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ESTIMATE OF PERTURBATION EFFECTS OF ASTEROID ORBITS DURING PROXIMITY

J. L. Simovljevitch

Summary. A simplified method is presented of estimate of the perturbation effect of an asteroid upon another asteroid's orbital plane elements during a high order proximity.

In the paper (**1**) we exposed a simplified procedure of calculation of first order special perturbations of the classical orbital elements of one asteroid produced by another, during their high order proximity. The simplification of the old, well known method effected on both the calculation of components of the perturbative acceleration and on the integration of differential equations of the selected osculating elements. The same simplifications, in principle, was also effected in the paper (**2**) on the vectorial-scalar elements **C** (double areal velocity), **D** (Laplace's integral of differential equation of the Keplerian motion) and T (time of perihelion passage). As a result the calculations proved reduced, at the cost, however, of our subsequent having to return to the astronomical orbital elements, in order to make perturbation effects clearly visualized. Making use of the main results of the paper quoted, we propose to go here a step furher in our simplification with the aim of obtaining true not accurate values, but only estimation of the perturbation effects, and even this not for all elements, but for those only where such estimate is quickly and easily attainable. Approximate effects obtained in this way will be very valuable in the preliminary investigation of every fairly close proximity. The confirmation that the possibility of existence of very high order proximities is quite a real thing has at last been provided by *J. Lazović* and *M. Kuzmanoski* in (**3**).

1. It has been shown in (**1**) that maximum perturbation effect, at least during proximities of the quasicomplanar asteroids, is to be expected in the longitude of the ascending node Ω and in the inclination of the orbital plane i. It is exactly these two elements that are involved in the forming of the vectorial element

$$\mathbf{C} = [\mathbf{r}\,\mathbf{v}] = C\,\mathbf{R} = C\,(\sin\Omega \sin i, \; -\cos\Omega \sin i, \; \cos i), \tag{1}$$

$$C = |\mathbf{C}| = \sqrt{\mu p}, \qquad \mu = k^2(1+m), \qquad p = a(1-e^2),$$

to which, as an osculating element of the planetary motion, the differential equation of the simplest form is attached

$$\frac{d\mathbf{C}}{dt} = [\mathbf{r}\,\mathbf{F}], \tag{2}$$

with the perturbative acceleration

$$\mathbf{F} = k^2 m_i \left(\rho^{-3}\, \rho - r_i^{-3}\, \mathbf{r}_i\right), \qquad \rho = \mathbf{r}_i - \mathbf{r}. \tag{3}$$

We have seen in (2) that the perturbative acceleration (3), caused by an asteroid with the mass m_i and the heliocentrical vector of position $\mathbf{r}_i$, during high order proximities can be substituted by

$$\mathbf{F} = U\,\rho, \qquad U = w\,k\,m_i\,\rho^{-3} \qquad \rho = \mathbf{r}_i - \mathbf{r}, \tag{4}$$

having regard to the requirements of the numerical integration, as well as the choice of an optimum time unit (mean Gaussian day). By inserting this in (2) and integration we obtain

$$\Delta\,\mathbf{C} = \int_{t_a}^{t_b} U\,[\mathbf{r}\,\mathbf{r}_i]\,dt, \tag{5}$$

where $\Delta\,\mathbf{C}$ denotes the perturbation of $\mathbf{C}$ in (1) within the time interval from the instant t_a to the instant t_b.

In (1) and (2) the simplification of the integration was performable thanks to the fact that the integration limits were very close, i.e. time interval of sensible perturbation very short. Within this short interval, which in most cases is less than half a day, cut in two by the instant t_p of the very proximity, only ρ^{-3} can undergo changes of substantial amount. Accordingly we put here, as we did in (1) and (2), for (5):

$$\Delta\,\mathbf{C} = 2\,[\mathbf{r}_p\,\mathbf{r}_{ip}] \int_{t_p}^{t_b} U\,dt, \tag{6}$$

where p indicates the poximity values of the position vectors of the perturbing ($\mathbf{r}_i$) and the perturbed ($\mathbf{r}$) asteroid. It is also evident that the variation of U is, by implication, strictly symmetrical before and after proximity. It should be noted, however, that the vector $[\mathbf{r}\,\mathbf{r}_i]$ is varying relatively rapidly, yet its variation is slower than that of U (i.e., of ρ^{-3}). Nevertheless we accept the expression (6) as the solution of our problem, as only estimate, and not exact value, of the perturbation is what we seek.

