SIMPLE APPROXIMATE METHOD OF BEAM SHEAR FLOWS ANALYSIS

by

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Introduction.

Hatcher [1] developed a rational shear analysis of a symmetrical rectangular box girder in the case where the cross-section of the box is constant along the span. Sibert [2] derived simple formulas for the shear distribution in a box spar of any shape and of constant cross-section along the span, whether or not any portion of the skin is effective in bending. Shanley and Cozzone [3] developed a unit method of beam analysis in the case where the cross-section of the beam is not constant along the span. Below, a different approximate method of beam analysis in the last case is presented. In the type of box chosen, the moment of inertia and the static moment vary continuously. Consequently the method of this note enables one to find only the average value of the shearing stress and shearing flow in a panel and presents an approximation. An example shows the procedure.

Box loaded by a transverse load.

Let it be assumed that the box of a trapezoidal cross-section, tapered in plan form and in depth, consists of heavier caps along the edges, of a certain number of light stringers, the covering sheet, and a number of stiffening ribs perpendicular to the longitudinal axis. All the stringers have the same constant cross-sectional area throughout the span and are symmetrically located with respect to the vertical plane $y_1 z_1$. The cross-sectional areas of the caps are constant in each bay. The cross-sectional areas of the stringers and caps included in the calculation are not those in the planes of the normal cross sections but those in the planes of the ribs. Throughout all the calculations, it is assumed that in the corners of the cross-sections of the box in the plane of a rib, the centers of gravity of the extreme stringers coincide with the centers of gravity of the caps. All of the longitudinal elements carry normal stresses, while the covering sheet carries shearing stresses only. The following assumptions were made: (a) that part of the cross-sectional area of the covering sheet which is considered to be effective in resisting normal stresses is added to the cross-sectional areas of stringers and caps. Thus the stringers, the caps, and the effective width of the covering sheet, form the total cross-sectional area carrying the normal stresses; (b) the covering sheet and the webs carry shear only and no normal stresses; (c) the stringers offer negligible resistance to bending, i. e., they resist only tension or compression; (d) the intensity of normal stresses across the areas of stringers and caps is constant; (e) from the condition of membrane equilibrium, it follows that the shear stress and the shear flow will be constant in each panel bounded by two adjacent stringers and two adjacent ribs (Figures 1, 2, 3).

The stresses are calculated separately in each bay as if each bay were an independent unit. Assume that the external load causes a bending

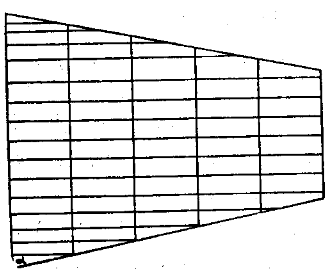

Fig. 1

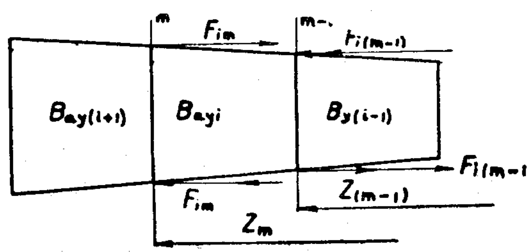

Fig. 2

moment and a shear force in each cross-section of the beam. From Fig. 2 one can easily see that in the first, rough approximation, the following relations are valid:

$$F_{im}=\frac{M_m}{h_m}=\frac{M_m \cdot y_{im}}{I_{im}} S_n , \tag{1}$$

$$\begin{aligned} q_{in}&=\frac{F_{im}-F_{i\,(m-1)}}{[z_m-z_{(m-1)}]}=\frac{S_n\,\sigma_{im}}{[z_m-z_{(m-1)}]}-\frac{S_n\,\sigma_{i\,(m-1)}}{[z_m-z_{(m-1)}]}= \\ &=\frac{P\,z_m\,y_{im}\,S_n}{I_{im}\,[z_m-z_{(m-1)}]}-\frac{P\,z_{(m-1)}\,y_{i\,(m-1)}\,S_n}{I_{i\,(m-1)}\,[z_m-z_{(m-1)}]}, \end{aligned} \tag{2}$$

or in dimensionless form:

$$\frac{q_{in}\,a}{P} = \frac{Q_{imn}\, z_m\, a}{I_{im}\,[z_m - z_{(m-1)}]} - \frac{Q_{i\,(m-1)\,n}\, z_{(m-1)}\, a}{I_{i\,(m-1)}\,[z_m - z_{(m-1)}]},$$

