FURTHER NOTE ON THE CALCULUS OF PERTURBATIONS OF ASTEROID ORBITS DURING PROXIMITY

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Summary. A simplification is made in calculations of relative coordinates of the perturbing asteroid during its proximity with the perturbed one.

In the paper (1) a transfc rmation has been performed of the classical firsl order special perturbations method of calculation of changes in the orbital elementt of an asteroid, caused by the gravitational action of another one during their pros ximity. This action becomes sensible for high degree proximities only (mutua- distances of the order of about 0.00005 *a.u.*, or less) and is of a very short duration (a couple of hours, or less). So one part of the transformations mentioned pertain to the calculating of relative coordinates of the perturbing asteroid $\xi_i - r$, η_i and ζ_i (in a coordinate system moving with the perturbed asteroid) by means of fast convergent time series. The other parts of transformation are not of interest in the present paper.

1. In (1) we firstly formed the Lagrangean series for the position vectors of both asteroids, $\mathbf{r}_i$ (perturbing) and $\mathbf{r}$ (perturbed) and then $\rho = \mathbf{r}_i - \mathbf{r}$. On the other hand, we developed series for the radius vector of perturbed asteroid r and its reciprocal r^{-1}, by the aid of

$$r = r_p + r_p' \tau + \frac{1}{2} r_p'' \tau^2 + \dots, \quad \tau = k(t - t_p),$$

and the differential equation $dr/d\tau = r^{-1} s$, $s = (\mathbf{r}\,\mathbf{v})$. The time unit is mean Gaussian day measured from t_p, the instant of proximity. Index p denotes the proximity values of all the variables. In this way we obtain, by multiplying necessary series,

$$\mathbf{a} = r^{-1}\mathbf{r}, \ \mathbf{b} = (C r)^{-1}[\mathbf{C}\,\mathbf{r}] = [\mathbf{R}\,\mathbf{a}], \ \mathbf{c} = C^{-1}\,\mathbf{C} = \mathbf{R},$$

i.e. the time series for the unit vectors **a** and **b** (**c** is constant in first order perturbations). By multiplying these series with those for ρ, we have finally

$$\xi_i - r = (\mathbf{a}\,\rho), \ \eta_i = (\mathbf{b}\,\rho) = ([\mathbf{R}\,\mathbf{a}]\,\rho), \ \zeta_i = (\mathbf{c}\rho) = (\mathbf{R}\,\rho). \quad (1)$$

All these computations are simple and the series easily evaluable, but the effective calculations may be reduced even more, which is what we intend to do in this paper.

2. If we put $\sigma_i = \xi_i - r$, η_i or ζ_i, we have generally

$$\sigma_i = \sigma_{ip} + \sigma'_{ip}\,\tau + \frac{1}{2}\,\sigma''_{ip}\,\tau^2 + \dots, \tag{2}$$

in which, according to (1),

$$\begin{aligned}
&(\xi_i - r)' = (\mathbf{a}'\,\rho) + (\mathbf{a}\,\rho'),\\
&(\xi_i - r)'' = (\mathbf{a}''\,\rho) + 2\,(\mathbf{a}'\,\rho') + (\mathbf{a}\,\rho''),\\
&\dots\dots\dots\dots\dots\dots\\
&\eta'_i = ([\mathbf{R}\,\mathbf{a}']\,\rho) + ([\mathbf{R}\,\mathbf{a}]\,\rho'),\\
&\eta''_i = ([\mathbf{R}\,\mathbf{a}'']\,\rho + 2\,([\mathbf{R}\,\mathbf{a}']\rho') + ([\mathbf{R}\,\mathbf{a}]\,\rho''),\\
&\dots\dots\dots\dots\dots\dots\\
&\zeta'_i = (\mathbf{R}\,\rho'),\quad \zeta''_i = (\mathbf{R}\,\rho''),\ \dots\dots
\end{aligned} \tag{3}$$

omitting for a moment the index p. The only variable vectors are $\mathbf{a}$ and ρ.

To form the derivatives $\mathbf{a}^{(j)}$ we note that the general form is

$$\mathbf{a}^{(j)} = \alpha_j\,\mathbf{r} + \beta_j\,\mathbf{v},\quad j = 0,\ 1,\ 2,\ \dots. \tag{4}$$

Making use of

$$\mathbf{a}^{(j+1)} = \alpha_{j+1}\,\mathbf{r} + \beta_{j+1}\,\mathbf{v} = (\alpha_j\,\mathbf{r} + \beta_j\,\mathbf{v})'$$

and the differential equation of Keplerian motion $\mathbf{r}'' = -r^{-3}\,\mathbf{r}$, we conclude that the scalar functions of time α_j and β_j follow from the recursive expressions

$$\alpha_{j+1} = \alpha'_j - r^{-3}\,\beta_j\,,\qquad \beta_{j+1} = \alpha_j + \beta'_j\,,$$

as the Lagrangean functions f and g in development of $\mathbf{r}$. The determination of α_j and β_j starts, obviously, with $\alpha_o = r^{-1}$, $\beta_o = 0$. The first couple terms are

$$\begin{aligned}
\mathbf{a}' &= -r^{-3}\,s\,\mathbf{r} + r^{-1}\,\mathbf{v},\\
\mathbf{a}'' &= r^{-3}\,(3\,r^{-2}\,s^2 - v^2)\,\mathbf{r} - 2\,r^{-3}\,s\,\mathbf{v},\\
&\dots\dots\dots\dots\dots\dots
\end{aligned}$$

in which

$$s = (\mathbf{r}\,\mathbf{v}),\quad s' = v^2 - r^{-1},\quad s'' = -r^{-3}\,s.$$

The general form for the derivatives $\rho^{(j)}$ is

$$\rho^{(j)} = \mathbf{r}_i^{(j)} - \mathbf{r}^{(j)} \tag{5}$$

with the well known derivatives of position vectors $\mathbf{r}_i$ and $\mathbf{r}$. In practice, however, most frequently we find that

$$\rho' = \mathbf{v}_i - \mathbf{v} \qquad \text{and} \qquad \rho'' = -(r_i^{-3}\,\mathbf{r}_i - r^{-3}\,\mathbf{r})$$

are quite sufficient.