This being so, the numerical integration in (6) will be performed in the roughest way. A set of values U_{jw} for the equidistant moments $t_p + jw, j = 0, 1, 2, \ldots, q$, is calculated, proceeding from the moment of proximity t_p, using a preselected „integration step" w. Next we simply adopt

$$\int_{t_p-qw}^{t_p+qw} U\,dt = 2 \int_{t_p}^{t_p+qw} U\,dt = U_0 + 2\,(U_w + U_{2w} + \ldots + U_{qw}). \tag{7}$$

It follows therefore that ρ is calculated, for the moments $t = t_p + jw$, $j = 0, 1, 2, \ldots, q$, from the difference of the Lagrangean development of $\mathbf{r}_i$ and $\mathbf{r}$:

$$\rho = \rho_p + (\mathbf{v}_{ip} - \mathbf{v}_p)\,\tau - \frac{1}{2}\,(r_{ip}^{-3}\,\mathbf{r}_{ip} - r_p^{-3}\,\mathbf{r}_p)\,\tau^2 + \ldots, \tag{8}$$

$$\tau = k\,(t - t_p) = k\,j\,w.$$

The calculation of ρ^{-3} and U is then carried out for each moment. Then we proceed to the „integration" (7) and to the application of the final formula (6). By employing the formulae

$$\Delta C = (\mathbf{R} \Delta \mathbf{C}), \qquad \Delta \mathbf{R} = \frac{1}{C}(\Delta \mathbf{C} - \mathbf{R} \Delta C), \qquad C = \sqrt{p},$$

$$\Delta \Omega = (\cos \Omega \cdot \Delta R_x + \sin \Omega \cdot \Delta R_y) \operatorname{cosec} i,$$

$$\Delta i = (\sin \Omega \cdot \Delta R_x - \cos \Omega \cdot \Delta R_y) \sec i,$$

known from (2), we arrive finally at the perturbation in the elements Ω and i. The values thus obtained are evidently only approximate ones, however as repeatedly stated — it is whether noticeable perturbation of a newly discovered proximity is possible at all (say up to 0".5 in the above elements) that we, first of all, are interested in. If so, then we have every reason to expect this method to afford quite satisfactory results.

2. In the list of high order proximities of the quasicomplanar asteroid orbits given in (3), we find data on the asteroid pair (205) *Martha* and (992) *Swasey*, used in (1) and (2) for illustrating the fashion of perturbation calculation described there. Let us make estimation, by the method described here, of the perturbations in the elements Ω and i of the asteroid (992) *Swasey*, produced by (205) *Martha* at the time of their fictive proximity.

It has been stated in (2) that the advantage of the calculation, whereby direct form of the perturbative acceleration (3), i.e. (4), is used, consisted in that the series (8) is fast convergent one. For the high degree proximities the coefficient of τ^2 is small, thus higher order terms are negligible, since the time interval τ is short. In our case

$$\boldsymbol{\rho} = \begin{cases} -0.0000\ 0313 - 0.0001\ 0233\ \tau, \\ +0.0000\ 0701 + 0.0568\ 3298\ \tau, \\ +0.0000\ 3713 - 0.0108\ 5231\ \tau, \end{cases} \qquad \text{(ecliptic, 1950.0)}$$

is quite satisfactory. We proceed next to the calculation of ρ and U; as in previous papers it is taken that $w = 0.01$ day and that the mass of (205) *Martha* is $m_i =$ $= 10^{-13}$ solar mass. The results thus obtained are listed in *Table* 1. Another advan-

TABLE I

jw		$10^6\boldsymbol{\rho}$		$10^6\rho$	10^6U
0.00	−3	+ 7	+37	38	31
1	3	17	35	39	29
2	3	27	33	43	22
3	3	36	32	48	16
4	3	46	30	55	10
0.05	−3	+ 56	+28	63	7
6	3	66	26	71	5
7	3	75	24	79	3
8	3	85	22	88	3
9	3	95	20	97	2
0.10	−3	+105	+18	107	1
11	3	115	17	116	1
12	3	124	15	125	1
13	3	134	13	135	1
14	3	144	11	144	1
0.15	−3	+154	+ 9	154	0

tage of the present procedure becomes now apparent: integration limits do not have to be determined beforehand by trials, as in previous papers. The calculation is started with the instant of proximity, when U is maximum, and is being prosecuted until this variable gets negligible values. Making use of the last column in the *Table 1* we find, by „integration" just described,

$$2 \int_{t_p}^{t_p+0.15} w\ k\ m_i\ \rho^{-3}\ dt = 0.00235.$$

With the known proximity values of the position vectors $\mathbf{r}$ and $\mathbf{r}_i$ we calculate

$$[\mathbf{r}_p\ \mathbf{r}_{ip}] = (-981, +425, -163) \times 10^{-8},$$

therefore

$$\Delta \mathbf{C} = (-2305, +999, -383) \times 10^{-10}.$$

We find further, by formulae already indicated and taking into account elements of motion of the perturbed asteroid (992) *Swasey*,

$$\Delta C = +15 \times 10^{-10}, \quad \Delta \mathbf{R} = (-1328, +575, -230) \times 10^{-10},$$

$$\Delta\Omega = +4314 \times 10^{-10} = +0.''089, \quad \Delta i = +1221 \times 10^{-10} = +0.''025.$$

In (1) and (2) for the same perturbations we have obtained

$$+0.''088, +0.''025 \quad \text{and} \quad +0.''090, +0.''026.$$

So good a result cannot, obviously, be expected in each case, this especially if perturbations are substantial. Nevertheless, a plausible order of magnitude will be acquired and this by a very simple and expedient calculation.

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