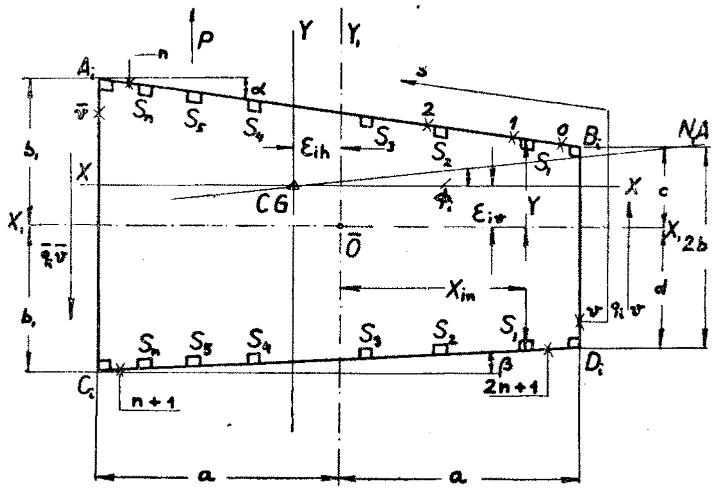

Fig. 3

where the symbols denote:

q_{in}, average shear flow in the i-th bay and n-th panel,

F_{tm}, σ_{im}, approximate normal force and normal stress in the i-th bay in the plane of the m-th rib,

$F_{i(m-1)}$, $\sigma_{i(m-1)}$, approximate normal force and normal stress in the i-th bay in the plane of the $(m—1)$-th rib,

S_n, cross-sectional area of an element resisting normal stresses; this area is in the plane of a rib,

I_{im}, y_{im}, moment of inertia of all the elements resisting normal stresses with respect to the neutral axis and the distance from the neutral axis in the plane of the m-th rib,

$I_{i(m-1)}$, $y_{i(m-1)}$, similar to above in the plane of the $(m—1)$-th rib.

Q_{imn}, $Q_{i(m-1)n}$, static moment with respect to the neutral axis of the element S_n in the plane of the m-th or $(m—1)$-th rib relatively,

M_m bending moment in the plane of the m-th rib,

z_m, distance of the m-th rib from the external load (arm of the moment),

P, shear force constant along the full span of the box,

a, length.

But the expressions $\frac{PQ_{imn}}{I_{im}}$ and $\frac{PQ_{i(m-1)n}}{I_{i(m-1)}}$ represent the values of the shear flow due to the dimensions existing in the plane of the m-th rib or (m—1)-th rib relatively. Consequently one may write:

$$\frac{q_{in}\,a}{P} = \frac{q_{imn}\,z_m\,a}{P[z_m - z_{(m-1)}]} - \frac{q_{i(m-1)n}\,z_{(m-1)}\,a}{P[z_m - z_{(m-1)}]}, \tag{4}$$

where the symbols denote:

q_{imn}, $q_{i(m-1)n}$, the values of the shear flows in the i-th bay and in the plane of the m-th or (m—1)-th rib relatively due to the dimensions in these planes. Thus the average shear flow in each panel may be approximately calculated as the difference between two shear flows in the planes of two consecutive ribs. These "component" shear flows may be calculated in the way explained in Reference [2]. The values of the average shear flows in the subsequent bays will, under the assumption that the external load P is attached to the first rib, be given by the formulas:

$$\begin{aligned}
&\text{a – bay,} && \frac{q_{an}\,a}{P} = \frac{q_{a2n}\,a}{P},\\
&\text{b – bay,} && \frac{q_{bn}\,a}{P} = \frac{2\,q_{b3n}\,a}{P} - \frac{q_{b2n}\,a}{P}\\
&\text{c – bay,} && \frac{q_{cn}\,a}{P} = \frac{3\,q_{c4n}\,a}{P} - \frac{2\,q_{c3n}\,a}{P},\\
&\text{d – bay,} && \frac{q_{dn}\,a}{P} = \frac{4\,q_{d5n}\,a}{P} - \frac{3\,q_{d4n}\,a}{P}, \text{ etc.}
\end{aligned} \tag{5}$$

In the case of a triangular panel, it is assumed that the average shear flow is equal to the shear flow in the plane of the rib where the panel has its finite width. Of course, some other assumption may be made.

