functions of the class* $w(z, \bar{z}) = Q'(z) - (Q + \bar{Q})\frac{\varphi'(z)}{\varphi+\bar{\varphi}}$ *it is possible to eliminate an arbitrary analytic function* $Q(z)$ *and form an adequate fundamentally finite-integrable Vekua CDE using methods* (4.6)–(4.9).

4.2. Boundary problems for the Vekua equation. Boundary problems play an important role in the theory of the Vekua CDE, and especially in the monograph [**1**]. The main method for solving these problems is reduction to the adequate boundary problems in the analytic functions. Having the fact that it is important to define the unknown analytic function using the given boundary condition, then the boundary problems can also be considered as certain inversion compared to the main problem for defining the general solution to the Vekua equation.

The mere reduction can sometimes be very complex, so in this paper we will analyze two examples when it can be simple.

EXAMPLE 4.3. In a monograph [**5**] by Gakhov there is a following formulation for the Hilbert boundary problem for the analytic function: A simple, smooth, closed contour L is given which divides the plain of the complex variable to an inner surface D^+ and outer D^-. The following functions $a(t)$, $b(t)$ and $c(t)$, $(t \in L)$ are given, which meet the condition of Hölder-type continuity on the contour L. A function $Q(z) = u(x, y) + iv(x, y)$ should be defined, which is analytic in D^+ and continuous in $\bar{D} = D^+ \cup L$ and which meets the condition on the contour

$$a(t)u(t) + b(t)v(t) = c(t) \tag{4.10}$$

Gakhov has solved this problem using regularized multiplier and Schwarz operator. He thoroughly analyzed the question of existence and the number of solutions for different cases, as well as various generalizations of the problem.

One of such generalizations is boundary problem (4.10) for solving the Vekua CDE. Vekua [**1**] has shown that some specific cases of this boundary problem have a great significance in the theory of the pressure status of the elastic shells.

In this example we will analyze the Hilbert boundary problem for the Vekua CDE.

Boundary problem H_r: In Vekua equation (4.1) the general solution, which is already known, has the form (4.2). However, we are looking for a particular solution

$$w_p(z, \bar{z}) = U(x, y) + iV(x, y)$$

where both real and imaginary parts on the contour L meet the Hilbert boundary condition

$$a(t)U(t) + b(t)V(t) = c(t); \quad t \in L \tag{4.11}$$

where $a(t)$, $b(t)$ and $c(t)$ are the given continuous functions.

If we introduce a new expressions in (4.2)

$$\begin{gathered} e^{-I(A)} = C(z,\bar{z}) = c_1 + ic_2; \\ -e^{-I(A)} \cdot I(Be^{I(A)}) = D(z,\bar{z}) = d_1 + id_2; \\ Q(z) = q_1 + iq_2 \end{gathered}$$

we get

$$\begin{aligned} w &= (c_1 + ic_2)(q_1 + iq_2) + d_1 + id_2 \\ &= c_1q_1 + c_1iq_2 + c_2iq_1 - c_2q_2 + d_1 + id_2 \\ &= (c_1q_1 - c_2q_2 + d_1) + i(c_1q_2 + c_2q_1 + d_2). \end{aligned} \tag{4.12}$$

Substituting real and imaginary parts in (4.12) into boundary condition (4.11) we get

$$\begin{gathered} a(c_1q_1 - c_2q_2 + d_1) + b(c_1q_2 + c_2q_1 + d_2) = c \quad \text{ie.} \\ (ac_1 + bc_2)q_1 + (bc_1 - ac_2)q_2 = c - ad_1 - bd_2 \end{gathered} \tag{4.13}$$

However, relation in (4.13) is the Hilbert boundary form for the analytic functions. At this moment we move to an inverse aspect of the main problem in defining the general solution to the Vekua equation. The general solution is familiar, and we now focus on defining the analytic function $Q(z)$ based on given boundary problem (4.13). This problem can be solved using the Gakhov method (see [**5**]).

In [**6**] Čanak, Stefanovski and Protić have analyzed some boundary problems for the finite-integrable Vekua CDE. In this paper we will analyze only one simple example

Cherskiy [**7**] has analyzed the following boundary problem.

Boundary problem *R*. Let us observe three continuous real functions $a(x)$, $b(x)$ and $G(x)$. We need to define analytic function $Q(z)$ in the upper complex half-plane, which meets the following boundary condition on x-axis

$$a(x)Q(x) + b(x)Q'(x) = G(x)$$

This problem can be used on the Vekua CDE too.

Let us analyze the next special example.

Example 4.4a. It is necessary to find the solution for the Vekua CDE

$$w'_{\bar{z}} = -\frac{1}{z+\bar{z}}\bar{w} \tag{4.14}$$

which meets the following boundary condition on the x-axis

$$w(x) + xw'_{\bar{z}}(x) = \frac{x}{2}. \tag{4.15}$$

SOLUTION. Using formula (1.5) the general solution to equation (4.14) is

$$w(z,\bar{z}) = Q'(z) - \frac{Q+\bar{Q}}{z+\bar{z}} \quad (Q(z) \in \mathcal{A}) \tag{4.16}$$

moreover

$$w'_{\bar{z}} = -\frac{\bar{Q}'_{\bar{z}}}{z+\bar{z}} + \frac{Q+\bar{Q}}{(z+\bar{z})^2}$$

From this we get

$$w(x) = Q'(x) - \frac{Q(x)}{x}$$
$$w'_{\bar{z}}(x) = -\frac{Q'(x)}{2x} + \frac{Q(x)}{2x^2}$$

Substituting these results into (4.15) we get

$$Q'(x) - \frac{Q(x)}{x} = x \tag{4.17}$$

Condition (4.17) is the boundary problem R for determining the analytic function $Q(z)$. The solution to this problem is $Q(z) = z^2 + cz$, where c is an arbitrary constant. In the end, substituting these values into (4.16) we get

$$w(z,\bar{z}) = 2z - \frac{z^2+\bar{z}^2}{z+\bar{z}}$$

which is the solution to the boundary problem (4.14)–(4.15).

5. Conclusion

The principle of inversion is present in the theory of the Vekua CDE. However, it came to full significance when the class of fundamentally finite integrable Vekua equations was discovered

$$w'_{\bar{z}} = \frac{-\varphi'(z)}{\varphi+\bar{\varphi}}\bar{w},$$

where $\varphi(z)$ is the given analytic function − the kernel of the coefficient $A = \frac{-\varphi'}{\varphi+\bar{\varphi}}$ and the whole equation. Above all, in this paper it is shown that this important class of equations can be reached using fixed points which are later analyzed.

Furthermore, the attention should be directed to the three main constitutive elements of the equation, which are: the general solution $w(z,\bar{z})$, the kernel of the equation $\varphi(z)$ and the arbitrary analytic function $Q(z)$ which appears in the general solution. Each of these constitutive elements have a certain form: $w \to \{w, \bar{w}, w'_{\bar{z}}\}$; $\varphi \to \{\varphi, \bar{\varphi}, \varphi'_z\}$; $Q \to \{Q, \bar{Q}, Q'_z\}$.

In a boarder sense, we can understand the inversion of the Vekua equation in a way where the main constitutive elements change their roles (what is given and what we are looking for) in different ways. The last example in this paper–boundary problems are also one form of the inversion.

The advantage in understanding the Vekua theory in this way is the fact that a great number of its most important problems can be interpreted as one of numerous inversions of the main problem of determining the general solution. This way, using the fundamentally finite integrable equation on a small scale, we understand that in the general Vekua theory on a large scale as well, inversion represents the main moving force.

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