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MASSES OF SOME NUMBERED MINOR PLANETS

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Summary. Linear radii and masses of 142 asteroids, constituting 77 different pairs, closely approaching each other during proximity, are estimated. The estimate was effected by way of the known absolute magnitudes of the asteroids and their relations with the radii and geometric albedos. The results are presented in two variants: the one bearing upon the dark (carbonaceous type) asteroids and the other upon the light (silicaceous type) ones. Mean values of albedo and density were employed: 0.04 and 2.6 g cm^{-3} for the dark, and 0.15 and 3.5 g cm^{-3} for the light asteroids.

In order to tackle the preliminary calculation of the mutual perturbations of asteroids at their passage through proximity it is necessary to know their masses, if only by their approximate values. In (**1**) 77 different quasicomplanar asteroid orbits are found, with minimum distances under 0.0004 AU (60000 km), formed by 142 numbered minor planets. Among them one single 1635 Bohrmann appears paired thrice, ten of them, numbered 227, 379, 452, 461, 848, 938, 1130, 1200, 1581, 1782, are paired twice, and the rest of them only once. The estimate of asteroid masses, necessary for our further investigation of their motions, implies our assuming their shape to be spherical and adopting some mean values of their albedos and densities. The corresponding calculation may be performed using known absolute magnitudes of asteroids and their relations with the geometric albedo and radius, assuming the phase angle (the Sun — the asteroid — the Earth) $i = 0$.

The well known Pogson's low

$$\log \frac{E}{E_{\odot}} = 0.4\ (m_{\odot} - m) \tag{1}$$

gives the relation of the illumination E, received on the Earth from the asteroid at the time of its being at a geocentric distance Δ and heliocentric distance r, the illumination $E_{\odot}$ received on the Earth from the Sun at the distance 1 AU, with the corresponding apparent magnitudes of the asteroid and the Sun, m and $m_{\odot}$ respectively. For $i = 0$ and $r = \Delta = 1$ AU the absolute magnitude g of the asteroid is represented by the formula

$$g = m - 5 \log r\Delta, \tag{2}$$

while its geometric albedo p is given by the expression, (**2**),

$$p = \frac{E}{E_{\odot}} \frac{r^2 \Delta^2}{R^2}, \tag{3}$$

where R is the linear radius of the asteroid. All the lengths in (3) are given in astronomical units. Upon finding the logarithm of (3) and the substituting of (1) and (2), assuming $\log R_{AU} = \log R_{km} - 8.1749$ and passing over to the magnitudes B of the UBV sistem, we obtain the corresponding expression for R in kilometers

$$\log R = 8.1749 + 0.2\, B_{\odot} - 0.5 \log p - 0.2\, B\,(1, 0), \tag{4}$$

where $B\,(1, 0)$ is absolute asteroid's magnitude, supposed at a distance of 1 AU simultaneously from the Sun and the Earth and an phase angle zero. Concerning the Sun we have the values $V_{\odot} = -26^{m}.77$ and $(B - V)_{\odot} = +0^{m}.63$, (3), thus (4) is turned into

$$\log R = 2.9469 - 0.5 \log p - 0.2\, B\,(1, 0). \tag{5}$$

For the geometric albedos (in B, blue) of the first four numbered minor planets in (3) the values found are: 0.12 for 1 Ceres, 0.11 for 2 Pallas, 0.23 for 3 Juno and 0.40 for the asteroid 4 Vesta. In (4) are presented the values of the geometric albedos of 13 light asteroids, determined from the polarimetric and radiometric observations, from which I found the mean values of these albedos 0.16 (from polarimetric) and 0.13 (from radiometric observations). However, we learn from the Reports (5) of the XVI IAU Assembly that asteroid albedos are ranging from 0.02 to 0.4, that they are darker and larger than previously believed, and that two classes of them are distinguishable: 1) those with a low albedo, chondritic surfaces and moving within the outer band (about 3 AU) of asteroid belt, and 2) those with higher albedo, metallic surfaces and semimajor axes of 2.6 AU on the average. It is pointed out in (5) that asteroid albedos displayed a bimodal distribution, tending to the values 0.03 to 0.06 for the dark asteroids, and to 0.10 to 0.20 for the light ones. With regard to the foregoing the following two values of albedo are adopted:

$$p_d = 0.04 \quad \text{and} \quad p_l = 0.15, \tag{6}$$

of which use will hereafter be made of. The first value, with the index $_d$, is referring to the dark, and the second, with the index $_l$ to the light asteroids. Using (5) and (6) we obtain the corresponding expressions by which the average, that is, hypothetical linear radii R_d of the dark and R_l of the light asteroids in kilometers are derived:

$$\log R_d = 3.6459 - 0.2\, B\,(1, 0), \quad \log R_l = 3.3589 - 0.2\, B\,(1, 0). \tag{7}$$

