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DURATIONS OF PROXIMITIES OF PARTICULAR PAIRS OF QUASICOMPLANAR ASTEROIDS

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Summary. Kinematic durations of proximities, related to the pairs of orbits of the quasicomplanar asteroids, found out earlier, with mutual inclinations not exceeding 0°.500 and minimum mutual distances under 0.0004 AU (60000 km) were determined. These kinematic durations may be useful in providing orientation on how long intervals could be expected during which eventual perturbing effects occur at the found proximities. Angular widths on the orbits, enclosing the proximities, are also determined. For those among the pairs which are particularly outstanding, positions are given of the outermost points of the corresponding sections. Data presented may be of practical interest in the observations of these minor planets.

By making use of the orbital elements of all the numbered minor planets listed in (1), we found in (2) 142 asteroids whose orbits can be grouped into 77 different pairs with the mutual inclinations which do not exceed 0°.500 and their minimum mutual distances being under 0.0004 AU (60000 km). These asteroid pairs were named quasicomplanar, since their motion proceeds in orbits laying almost in the same plane. In (2) we specified the reason for our choosing this upper limit of distances between the quasicomplanar asteroid orbits: in our view, the mutual perturbations of the most of asteroids might be „measurable", that is, their perturbations could grow sensible, provided these bodies were approaching each other to distances less than 0.0004 AU.

The term „kinematic duration of proximity" was attached to the time interval elapsed between two moments at which both asteroids are at mutual distance 0.000400 AU before and after passing the proximity position. It is non-perturbed motion of asteroids that is held in view and herein our interest bears upon the duration of proximity, as defined above, connected with the limiting distance adopted in our investigations so far. In (2) the proximity positions are given of particular pairs (j, k) of the numbered quasicomplanar minor planets. If the moment of proximity t_p is taken to be $t_p = 0$, and the moments, the one of which is preceding and the second following the proximity, at which the two asteroids are at distance 0.000400 AU, are denoted by t_- and t_+ respectively, then the kinematic duration of proximity Δt is given by the difference $\Delta t = t_+ - t_-$; that is the times t_+ and t_- are reckoned from the moment $t_p = 0$, and are expressed in days as units. True anomalies of asteroids at these instants are denoted by v_+ and v_-, whereby indexes j or k are added, depending upon whether the asteroid j or k is concerned. The difference $\Delta v = v_+ - v_-$ furnishes the angular width of the section of the related orbit about the proximity position, within which eventual perceivable mutual perturbing effects of asteroids could be expected.

The determination of the needed quantities v_{i-}, v_{i+}, t_- and t_+, where $i = j, k$, was performed by the known formulae, used already in (**2**). The calculation was inasmuch easier as the values of the true anomalies v_j and v_k were used, found in (**2**), which correspond to the proximity distance of the orbits of asteroids j and k. We further proceeded from the already known values related to the proximity positions to calculate the corresponding values of the distance ρ between the two asteroids considered for a set of instants, preceding and following the instant of proximity, stopping at the above indicated distance of 0.000400 AU. The instants t_- and t_+ were determined with an accuracy of 0.0002 mean days. This accuracy proved adequate to our calculation distances to six decimals, in analogy to our procedure in (**2**).

TABLE I

j	k	$10^6 \rho_{min}$	Δt	Δv_j	Δv_k
16	1245	181	$1^d.0056$	$0^\circ.22217$	$0^\circ.22268$
21	367	265	0.2148	0.06512	0.06267
24	1462	391	0.1632	0.02382	0.02392
39	251	218	0.7390	0.15036	0.15927
43	211	250	0.3420	0.07620	0.08958
47	1541	178	0.3808	0.07528	0.07435
50	1335	158	0.9186	0.39247	0.37218
76	1692	89	0.2824	0.05407	0.04958
79	1200	273	0.2586	0.05193	0.05887
84	227	254	0.2048	0.03582	0.04149
110	1393	63	0.7450	0.16685	0.15705
111	1092	196	0.4420	0.08945	0.09478
143	469	99	0.8592	0.20990	0.22121
163	1874	288	0.1650	0.03101	0.03495
171	1581	112	1.0300	0.22060	0.22248
205	992	35	0.8006	0.16134	0.16800
212	406	359	0.2336	0.05117	0.04899
215_{-}	1851	4	0.5726	0.12390	0.12897
$\bar{215}$	1851	282	0.4074	0.09244	0.09622
227	1737	7	0.4536	0.09257	0.09273
243	1848	303	0.5792	0.12035	0.12055
263	848	345	0.3814	0.07030	0.07216
277	586	291	0.4604	0.08068	0.08304
280	1325	311	0.1808	0.04400	0.03966
311	1397	66	0.3206	0.06385	0.05948
335	873	149	1.0440	0.27684	0.28678
355	1700	373	0.1156	0.02466	0.02328
376	1374	394	0.0332	0.01305	0.01262
379	461	369	0.1176	0.02601	0.02601
379	1635	328	0.4122	0.08525	0.08232
384	722	280	0.2324	0.06168	0.05587
389	972	9	0.2780	0.06165	0.06511
400	1187	175	0.2596	0.05488	0.04934
412	891	65	1.1728	0.23835	0.24266
452	534	168	1.3028	0.24963	0.25014
452	534	364	0.5976	0.12875	0.12902
452	1131	381	0.1160	0.02458	0.02079
460	1200	254	0.4030	0.07329	0.07770