As mentioned above the "component" shear flows may be calculated by the use of the method explained in Reference [2].

Here only the results will be given according to [2]. The shear flow at the point n is given by the formula (Fig. 3, 4):

$$q_{in} = q_{iv} - \frac{dR_{in}}{dz} = q_{iv} - \frac{PQ_{ivn}}{I_{im}}, \tag{6}$$

or

$$\frac{q_{in}\,a}{P}=\frac{q_{iv}\,a}{P}-\frac{Q_{ivm}\,a}{I_{im}}\,, \tag{7}$$

where q_{iv} denotes the value of the shear flow at the point v, and q_{in} the value of the shear flow at the point n following the running coordinate s from the point v; Q_{ivn} denotes the static moment about the neutral axis of all the elements resisting the normal stresses between the points v and n; dR_{in} denotes all the unbalanced normal forces between v and n.

In case the skin resists only shearing stresses, the value of q_{iv} for the homogeneous material of the covering sheet, is given by the formula in dimensionless form:

$$\frac{q_{iv}\,a}{P}=\frac{a}{(I_{im}\,\Sigma\,k_n)}(\Sigma\,k_n\,Q_{ivn})\,, \tag{8}$$

$$k_n=\frac{L_n}{t}\,, \tag{9}$$

where L_n denotes the length of the arc between two specified points n and $(n-1)$. Attention should be called to the fact that although L_n is always taken between two consecutive points, the value of Q_{ivn} must be referred to all the elements resisting normal stresses located between v and n. For the uniform thickness t of the skin, the last equation takes the form:

$$\frac{q_{iv}\,a}{P}=\frac{a}{(I_{im}\,\Sigma\,L_n)}(\Sigma\,L_n\,Q_{ivn})\,. \tag{10}$$

The shearing forces in each panel along the perimeter of the cross-section are obtained by multiplying the shear flow in this panel by the corresponding length L_n of the perimeter. Such shear forces must fulfill two equilibrium conditions

$$\Sigma\left(\frac{V}{P}\right)=1,\quad \frac{2\,q_{iv}\,b_2}{P}-\frac{2\,q_{i\bar{v}}\,b_1}{P}+$$

$$+\left[\frac{\Sigma\,(q_{in}\,L_n)}{P}\right]_{UP}\sin\alpha+\left[\frac{\Sigma\,(q_{in}\,L_n)}{P}\right]_{LP}\sin\beta=1\,, \tag{11}$$

$$\Sigma\left(\frac{H}{P}\right)=1,\quad \left[\frac{\Sigma\,(q_{in}\,L_n)}{P}\right]_{UP}\cos\alpha+\left[\frac{\Sigma\,(q_{in}\,L_n)}{P}\right]_{LP}\cos\beta=0\,, \tag{12}$$

where the subscripts UP, and LP, mean that the corresponding sums include all the shearing forces acting in the corresponding plane (upper or lower). The values of the shearing forces should be taken with the proper signs, i. e. positive if the vertical component is acting upwards or if the horizontal component is acting towards the left. Because of the assumption that the skin does not resist the normal stresses, the shear flow in each panel as well as in the vertical webs will be constant in each bay.

Box loaded by the longitudinal forces.

The method explained above may be applied to the case of a box loaded by the longitudinal forces perpendicular to the planes of the ribs. The forces in stringers will be denoted by R_{mn}, in the caps by T_{mA}, T_{mB}, etc. The subscript m denotes the rib considered. Let it be assumed that the sum of all the forces R_{mn}, T_{mA}, T_{mB}, etc. along the entire perimeter of the cross-section is equal to zero, and apply the calculation to the second bay. The forces R_{2n} in the upper plane and the forces T_{2C}, T_{2D}, are assumed to act towards the fixed end of the box; the forces R_{2n} in the lower plane and the forces T_{2A}, T_{2B}, are assumed to act from the fixed end. As a positive direction of forces, assume the direction towards the fixed end.