These formulae are differing from the known formula $\log R = 3.3135 - 0.2 g$, which I employed earlier in (6), where the mean albedo of asteroids adopted was 0.24. The formulae (7) differ also from that in (7) and (8).

Assuming the asteroids to be, on the average, spherical in shape, their mean density to be σ, and using the values of R obtained by (7), their corresponding mass M can be derived by means of

$$M = \frac{4}{3} \pi \sigma R^3. \tag{8}$$

By finding the logarithm of (8) and after making the substitution (5), a general expression for the asteroid mass in grams is derived as a function of its density σ (in g cm^{-3}), of its geometric albedo p and absolute magnitude $B\,(1, 0)$

$$\log M = 24.4628 + \log \sigma - 1.5 \log p - 0.6\, B\,(1, 0). \tag{9}$$

Taking the Sun's mass to be 1.9891×10^{33} g, (**9**), we find from (9) the corresponding general expression for the mass of any asteroid in terms of solar mass

$$\log M = -8.8359 + \log \sigma - 1.5 \log p - 0.6\, B(1,0), \qquad (10)$$

where σ is given in $\mathrm{g\,cm^{-3}}$ units.

Currently, for the masses of three largest minor planets, expressed in terms of solar mass, the following values are adopted, (**9**), (**10**),

1 Ceres	5.9×10^{-10}
2 Pallas	1.1×10^{-10}
4 Vesta	1.2×10^{-10}

Using these masses and the latest values of their radii, derived from albedos, the following densities in $\mathrm{g\,cm^{-3}}$ are obtained: 2.6 for Ceres, 2.5 for Pallas and 3.6 for the asteroid Vesta, (**5**). Earlier, the mean asteroid density I adopted was 3.3 $\mathrm{g\ cm^{-3}}$, (**6**). Having regard to the two previous and two subsequent values, I am now going to adopt the following two mean values of asteroid density:

$$\sigma_d = 2.6\ \mathrm{g\,cm^{-3}} \quad \text{and} \quad \sigma_l = 3.5\ \mathrm{g\,cm^{-3}}. \qquad (11)$$

The first value is relating to the dark and the second to the light asteroid class, the distinction made according to their albedos. From (9) and (10), following the substitutions (6) and (11), the expressions are derived for the asteroid mass in grams:

$$\log M_d = 26.9748 - 0.6\, B(1,0), \quad \log M_l = 26.2429 - 0.6\, B(1,0), \qquad (12)$$

and in terms of solar mass:

$$\log M_d = -6.324 - 0.6\, B(1,0), \quad \log M_l = -7.056 - 0.6\, B(1,0), \qquad (13)$$

the quantities with the index $_d$ referring to the dark and those with the index $_l$ to the light asteroids.

TABLE I

$B(1,0)$	km		$M_\odot = 1$	
	R_d	R_l	M_d	M_l
5.0	442	229	5.0×10^{-10}	0.9×10^{-10}
6.0	279	144	1.2×10^{-10}	2.2×10^{-11}
7.0	176	91	3.0×10^{-11}	5.5×10^{-12}
8.0	111	57	7.5×10^{-12}	1.4×10^{-12}
9.0	70	36	1.9×10^{-12}	3.5×10^{-13}
10.0	44	23	4.7×10^{-13}	8.8×10^{-14}
11.0	28	14	1.2×10^{-13}	2.2×10^{-14}
12.0	18	9.1	3.0×10^{-14}	5.5×10^{-15}
13.0	11	5.7	7.5×10^{-15}	1.4×10^{-15}
14.0	7.0	3.6	1.9×10^{-15}	3.5×10^{-16}
15.0	4.4	2.3	4.7×10^{-16}	8.8×10^{-17}
16.0	2.8	1.4	1.2×10^{-16}	2.2×10^{-17}
17.0	1.8	0.9	3.0×10^{-17}	5.5×10^{-18}
18.0	1.1	0.6	7.5×10^{-18}	1.4×10^{-18}

