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PERTURBING EFFECTS OF THE ASTEROID 215 OENONE ON THE ASTEROID 1851 ≡ 1950 *VA* DURING THEIR PROXIMITY

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Summary. The smallest mutual distance, hitherto known to us, of any two natural celestial bodies coming at, is associated with the asteroids 215 Oenone and 1851 ≡ 1950 *VA*. The perturbing effect of the former minor planet, as the more massive, upon the latter, smaller, at their proximity, is estimated. This is the first time ever, that the perturbing effects of measurable amount have been stated with asteroids at their close proximity, qualifying thereby such proximities as suitable for the determination of the masses of the minor planets.

The possibility of determination of the masses of the minor planets, from the dynamical effects due to the mutual perturbations during proximity in their orbits, has been emphasised in (1) as early as 1964. In (1) and (2) corresponding perturbations of some selected asteroids have been derived, yet the values obtained were still falling short of being measurable, although they were coming very close to. Accordingly, still closer proximities had to be sought out, such that noticeable, that is measurable, effects could have been expected. In (3) pairs of the quasicomplanar asteroids have been found with orbits of mutual inclinations $I \leqslant 0\overset{\circ}{.}500$ and minimum mutual distances $\rho < 0.0004$ AU. Among these pairs are asteroids 215 Oenone and 1851 ≡ 1950 *VA*, approaching each other to the smallest as yet known proximity distance of two natural celestial bodies, of barely 0.000004 AU, or 600 km. The inclination between their orbits was found to be $0\overset{\circ}{.}007$, another smallest value found for two asteroid orbits. With such a unique asteroid pair, we surmised, measurable perturbing effects, exerted by the larger, during their proximity, in the motion of the smaller one, might evolve. It was therefore quite natural to analyze the mutual perturbations first of this curious asteroid pair.

We employed here, as in (3), the orbital elements from (4), which are the same as in Ephemerides of Minor Planets for 1979. For comparison, they are given in Table I. In the last row of this Table differences are presented of the related orbital elements. Through them, already, even before we calculated the inclination I between their orbits, one was able to infere that one had to do with a quasicomplanar asteroid pair, in view of the small values of the differences of longitudes of ascending nodes ($0\overset{\circ}{.}146$) and of inclinations of their orbits ($0\overset{\circ}{.}005$). We see that there exists a big difference of the perihelion directions, about 20°, and that the second orbit is the more elongated one. In view of the very small angle between their orbits, $0\overset{\circ}{.}007$, it is evident that we are faced with a strikingly quasicomplanar asteroid pair, moving in almost one and the same plane. It has been found in (3) that the proximity positions, involving a distance of 0.000004 AU, was taking place with the true anomalies of the respective asteroids reach the values $v_{215} = 83^\circ.09412$ and $v_{1851} = 62^\circ.43319$. The difference of their heliocentric radii vectors at proximity is $r_{215} - r_{1851} = 0.0000000$ AU. The proximity distance between asteroids 589 Croatia and 1564 Srbija, found in (2), was over hundred times greater (0.000498 AU), than that of pair treated here. There, the role of the perturbing asteroid went to the formerly quoted asteroid as the more massive, its mass being evaluated at $m_{589} = 6 \times 10^{-14}$ of the solar mass. As already stated, the first order perturbations obtained for this pair were at the very limit of the measurable.

TABLE I

Asteroid	Epoch 0^h E. T.	M	ω	Ω	i	φ	n	a
			Ecl. and Equin. 1950.0					
215 Oenone	64 IX 2	347°.500	318°.572	25°.131	1°.700	1°.909	771''.476	2.7656
1851≡1950 *VA*	50 XI 15	24.536	339.087	25.277	1.705	10.940	648.498	3.1050
Δ (1851−215)		—	20.515	0.146	0.005	9.031	−122.978	0.3394

From (4), as well as from Ephemerides of Minor Planets for 1979, we find that the absolute magnitudes $B(1, 0)$ of the asteroid 215 and 1851, have the values 10.9 and 13.1 respectively. With these, the radii and the masses of the asteroids could be evaluated. Assuming both asteroids to belong to the dark or carbonaceous class, the following values of their radii and masses, in terms of solar mass, are found (5): $R_{215} = 29$ km, $R_{1851} = 11$ km, $m_{215} = 1.4 \times 10^{-13}$, $m_{1851} = 6.5 \times 10^{-15}$. Consequently, the asteroid 215, numbered earlier, is larger and more massive compared with the latter numbered asteroid 1851. We are therefore going to assume here the asteroid 215 Oenone as the perturbing one, its mass being evaluated at 1.4×10^{-13} solar mass. By using this mass we are going to calculate the first order perturbations, exercised by this asteroid during proximity upon the asteroid 1851 ≡ ≡ 1950 *VA*. The calculations have been carried out according to the formulae given in (2) and (6).