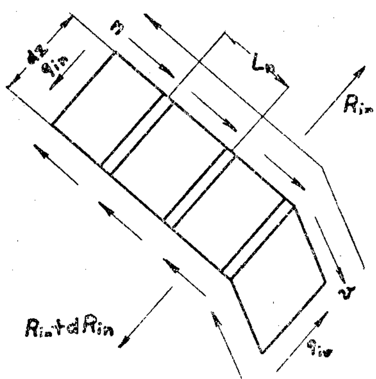

Fig. 4

To calculate the shear flows in the consecutive panels, the modified equations (6) and (10) will be used. Namely, instead of (PQ_{in}/I_{im}) one has to apply the expression $(R_2)_{vn}/(z_3-z_2)$ where the symbols $(R_2)_{vn}$ denote the sum of all the forces R_2 and T_2 located between the point v and the specified point n taken with the proper signs. Since $(z_3-z_2)=z_2$ one obtains:

$$\frac{\bar{q}_{in}\,a}{P}=\frac{\bar{q}_{iv}\,a}{P}-\frac{(R_2)_{vn}\,a}{P\,z_2}, \tag{13}$$

$$\frac{\bar{q}_{iv}\,a}{P}=\frac{a}{(z_2\Sigma L_n)}\sum\frac{(R_2)_{vn}}{P}L_n=$$

$$=\frac{a}{z_2\Sigma L_n}\sum\Upsilon_{2n}. \tag{14}$$

The calculation of the shearing forces performed in the same way as in (11) and (12), must give in the present case

$$\sum\left(\frac{V}{P}\right)=0, \quad \text{and} \quad \sum\left(\frac{H}{P}\right)=0.$$

The shear flow in the second bay is assumed to be equal to the mean value from two "component" shear flows, each of which is calculated for the dimensions in the plane of two consecutive ribs, at the beginning and at the end of the bay. Each of these two "component" shear flows is calculated in the way explained above under the condition that the forces R_{2n} , T_{2A} , T_{2B} , etc. are the same for these two component shear flows i. e. the forces are equal at both ends of the bay. In the case of a triangular panel the same assumption may be valid as in the previous chapter.

Examples:

(1) The numerical subscripts in the symbols used below refer to the ribs. The symbols used are explained in Fig. 3. Assume: Bay b, rib 2: $(2a)_2=28''$, $(2b_1)_2=12.6''$, $(2b_2)_2=9.8''$, $(c)_2=4.2''$, $(d)_2=5.6''$. Seven stringers in each plane, dimensions are given in Table 1a; $z_2=6''$, $z_3=12''$, $\tan\alpha=0.075$, $\tan\beta=0.025$, $\alpha=4^0\,17'21''$, $\beta=1^0\,25'56''$. The following values were calculated in the usual way: $(\varepsilon_{bh})_2=-2{,}8''$, $(\varepsilon_{bv})_2=0.28''$, $(\tan\Phi_b)_2=-0.02052$, $I_{b2}=98.93582''^4$. Bay b, rib 3: $(2a)_3=36''$, $(2b_1)_3=16.2''$, $(2b_2)_3=12.6''$, $(c)_3=5.4''$, $(d)_3=7.2''$. The following values were calculated: $(\varepsilon_{bh})_3=-3.6''$, $(\varepsilon_{bv})_3=0.36''$, $(\tan\Phi_b)_3=-0.02044$, $I_{b3}=163.35719''^4$. The external force P is attached to the first rib. The results of the calculations are given in Table 1a, 1b, 1c.

(2) Assume that the bay b is loaded by the longitudinal forces being in equilibrium. The values of these forces are given in the last row of the Table 1a. Their sum must be equal to zero. In the case given the error of the sum amounts of 0.054 percent. The results of the calculation are given in the lower part of the Table 1c. All the calculations were performed to the tenth decimal place. The results in the text are given only to four or five decimal places. Some deviations in the last decimal place may exist because of this cut—off. The forces in the corners of a cross-section, in the plane of a rib, are assumed to be distributed between the cap and the stringer, proportionally to their areas.

Various refinements may be introduced into this method similarly as is explained in Reference [3].

Accepting the distance between two consecutive ribs as a unit of the beam, the explained method presents a simple „unit“ method of shear flow analysis.

REFERENCES

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[2] H. W. Siber — Shear Distribution in a Sheet—Metal Box Spar. *Journal of the Aeronautical Sciences*, Vol. 5, №. 4. February, 1938, p. 134—137.