Table I summarizes the corresponding values of radii in kilometers and the masses in terms of solar mass, as derived according to the formulae (7) and (13) for different equidistant values of absolute magnitudes $B(1,0)$. Table II presents the corresponding data on 142 numbered asteroids, whose small mutual distances during proximities were found earlier, (1). The absolute magnitudes from (11) of these asteroids, figuring in the first column of Table II, were then used to obtain, by means of the formulae (7) and (13), two values of the radius and the mass to each one of the asteroids. The values corresponding to the quantities with the index $_d$ are relating to the asteroid assumed to belong to the dark class, while those with the index $_l$ relate to the light class. The values of radii are roundedoff to 1 km for $R \geqslant 10$ km, and to 0.1 km for $R < 10$ km. From here adopted albedos (6) and densities (11), employing the values from Tables I and II, it is inferred that $R_d \approx \approx 2R_l$ and $M_d \approx 5M_l$. It can be seen from Table II that the maximum values of the radii and the masses in terms of solar mass: Max $R_d = 184$ km, Max $R_l = = 95$ km, Max $M_d = 3.4 \times 10^{-11}$, Max $M_l = 6.4 \times 10^{-12}$, are associated here with the lightest of examined asteroids 16 Psyche, which has the lowest absolute magnitude $B(1,0) = 6.9$. The minimum values obtained of the radius and the mass are: Min $R_d = 3.7$ km, Min $R_l = 1.9$ km, Min $M_d = 2.7 \times 10^{-16}$, Min $M_l = 5.1 \times \times 10^{-17}$, which are associated with the asteroid 1131 Porzia, which is standing out here by the highest absolute magnitude $B(1,0) = 15.4$.

TABLE II

Asteroid №	km		$M_\odot = 1$	
	R_d	R_l	M_d	M_l
16	184	95	3.4×10^{-11}	6.4×10^{-12}
21	81	42	2.9×10^{-12}	5.3×10^{-13}
24	101	52	5.7×10^{-12}	1.1×10^{-12}
39	147	76	1.7×10^{-11}	3.2×10^{-12}
43	64	33	1.4×10^{-12}	2.7×10^{-13}
47	64	33	1.4×10^{-12}	2.7×10^{-13}
50	37	19	2.7×10^{-13}	5.1×10^{-14}
76	70	36	1.9×10^{-12}	3.5×10^{-13}
79	61	32	1.2×10^{-12}	2.3×10^{-13}
84	39	20	3.1×10^{-13}	5.8×10^{-14}
110	84	44	3.3×10^{-12}	6.1×10^{-13}
111	67	35	1.6×10^{-12}	3.0×10^{-13}
143	34	17	2.1×10^{-13}	3.8×10^{-14}
163	35	18	2.4×10^{-13}	4.4×10^{-14}
171	49	25	6.3×10^{-13}	1.2×10^{-13}
205	35	18	2.4×10^{-13}	4.4×10^{-14}
211	70	36	1.9×10^{-12}	3.5×10^{-13}
212	56	29	9.5×10^{-13}	1.8×10^{-13}
215	29	15	1.4×10^{-13}	2.5×10^{-14}
227	40	21	3.6×10^{-13}	6.7×10^{-14}
243	25	13	9.0×10^{-14}	1.7×10^{-14}
251	24	13	7.9×10^{-14}	1.5×10^{-14}
263	20	10	4.5×10^{-14}	8.4×10^{-15}
277	25	13	9.0×10^{-14}	1.7×10^{-14}
280	18	9.1	3.0×10^{-14}	5.5×10^{-15}
311	25	13	9.0×10^{-14}	1.7×10^{-14}
335	44	23	4.7×10^{-13}	8.8×10^{-14}
355	21	11	5.2×10^{-14}	9.6×10^{-15}
367	17	8.7	2.6×10^{-14}	4.8×10^{-15}
376	34	17	2.1×10^{-13}	3.8×10^{-14}
379	42	22	4.1×10^{-13}	7.7×10^{-14}
384	32	17	1.8×10^{-13}	3.3×10^{-14}

TABLE II (continued)