Kinematic duration of proximity, as defined and specified in (7), facilitated the finding out of the limits of the time interval, for which the calculation of the perturbation effects was warrantable. The integration step w, in the mean days units was chosen such, that the third differences in the corresponding integration table became practically constant. As the proximity distance in the present case is considerable smaller than the one we had in (2), a substantially narrower integration step could have been expected than that in (2). Our trials in chosing such a step showed that $w = 0.0002$ mean days could be adopted. The proximity time was taken as zero, $t_p = 0$, and it was from this that other times were reckoned, expressed in mean days with the sign − or +, according to wether they preceded or followed the instant of proximity.

Table II summarizes the calculated values of the changes of the orbital elements of the asteroid 1851 ≡ 1950 *VA*, as a result of the perturbing action of the asteroid 215 Oenone, during their proximity. The second column of this Table comprises the changes obtained for the intervals, reckoned from the moment the perturbing effects became measurable, up to the time of proximity. In the third column are changes for the intervals reckoned from the time of proximity up to the time after proximity, at which further perturbing effects on the corresponding orbital element had ceased being measurable. The forth column gives the total effects, exerted by the asteroid 215 upon the asteroid 1851 in the corresponding intervals, comprising the instant of proximity. We see that the moments at which the perturbing effects begin to make themselves felt, or at which they vanish, are different for different elements, being asymetrical with respect to the instant of proximity.

TABLE II

Δt	$-0^{d}.0228$ to 0	0 to $+0^{d}.0334$	$-0^{d}.0228$ to $+0^{d}.0334$
$\Delta\Omega$	$-3''.55$	$-3''.39$	$-6''.94$
Δt	$-0^{d}.0228$ to 0	0 to $+0^{d}.0334$	$-0^{d}.0228$ to $+0^{d}.0334$
$\Delta\omega$	$+3''.51$	$+3''.22$	$+6''.73$
Δt	$-0^{d}.0078$ to 0	0 to $+0^{d}.0192$	$-0^{d}.0078$ to $+0^{d}.0192$
$\Delta\tilde{\omega}$	$-0''.04$	$-0''.17$	$-0''.21$
Δt	$-0^{d}.0048$ to 0	0 to $+0^{d}.0066$	$-0^{d}.0048$ to $+0^{d}.0066$
Δi	$-0''.12$	$-0''.11$	$-0''.23$
Δt	$-0^{d}.0242$ to 0	0 to $+0^{d}.0320$	$-0^{d}.0242$ to $+0^{d}.0320$
ΔM	$-0''.24$	$+0''.43$	$+0''.19$
Δt	$-0^{d}.0186$ to 0	0 to $+0^{d}.0358$	$-0^{d}.0186$ to $+0^{d}.0358$
$\Delta\varphi$	$+0''.18$	$-0''.20$	$-0''.02$
Δt	$-0^{d}.0312$ to 0	0 to $+0^{d}.0342$	$-0^{d}.0312$ to $+0^{d}.0342$
Δn	$-0''.00065$	$+0''.00086$	$+0''.00021$

The perturbing effect of the asteroid 215 is reflected mostly in the longitude of the ascending node of the asteroid 1851. This change amounts to $-6''.94$, and it is the largest value obtained of the change of any orbital element, produced by the mutual perturbations of the asteroids during proximity of their orbits. In the case under consideration the perturbing effect on the node starts at -0.0228 mean days or 33 minutes ahead of proximity and ceases being noticeable at the moment $+0.0334$ mean days or 48 minutes after the instant of proximity. We gather from Table II that this angle is diminishing alike in the interval preceding and in that following the proximity. The change in the argument of perihelion $\Delta\omega$ is second by magnitude. It was derived from the calculated changes of the longitude of perihelion and the longitude of ascending node using the formula $\Delta\omega = \Delta\tilde{\omega} - \Delta\Omega$. The argument of perihelion is increasing in consequence of this perturbation. It could be stated here that its change starts and vanishes at the same instants as those relating to the longitude of the ascending node. The longitude of perihelion $\tilde{\omega}$ of the asteroid 1851 starts changing at $-0\overset{d}{.}0078$ or 11 minutes ahead of the instant of proximity, its further change vanishing at $+0\overset{d}{.}0192$ or 28 minutes following the proximity. The change of the longitude of perihelion is about four times greater in the interval subsequent to the proximity, than in the interval preceding it. The inclination i of the orbit of the asteroid 1851 decreases by almost equal amounts before and after passing the proximity. As is known, the change of the mean anomaly ΔM, produced by the perturbations, can be expressed as a sum of two terms, the first being due to the variation of the mean anomaly and the second is originating from the double integral of the derivative of the mean motion. In the present case the second term can be omitted as the value of it found was $0\overset{''}{.}00$. Thus, the first term alone embodies the sensible change of the mean anomaly. This change starts at $-0\overset{d}{.}0242$ or 35 minutes before proximity and vanishes at $+0\overset{d}{.}0320$ or 46 minutes after the instant of proximity. This change in the interval preceding the proximity amounts to $-0\overset{''}{.}24$, while the one following the proximity is $+0\overset{''}{.}43$, accordingli its total effect is below $+0\overset{''}{.}19$. The angle of eccentricity φ of the orbit of the asteroid 1851 starts changing at $-0\overset{d}{.}0186$ or 27 minutes ahead of proximity, whereby this changing lastuntil $+0\overset{d}{.}0358$ or 52 minutes after the proximity. The change before proximity and the one following it have opposite signs, the final result being that the total change of this element is close to the unmeasurable. The changes of the mean daily motion Δn before and after the proximity have also opposite signs.

