DIAGONAL STIFFENING OF A SIMPLY SUPPORTED SQUARE PLATE SUBMITTED TO SHEARING STRESSES

by

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A simply supported square plate is submitted to the action of uniformly distributed shearing stresses and stiffened by a diagonal rib.

If the rigidity of the stiffener is not sufficient, the inclined waves of the buckled plate run across the stiffener and buckling of the plate is accompanied by bending of the rib. By subsequent increase of the rigidity of the rib, we may finally attain a condition where each half of the plate will buckle as a triangular plate with simply supported edges and the rib will remain straight.

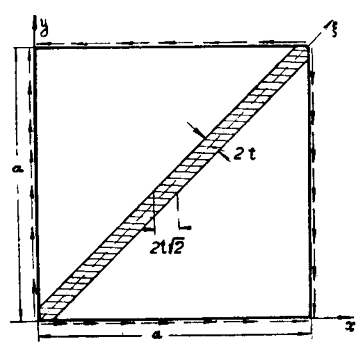

Fig. 1

The relation between the cross-sectional dimensions of the rib and the critical value of stresses in the plate can be obtained by using the energy method.

The boundary conditions at the supported edges are satisfied by taking for the deflection surface of the buckled plate the expression

$$w = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} a_{mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{a} \quad . \tag{1}$$

The strain energy of bending of the plate is

$$\Delta V = \frac{1}{2} D \int\int \left\{ \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right)^2 - 2(1-\nu) \left[\frac{\partial^2 w}{\partial x^2} \frac{\partial^2 w}{\partial y^2} - \left(\frac{\partial^2 w}{\partial x \partial y} \right)^2 \right] \right\} dx\, dy,$$

where D is the flexural rigidity of the plate.

Substituting expression (1) for w we obtain

$$\Delta V = \frac{1}{2} D \int_0^a \int_0^a \left\{ \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} a_{mn} \left(\frac{m^2 \pi^2}{a^2} + \frac{n^2 \pi^2}{a^2} \right) \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{a} \right\}^2 dx\, dy \,.$$

Only the squares of terms of the infinite series in the parenthesis yield integrals different from zero.

Observing then that

$$\int_0^a \int_0^a \sin^2 \frac{m\pi x}{a} \sin^2 \frac{n\pi y}{a}\, dx\, dy = \frac{a^2}{4} \;,$$

we obtain

$$\Delta V = \frac{D\pi^4}{a^2} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} a_{mn}^2 (m^2 + n^2)^2 \,. \tag{2}$$

The strain energy of bending of the rib, buckled together with the plate, is

$$\Delta V_1 = \frac{B}{2} \int_0^{a\sqrt{2}} \left(\frac{\partial^2 w}{\partial s^2} \right)^2 ds \;, \tag{2'}$$

denoting by B the flexural rigidity of a rib.

With the relations

$$s = x\sqrt{2}\,, \quad ds = dx\sqrt{2}$$

$$\frac{\partial^2}{\partial s^2} = \frac{\partial^2}{\partial x^2} \frac{1}{2} \;,$$

and substituting expression (1) for w, we obtain

$$\Delta V_1 = \frac{B}{2} \int_0^a \left(\frac{1}{2} \frac{\partial^2 w}{\partial x^2}\right)^2 \sqrt{2}\, dx =$$

$$= \frac{B\sqrt{2}\,\pi^4}{8\,a^4} \int_0^a \left\{ \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} a_{mn} \left[2\, mn \cos\frac{m\pi x}{a} \cos\frac{n\pi x}{a} - \left(m^2 + n^2\right) \sin\frac{m\pi x}{a} \sin\frac{n\pi x}{a} \right]\right\}^2 dx \;.$$

Taking into consideration that

$$\int_0^a \cos^2\frac{m\pi x}{a} \cos^2\frac{n\pi x}{a}\, dx = \frac{a}{4} \;,$$

$$\int_0^a \cos\frac{m\pi x}{a} \cos\frac{n\pi x}{a} \sin\frac{m\pi x}{a} \sin\frac{n\pi x}{a}\, dx = 0 \;, \quad \text{for } m \neq n \;,$$

and

$$\int_0^a \sin^2\frac{m\pi x}{a} \sin^2\frac{n\pi x}{a}\, dx = \frac{a}{4} \;,$$

one obtains

$$\Delta V_1 = \frac{B\pi^4\sqrt{2}}{32\,a^3} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} a_{mn}^2 \left(m^2 + 6\, m^2 n^2 + n^4\right) \;. \tag{3}$$

The work done by external forces during buckling of the plate is

$$\Delta T = T \int_0^a \int_0^a \frac{\partial w}{\partial x} \frac{\partial w}{\partial y}\, dx\, dy \;, \tag{3'}$$

where T is the shearing force per unit length.