Asteroid №	km		$M_{\odot}=1$	
	R_d	R_l	M_d	M_l
389	61	32	1.2×10^{-12}	2.3×10^{-13}
400	23	12	6.9×10^{-14}	1.3×10^{-14}
406	23	12	6.9×10^{-14}	1.3×10^{-14}
412	39	20	3.1×10^{-13}	5.8×10^{-14}
452	9.2	4.8	4.3×10^{-15}	8.0×10^{-16}
460	18	9.1	3.0×10^{-14}	5.5×10^{-15}
461	21	11	5.2×10^{-14}	9.6×10^{-15}
469	44	23	4.7×10^{-13}	8.8×10^{-14}
534	28	14	1.2×10^{-13}	2.2×10^{-14}
548	13	6.9	1.3×10^{-14}	2.4×10^{-15}
554	56	29	9.5×10^{-13}	1.8×10^{-13}
557	11	5.5	6.5×10^{-15}	1.2×10^{-15}
577	31	16	1.6×10^{-13}	2.9×10^{-14}
586	37	19	2.7×10^{-13}	5.1×10^{-14}
650	8.4	4.4	3.3×10^{-15}	6.1×10^{-16}
685	11	5.7	7.5×10^{-15}	1.4×10^{-15}
703	8.1	4.2	2.9×10^{-15}	5.3×10^{-16}
722	10	5.2	5.7×10^{-15}	1.1×10^{-15}
750	11	5.7	7.5×10^{-15}	1.4×10^{-15}
753	18	9.5	3.4×10^{-14}	6.4×10^{-15}
763	7.3	3.8	2.2×10^{-15}	4.0×10^{-16}
765	6.7	3.5	1.6×10^{-15}	3.0×10^{-16}
794	15	7.6	1.7×10^{-14}	3.2×10^{-15}
799	22	11	6.0×10^{-14}	1.1×10^{-14}
813	10	5.2	5.7×10^{-15}	1.1×10^{-15}
848	18	9.5	3.4×10^{-14}	6.4×10^{-15}
873	15	7.6	1.7×10^{-14}	3.2×10^{-15}
891	24	13	7.9×10^{-14}	1.5×10^{-14}
938	15	7.6	1.7×10^{-14}	3.2×10^{-15}
954	19	10	3.9×10^{-14}	7.3×10^{-15}
960	6.4	3.3	1.4×10^{-15}	2.7×10^{-16}
962	13	6.6	1.1×10^{-14}	2.1×10^{-15}
972	31	16	1.6×10^{-13}	2.9×10^{-14}
985	6.4	3.3	1.4×10^{-15}	2.7×10^{-16}
991	18	9.5	3.4×10^{-14}	6.4×10^{-15}
992	15	7.9	2.0×10^{-14}	3.7×10^{-15}
993	9.2	4.8	4.3×10^{-15}	8.0×10^{-16}
1037	4.0	2.1	3.6×10^{-16}	6.7×10^{-17}
1044	15	7.9	2.0×10^{-14}	3.7×10^{-15}
1060	5.8	3.0	1.1×10^{-15}	2.0×10^{-16}
1078	12	6.0	8.6×10^{-15}	1.6×10^{-15}
1079	17	8.7	2.6×10^{-14}	4.8×10^{-15}
1082	20	10	4.5×10^{-14}	8.4×10^{-15}
1092	19	10	3.9×10^{-14}	7.3×10^{-15}
1100	15	7.6	1.7×10^{-14}	3.2×10^{-15}
1130	8.8	4.6	3.8×10^{-15}	7.0×10^{-16}
1131	3.7	1.9	2.7×10^{-16}	5.1×10^{-17}
1135	19	10	3.9×10^{-14}	7.3×10^{-15}
1137	17	8.7	2.6×10^{-14}	4.8×10^{-15}
1142	21	11	5.2×10^{-14}	9.6×10^{-15}
1169	5.8	3.0	1.1×10^{-15}	2.0×10^{-16}
1187	12	6.0	8.6×10^{-15}	1.6×10^{-15}
1200	19	10	3.9×10^{-14}	7.3×10^{-15}
1203	10	5.2	5.7×10^{-15}	1.1×10^{-15}
1245	28	14	1.2×10^{-13}	2.2×10^{-14}
1251	18	9.5	3.4×10^{-14}	6.4×10^{-15}
1289	21	11	5.2×10^{-14}	9.6×10^{-15}

TABLE II (continued)