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B = bay
R = rib
a = 10″

UPPER PLANE

STRINGERS AND CAPS

	B_i	S_1	S_2	S_3	S_4	S_5	S_6	S_7	A_i
B. b, cross section dimensions, sq. in.	0.36	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.66
B. b, R. 2, x_{1n} inch	14	14	10	5	0	−5	−10	−14	−14
B. b, R. 3, x_{1n} inch	18	14	10	5	0	−5	−10	−14	−18
B. b, R. 2, y_{b2} inch	4.26392	4.26392	4.48177	4.75409	5.02640	5.29872	5.57104	5.78889	5.78889
B. b, R. 3, y_{b3} inch	5.48051	5.69867	5.91684	6.18954	6.46225	6.73496	7.00766	7.22583	7.44399
B. b, R. 2, $s_n y_{b2}$ in³	1.53501	0.38375	0 40336	0.42786	0.45237	0.47688	0.50139	0.52100	3.82067
B. b, R. 3, $s_n y_{b3}$ in³	1.97298	0.51288	0.53251	0.55706	0.58160	0.60614	0.63069	0.65032	4.91303
R. 2, T_{2A}/P, P_{2n}/P	−0.02639	−0.00659	0.01314	0.01400	0.01485	0.01571	0.01657	−0.00489	−0.03586

Table 1-a

U. PLANE, PANELS

	Web v	$B_i - S_1$	$B_i - S_2$ / $S_1 - S_2$	$B_i - S_3$ / $S_2 - S_3$	$S_3 - S_4$	$S_4 - S_5$	$S_5 - A_i$ / $S_5 - S_6$	$S_6 - A_i$ / $S_6 - S_7$	$S_7 - A_i$
R. 2 dimensions in.	9.8		4.01123	5.01403	5 01403	5.01403	5.01403	4.01123	
R. 3, dimensions in.	12.6	4.01123	4.01123	5.01403	5.01403	5.01403	5.01403	4.01123	4.01123
R. 2, $\frac{Q_{bvn}a}{I_{b2}}$	0		0.19394	0.23471	0.27795	0.32368	0.37188	0.42256	
R. 3, $\frac{Q_{bvn}a}{I_{b3}}$	0	0.12077	0.15217	0.18477	0.21887	0.25447	0.29158	0.33018	0.36999
R. 2, $\frac{L_n Q_{bvn}a}{I_{c2}}$	0		0.77793	1.17684	1.39368	1.62294	1.86463	1.69499	
R. 3, $\frac{L_n Q_{bvn}a}{I_{b3}}$	0	0.48446	0.61040	0.92645	1.09743	1.27594	1.46199	1.32446	1.48415

Table 1-b

LOWER PLANE

STRINGERS AND CAPS

Σ	C_i	S_7	S_6	S_5	S_4	S_3	S_2	S_1	D_i	Σ
1.65	0.51	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.21	1.35
	−14	−14	−10	−5	0	5	10	14	14	
	−18	−14	−10	−5	0	5	10	14	18	
	−6.80844	−6.80844	−6.62638	−6.39880	−6.17122	−5.94365	−5.71607	−5.53401	5.53401	
	−8.75260	−8.57085	−8.38910	−8.16191	−7.93472	−7.70753	−7.48034	−7.29859	7.11684	
8.52229	3.47230	0.61276	0.59637	0.57589	0.55541	0.53492	0.51444	0.49806	1.16214	8.52229
10.95724	4.46382	0.77137	0.75502	0.73457	0.71412	0.69367	0.67323	0.65687	1.49453	10.95724
0.00054	0.04200	0.00741	−0.01990	−0.01926	−0.01862	−0.01799	−0.01735	0.01295	0.03022	0.00054

L. PLANE, PANELS

Web		C_i-S_6	C_i-S_5			S_3-D_i	S_2-D_i		
$\bar{v}$	C_i-S_7	S_7-S_6	S_6-S_5	S_5-S_4	S_4-S_3	S_3-S_2	S_2-S_1	S_1-D_i	Σ
12.6		4.00128	5.0016	5.0016	5.0016	5.0016	4.00128		78.48754
16.2	4.00128	4.00128	5.0016	5.0016	5.0016	5.0016	4.00128	4.00128	100.91256
0.86139		0.44849	0.38821	0.33001	0.27387	0.21980	0.16780		
0.67075	0.39749	0.35027	0.30405	0.25909	0.21537	0.17291	0.13169	0.09148	
10.85362		1.79456	1.94171	1.65058	1.36979	1.09937	0.67143		27.91214
10.86620	1.59050	1.40155	1.52077	1.29587	1.07722	0.86483	0.52696	0.36607	28.17533