Table III, for the corresponding change ΔE of the orbital element in the first column, gives in its second and third columns the times t_- and t_+ at which the perturbing effect, upon the determinate element E of the orbit of the asteroid 1851, starts and vanishes. In the fourth and fifth columns are mutual distances of the asteroid 215 and 1851 at the same times, expressed in the sixth decimal units of the astronomical unit. The true anomalies of both asteroids concerned at these times are given in the 6th, 7th, 8th and 9th columns. These data can be of practical use in observing these asteroids. The angular widths Δv of the parts of the orbits around proximity, whithin which it is assessed that the perturbing effect on particular orbital elements did exist, are given in the 10th and 11th columns. These widths are termed dynamical widths of proximity, for they represent the angular widths in the orbits around the proximity within which there is a measurable gravitational effect of the asteroid 215 on the particular orbital elements of the asteroid 1851. These have been calculated as differences of the previously cited true anomalies. The quantities with the index $_-$ are refered to the times preceding the proximity, at which the perturbing effect on determinate element do start, while those with the index $_+$ are related to the times after proximity, at which this effect vanishes. The last column of Table III contains, as we call them, dynamical durations of the proximity $\Delta t = t_+ - t_-$ which represent the durations of the perturbing effect, around proximity, on orbital elements of the asteroid 1851. The data in the last three columns too may be of practical interest in the corresponding observations as well as in further investigations.

TABLE III

ΔE	t_-	t_+	$10^6\rho_-$	$10^6\rho_+$	v^-_{215}	v^+_{215}	v^-_{1851}	v^+_{1851}	Δv_{215}	Δv_{1851}	Δt
Δn	$-0^{d}.0312$	$+0^{d}.0342$	44	48	$83^{\circ}.08737$	$83^{\circ}.10152$	$62^{\circ}.42616$	$62^{\circ}.44089$	$0^{\circ}.01415$	$0^{\circ}.01473$	$1^{h}34^{m}$
ΔM	−0.0242	+0.0320	34	45	83.08888	83.10104	62.42774	62.44040	0.01216	0.01266	1 21
$\Delta \Omega$	−0.0228	+0.0334	32	47	83.08919	83.10135	62.42805	62.44071	0.01216	0.01266	1 21
$\Delta \varphi$	−0.0186	+0.0358	26	50	83.09010	83.10187	62.42900	62.44125	0.01177	0.01225	1 18
$\Delta \tilde{\omega}$	−0.0078	+0.0192	11	27	83.09243	83.09827	62.43143	62.43751	0.00584	0.00608	0 39
Δi	−0.0048	+0.0066	8	10	83.09308	83.09555	62.43211	62.43468	0.00247	0.00257	0 16

The maximum dynamical width of this proximity is $0^\circ\!.01473$, while its longest dynamical duration is 1^h34^m, corresponding to a change in the mean daily motion $\Delta n = +0''\!.00021$ of the asteroid 1851. On comparing these values with those in (7), relating to the kinematic width and the kinematic duration of proximity of the same pair, where the parts of their orbits, comprising the proximity, were delimited by their distances of 0.0004 AU, one realizes that the dynamical width along with the dynamical duration of that proximity is about 9 times smaller than its kinematic width and kinematic duration. This is quite understandable considering that the upper limit of the mutual distance of this pair, at which the perturbing effect begins to be sensible, is 8 times smaller than the upper limiting value of the mutual distance of the two asteroids, adopted in (7) and (3). It should be noted that the values of the changes $\Delta\omega$ of the argument of perihelion are omitted from Table III, as these in the present case are equal to those of $\Delta\Omega$.

We contented ourselves here with the first order perturbations. According to the differential equations relating to the elliptical orbital elements, the amounts of the perturbations depend upon $m_i\,\rho^{-3}$. Consequently, with the very close passing-by of two minor planets or small their mutual distance ρ, this quantity depends on the amount of the mass m_i of the perturbing asteroid for the perturbing effects to become measurable. Were we to adopt for the mass of the perturbing asteroid 215 some other value m', instead of the one valued here at 1.4×10^{-13} of the solar mass, then the new corresponding changes of the orbital elements of the perturbed asteroid 1851, for the same time intervals, would have been obtained by multiplying by $m'\times10^{13}\times1.4^{-1}$ the values here obtained.

The results obtained suggest the conclusion that the observations of the singled out asteroids at their high order proximities might yield measurable effects on the orbital elements of the perturbed asteroid, from which the mass of the perturbing asteroid might be found out. Thus, the dynamical effects produced by the mutual actions of the minor planets during their proximities offer the prospect of deriving the gravitational estimates of the masses of these interesting celestial bodies, which would be more reliable than those found by various empirical formulae.

The computation has been performed on IBM 360/44 in the Computing Centre of the Institute for Mathematics in Beograd.

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