Observing that

$$\int_0^a \cos\frac{m\pi x}{a} \sin\frac{p\pi x}{a}\, dx = 0 \;,$$

if $p \pm m$ is an even number and

$$\int_0^a \cos\frac{m\pi x}{a}\sin\frac{p\pi x}{a}\,dx = \frac{2a}{\pi}\frac{p}{p^2-m^2}$$

if $p\pm m$ is an odd number, we obtain

$$\Delta T = 8T\sum_m^\infty\sum_n^\infty\sum_p^\infty\sum_q^\infty a_{mn}\,a_{pq}\frac{m\,n\,p\,q}{(m^2-p^2)\,(q^2-n^2)}\,, \tag{4}$$

in which m, n, p, q are such integers that $p\pm m$ and $n\pm q$ are odd numbers.

The general equation for the calculation of critical stresses is

$$\Delta V+\Delta V_1=\Delta T\,. \tag{5}$$

Now from eq. (5) we obtain

$$\frac{D\pi^4}{8\,a^2}\sum_{m=1}^\infty\sum_{n=1}^\infty a_{mn}^2\,(m^2+n^2)^2+\frac{\pi^4 B\sqrt{2}}{32\,a^3}\sum_{m=1}^\infty\sum_{n=1}^\infty a_{mn}^2\,(m^4+6\,m^2\,n^2+n^4)=$$

$$=8\,T\sum_m^\infty\sum_n^\infty\sum_p^\infty\sum_q^\infty a_{mn}\,a_{pq}\frac{m\,n\,p\,q}{(m^2-p^2)\,(q^2-n^2)}\,. \tag{6}$$

It is necessary now to select such a system of constants a_{mn} and a_{pq} as to make T a minimum. Equating the derivatives of the expression (6) with respect to each of the coefficients a_{mn}, to zero, we obtain a system of homogeneous linear equations in a_{mn}

$$a_{mn}\,(m^2+n^2)^2+\frac{B\sqrt{2}}{4a\,D}\,a_{mn}\,(m^4+6m^2n^2+n^4)=$$

$$=\frac{32\,T a^2}{\pi^4 D}\,mn\sum_p^\infty\sum_q^\infty\frac{p\,q}{(m^2-p^2)\,(q^2-n^2)}\,.$$

Using the notations

$$\gamma=\frac{B}{a\,D}\,,\qquad \frac{1}{\lambda}=\frac{32\,a^2T}{D\,\pi^4}\,, \tag{7}$$

we have

$$\lambda\,a_{mn}\,(m^2+n^2)^2+\frac{\lambda\gamma\sqrt{2}}{4}\,a_{mn}\,(m^4+6\,m^2\,n^2+n^4)+$$

$$+\,mn\sum_p^\infty\sum_q^\infty a_{pq}\frac{p\,q}{(m^2-p^2)\,(n^2-q^2)}=0\,. \tag{7'}$$

The conditions that these equations should have a non-zero solution give the infinite determinantal equation

$$\begin{vmatrix} 5^2\lambda+\frac{41}{4}\lambda\gamma\sqrt{2}-\frac{4}{9} & +\frac{4}{5} & -\frac{8}{45} & +\frac{8}{25} & \cdots \\ +\frac{4}{5} & 13^2\lambda+\frac{313}{4}\lambda\gamma\sqrt{2}-\frac{36}{25} & -\frac{8}{7} & +\frac{72}{35} & \cdots \\ -\frac{8}{45} & -\frac{8}{7} & 17^2\lambda+\frac{353}{4}\lambda\gamma\sqrt{2}-\frac{16}{225} & -\frac{16}{35} & \cdots \\ +\frac{8}{25} & +\frac{72}{35} & -\frac{16}{35} & 25^2\lambda+\frac{1101}{4}\lambda\gamma\sqrt{2}-\frac{144}{49} & \cdots \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{vmatrix}=0$$

Taking for $\lambda=0{,}008$ [Cf. Prof. J. M. Klitchieff's: Buckling of a triangular plate by shearing forces — *Quart. Journ. Mech. and Applied Math.*, Vol. **IV**, Pt. 3 (1951)], we obtain

$$\begin{vmatrix} 0{,}082\sqrt{2}\,\gamma-\frac{11}{45} & +\frac{4}{5} & -\frac{8}{45} & +\frac{8}{25} & \cdots \\ +\frac{4}{5} & 0{,}626\sqrt{2}\,\gamma-\frac{11}{125} & -\frac{8}{7} & +\frac{72}{35} & \cdots \\ -\frac{8}{45} & -\frac{8}{7} & 0{,}706\sqrt{2}\,\gamma+\frac{2521}{1125} & -\frac{16}{35} & \cdots \\ +\frac{8}{25} & +\frac{72}{35} & -\frac{16}{35} & 2{,}202\sqrt{2}\,\gamma+\frac{101}{49} & \cdots \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{vmatrix}=0$$

The determinantal equation formed by neglecting all elements except those common to the first three rows and columns yields

$$\gamma=0{,}5806\,.$$

From (7) we have

$$B \geqslant a D \gamma,$$

$$B \geqslant 0{,}5806 \frac{a h^3 E}{12(1-\mu^2),} \quad . \tag{8}$$

Thus we have finally obtained the necessary relation between the flexural rigidity of the diagonal rib and the dimensions of the plate. As long as the inequality (8) is satisfied the rib retains its straight shape although the triangular parts of the square plate may begin to buckle.