Asteroid №	km		$M_\odot = 1$	
	R_d	R_l	M_d	M_l
1305	21	11	5.2×10^{-14}	9.6×10^{-15}
1307	9.7	5.0	5.0×10^{-15}	9.2×10^{-16}
1325	9.7	5.0	5.0×10^{-15}	9.2×10^{-16}
1331	21	11	5.2×10^{-14}	9.6×10^{-15}
1335	4.4	2.3	4.7×10^{-16}	8.8×10^{-17}
1363	13	6.6	1.1×10^{-14}	2.1×10^{-15}
1374	4.9	2.5	6.3×10^{-16}	1.2×10^{-16}
1381	11	5.7	7.5×10^{-15}	1.4×10^{-15}
1393	10	5.2	5.7×10^{-15}	1.1×10^{-15}
1397	12	6.3	9.9×10^{-15}	1.8×10^{-15}
1443	15	7.6	1.7×10^{-14}	3.2×10^{-15}
1446	7.0	3.6	1.9×10^{-15}	3.5×10^{-16}
1462	16	8.3	2.3×10^{-14}	4.2×10^{-15}
1487	18	9.5	3.4×10^{-14}	6.4×10^{-15}
1492	5.8	3.0	1.1×10^{-15}	2.0×10^{-16}
1527	8.1	4.2	2.9×10^{-15}	5.3×10^{-16}
1534	11	5.7	7.5×10^{-15}	1.4×10^{-15}
1539	17	8.7	2.6×10^{-14}	4.8×10^{-15}
1541	13	6.9	1.3×10^{-14}	2.4×10^{-15}
1551	8.1	4.2	2.9×10^{-15}	5.3×10^{-16}
1560	12	6.0	8.6×10^{-15}	1.6×10^{-15}
1581	23	12	6.9×10^{-14}	1.3×10^{-14}
1590	11	5.5	6.5×10^{-15}	1.2×10^{-15}
1630	13	6.9	1.3×10^{-14}	2.4×10^{-15}
1634	5.3	2.7	8.2×10^{-16}	1.5×10^{-16}
1635	12	6.3	9.9×10^{-15}	1.8×10^{-15}
1644	16	8.3	2.3×10^{-14}	4.2×10^{-15}
1651	8.8	4.6	3.8×10^{-15}	7.0×10^{-16}
1667	8.8	4.6	3.8×10^{-15}	7.0×10^{-16}
1676	6.4	3.3	1.4×10^{-15}	2.7×10^{-16}
1692	14	7.2	1.5×10^{-14}	2.8×10^{-15}
1700	8.1	4.2	2.9×10^{-15}	5.3×10^{-16}
1736	9.7	5.0	5.0×10^{-15}	9.2×10^{-16}
1737	17	8.7	2.6×10^{-14}	4.8×10^{-15}
1740	5.8	3.0	1.1×10^{-15}	2.0×10^{-16}
1759	6.7	3.5	1.6×10^{-15}	3.0×10^{-16}
1774	8.8	4.6	3.8×10^{-15}	7.0×10^{-16}
1782	13	6.6	1.1×10^{-14}	2.1×10^{-15}
1791	11	5.5	6.5×10^{-15}	1.2×10^{-15}
1802	11	5.7	7.5×10^{-15}	1.4×10^{-15}
1810	7.7	4.0	2.5×10^{-15}	4.6×10^{-16}
1815	15	7.6	1.7×10^{-14}	3.2×10^{-15}
1818	4.0	2.1	3.6×10^{-16}	6.7×10^{-17}
1821	4.9	2.5	6.3×10^{-16}	1.2×10^{-16}
1827	13	6.6	1.1×10^{-14}	2.1×10^{-15}
1829	8.1	4.2	2.9×10^{-15}	5.3×10^{-16}
1836	13	6.9	1.3×10^{-14}	2.4×10^{-15}
1848	19	10	3.9×10^{-14}	7.3×10^{-15}
1850	6.4	3.3	1.4×10^{-15}	2.7×10^{-16}
1851	11	5.5	6.5×10^{-15}	1.2×10^{-15}
1856	9.2	4.8	4.3×10^{-15}	8.0×10^{-16}
1874	17	8.7	2.6×10^{-14}	4.8×10^{-15}
1898	9.7	5.0	5.0×10^{-15}	9.2×10^{-16}

Teh estimates of the masses of some numbered minor planets may serve a useful purpose as the first approximation in our acquiring a better knowledge of these celestial bodies and in our further investigation of their motion.

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