L O W E R P L A N E, P A N E L S

Web $\overline{v}$	$C_1 - S_7$	$C_1 - S_6$ $S_7 - S_6$	$C_1 - S_5$ $S_6 - S_5$	$S_5 - S_4$	$S_4 - S_3$	$S_3 - D_1$ $S_3 - S_2$	$S_2 - S_1$ $S_2 - S_1$	$S_1 - D_i$	Σ
−0.50577		−0.09287	−0.03259	0.02561	0.08175	0.13582	0.18781		$\Sigma V = 1.00024$ $\Sigma H = 0.00175$
−0.39154	−0.11829	−0.07107	−0.02485	0.02011	0.06382	0.10629	0.14750	0.18771	$\Sigma V = 1.00113$ $\Sigma H = -0.00000$
−0.17732	−0.11829	−0.04927	−0.01711	0.01461	0.04590	0.07676	0.10719	0.18771	
0.00053		0.04995	0.03005	0.01078	−0.00784	−0.02583	−0.04318		
0.00053	0.04254	0.04995	0.03005	0.01078	−0.00784	−0.02583	−0.04318	−0.03022	
0.00067		0.19988	0.15030	0.05394	−0.03922	−0.12920	−0.17278		0.10416
0.00086	0.17022	0.19988	0.15030	0.05394	−0.03922	−0.12920	−0.17278	−0.12095	0.22675
0.00132		−0.08104	−0.04787	−0.01576	0.01528	0.04526	0.07418		$\Sigma V = -0.00006$ $\Sigma H = -0.00175$
0.00285	−0.06715	−0.07951	−0.04633	−0.01422	0.01681	0.04679	0.07571	0.05412	$\Sigma V = -0.00044$ $\Sigma H = +0.00147$
0.00208	−0.06715	−0.08027	−0.04710	−0.01499	0.01604	0.04602	0.07494	0.05412	

UPPER PLANE, PANELS

B = bay
R = rib
$a = 10''$

	Web		$B_i - S_2$	$B_i - S_3$			$S_5 - A_t$	$S_6 - A_t$	
	ν	$B_i - S_1$	$S_1 - S_2$	$S_2 - S_3$	$S_3 - S_4$	$S_4 - S_5$	$S_5 - S_6$	$S_6 - S_7$	$S_7 - A_t$
R. 2, $\frac{q_{b2n}a}{P}$	0.35562		0.16168	0.12091	0.07766	0.03194	−0.01625	−0.06693	
R. 3, $\frac{q_{b3n}a}{P}$	0.27920	0.15842	0.12703	0.09443	0.06033	0.02472	−0.01237	−0.05098	−0.09079
B. b, $\frac{q_{bn}a}{P}$	0.20278	0.15842	0.09237	0.06795	0.04299	0.01751	−0.00849	−0.03503	−0.09079
R. 2, $\left[\frac{(R_2)\nu n}{P}\right]_2$	0		−0.03299	−0.01984	−0.00584	0.00900	0.02472	0.04129	
R. 3, $\left[\frac{(R_2)\nu n}{P}\right]_3$	0	−0.02639	−0.03299	−0.01984	−0.00584	0.00900	0.02472	0.04129	0.03640
R. 2, $\left[\frac{L_n(R_2)\nu n}{P}\right]_2$	0		−0.13233	−0.09951	−0.02932	0.04516	0.12395	0.16562	
R. 3, $\left[\frac{L_n(R_2)\nu n}{P}\right]_3$	0	−0.10586	−0.13233	−0.09951	−0.02932	0.04516	0.12395	0.16562	0.14600
R. 2, $\left[\frac{\bar{q}_{b2n}a}{P}\right]_2$	0.00221		0.05719	0.03529	0.01195	−0.01280	−0.03899	−0.06660	
R. 3, $\left[\frac{\bar{q}_{b3n}a}{P}\right]_3$	0.00374	0.04773	0.05873	0.03682	0.01349	−0.01126	−0.03745	−0.06507	−0.05692
B. b, $\left[\frac{\bar{q}_{bn}a}{P}\right]$	0.00297	0.04773	0.05796	0.03605	0.01272	−0.01203	−0.03822	−0.06583	−0.05692

Table 1-c