With proximity values of $\mathbf{r}_i$, $\mathbf{v}_i$, $\mathbf{r}$ and $\mathbf{v}$ we calculate all the necessary terms in (4) and (5), so that (3) leads to the needed coefficients in (2).

3. In a high degree proximity the coordinates of vectors ρ_p and ρ_p'' are very small. In consequence we may neglect in (3) and (2) the terms of the form $(\mathbf{u}\,\rho_p)\,\tau^n$

and $(\mathbf{u}\,\rho_p'')\,\tau^n$, $n \geqslant 1$, $\mathbf{u}$ being an arbitrary vector. Accordingly we may use approximate expressions, which often give us sufficient accuracy:

$$\begin{aligned} \xi_i - r &= (\mathbf{a}_p\,\rho_p) + (\mathbf{a}_p\,\rho_p')\,\tau + (\mathbf{a}_p'\,\rho_p')\,\tau^2, \\ \eta_i &= (\mathbf{b}_p\,\rho_p) + (\mathbf{b}_p\,\rho_p')\,\tau + (\mathbf{b}_p'\,\rho_p')\,\tau^2, \\ \zeta_i &= (\mathbf{R}\,\rho_p) + (\mathbf{R}\,\rho_p')\,\tau, \end{aligned} \tag{6}$$

where

$$\mathbf{a} = r^{-1}\,\mathbf{r}, \quad \mathbf{b} = [\mathbf{R}\,\mathbf{a}], \quad \mathbf{a}' = \sqrt{p}\,r^{-2}\,\mathbf{b}, \quad \mathbf{b}' = -\sqrt{p}\,r^{-2}\,\mathbf{a}.$$

4. In a paper previously mentioned we have had, as an illustration, the computation of perturbative action of asteroid (205) *Martha* upon the orbit of asteroid (992) *Swasey*. The proximity of asteroid orbits was of the order of 0.000038 *a.u.*, as found by J. Lazović and M. Kuzmanoski in (**2**). From this paper we use the true anomalies

$$v_{ip} = 219^\circ\!.65970, \quad v_p = 51^\circ\!.28628,$$

corresponding to the fictive instant t_p, and, with the orbital elements from (**3**), we calculate

$$\begin{aligned} \mathbf{r}_{ip} &= (-\ 1.1696\ 4983,\ -\ 2.5861\ 0072,\ +\ 0.2961\ 9889), \\ \mathbf{v}_{ip} &= (+\ 0.5349\ 1962,\ -\ 0.2162\ 1942,\ +\ 0.0881\ 5062), \\ \mathbf{r}_p &= (-\ 1.1696\ 4670,\ -\ 2.5861\ 0773,\ +\ 0.2961\ 6176), \\ \mathbf{v}_p &= (+\ 0.5350\ 2195,\ -\ 0.2730\ 5240,\ +\ 0.0990\ 0293). \end{aligned} \quad \begin{aligned} &\text{(ecl.,} \\ &\text{1950.0)} \end{aligned}$$

From this and

$$\mathbf{R} = (-\ 0.1010\ 2860,\ +\ 0.1581\ 7837,\ +\ 0.9822\ 2850)$$

we have

$$\begin{aligned} \mathbf{a}_p &= (-\ 0.4098\ 6685,\ -\ 0.9062\ 2222,\ +\ 0.1037\ 8081), \\ \mathbf{b}_p &= (+\ 0.9065\ 3317,\ -\ 0.3920\ 9807,\ +\ 0.1563\ 8643), \\ \mathbf{a}_p' &= (+\ 0.1930\ 0200,\ -\ 0.0834\ 7815,\ +\ 0.0332\ 9485), \\ \mathbf{b}_p' &= (+\ 0.0872\ 6114,\ +\ 0.1929\ 3580,\ -\ 0.0220\ 9506), \end{aligned}$$

and

$$\begin{aligned} \rho_p &= (-\ 0.0000\ 0313,\ +\ 0.0000\ 0701,\ +\ 0.0000\ 3713), \\ \rho_p' &= (-\ 0.0001\ 0233,\ +\ 0.0568\ 3298,\ -\ 0.0108\ 5231), \\ \rho_p'' &= (+\ 0.0000\ 0020,\ -\ 0.0000\ 0015,\ -\ 0.0000\ 0162). \end{aligned}$$

Finally, according to (6),

$$\begin{aligned} \xi_i - r &= -\ 0.0000\ 0122 - 0.052\ 5876\ \tau - 0.005\ 1254\ \tau^2, \\ \eta_i &= +\ 0.0000\ 0022 - 0.024\ 0740\ \tau + 0.011\ 1960\ \tau^2, \\ \zeta_i &= +\ 0.0000\ 3790 - 0.001\ 6594\ \tau. \end{aligned}$$

These equations give the same result as in (**1**), as in our example we had $|t - t_p| \leqslant 0^d\!.15$ (i.e. $|\tau| < 0.003$).

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