TABLE I (continued)

j	k	$10^6 \rho_{min}$	Δt	Δv_j	Δv_k
461	1782	238	$0^{d}.2283$	$0^{\circ}.03987$	$0^{\circ}.03995$
461	1782	360	0.1240	0.02417	0.02422
548	1551	292	0.2584	0.07552	0.07855
554	557	356	0.2288	0.07159	0.07305
577	765	245	0.2194	0.05394	0.04743
650	1130	207	0.3278	0.12426	0.11807
685	1307	268	0.1940	0.06445	0.06562
703	1130	14	0.3944	0.16033	0.16063
750	1850	227	0.2730	0.06895	0.06627
753	1534	369	0.0616	0.01479	0.01589
763	985	14	0.4380	0.09596	0.09473
794	799	344	0.1540	0.03812	0.03560
813	1676	328	0.2012	0.06286	0.06196
848	1363	210	0.7870	0.18039	0.17619
938	1815	43	0.2492	0.05475	0.05424
938	1331	140	0.6874	0.08895	0.08774
954	1898	51	0.3312	0.07723	0.07706
954	1898	291	0.2292	0.03328	0.03321
960	1818	10	3.1244	0.92032	0.90094
962	1802	294	0.5532	0.10603	0.10539
991	1305	327	0.2556	0.04264	0.04217
993	1635	183	0.7788	0.15012	0.14983
993	1635	342	0.4540	0.09894	0.09875
1037	1791	387	0.1172	0.02981	0.03365
1044	1630	380	0.0940	0.02186	0.02360
1060	1203	262	0.1988	0.03973	0.04458
1078	1634	147	0.2252	0.06177	0.06124
1079	1100	98	0.8066	0.15242	0.15292
1082	1782	112	0.4308	0.10034	0.10054
1135	1381	228	0.8906	0.16502	0.15780
1137	1667	232	0.4068	0.09846	0.09289
1142	1539	360	0.1264	0.02491	0.02445
1169	1810	325	0.5126	0.13870	0.13691
1251	1492	70	0.3948	0.10815	0.09729
1289	1635	370	2.0818	0.48093	0.48036
1443	1774	346	0.4570	0.09942	0.09832
1446	1527	399	0.0200	0.00518	0.00508
1487	1581	168	3.7444	0.52940	0.53306
1560	1829	325	0.4152	0.14522	0.13517
1590	1827	192	0.2148	0.05511	0.06053
1644	1836	205	0.1864	0.04297	0.04468
1651	1856	125	0.7508	0.25280	0.25585
1736	1759	22	0.3412	0.07880	0.08276
1740	1821	237	0.2132	0.04410	0.04315

In Table I the first and the second columns give the numbers j and k of the numbered minor planets, discriminated into 77 different pairs with the quasicomplanar orbits, whose mutual inclinations are not above $0^{\circ}.500$ and minimum mutual distances ρ_{min} not exceeding 0.000400 AU. There are among these pairs five (215, 1851), (452, 534), (461, 1782), (954, 1898) and (993, 1635) where minimum distances just defined appear twice, consequently two such values are given, the first being the smaller one. The third column gives the minimum distances ρ_{min} in units of the sixth decimal of the astronomical unit. In the fourth column are kinematic durations of proximities Δt in mean days, and in the two last columns are the values of the angular widths Δv, comprising the orbital proximity of asteroids j and k. The values contended in the fourth, fifth and sixth column correspond to the adopted limiting value 0.000400 AU for distance between the asteroids.

For 27 asteroid pairs was found $|t_-| \neq t_+$, and the maximum difference of these time intervals stated $\text{Max}(|t_-| - t_+) = 0.0008$ mean days is related to the pair (960, 1818). With 14 pairs $|t_-| > t_+$ and with further 13 pairs $t_+ > |t_-|$; with all the rest $|t_-| = t_+$. This is an indication that the pair concerned is at equal or nearly equal distances at times, symmetrically disposed before and after proximity.

The maximum kinematic duration of proximity is found with the pair (1487 Boda, 1581 Abanderada), and it amounts to 3.7444 mean days; the pair reaches the limiting mutual distance of 0.000400 AU 1.8724 mean days ahead of proximity. This is the maximum interval, separating an instant of occupation by an asteroid pair of the limiting distance from the instant of their proximity. The minimum separation of the two instants is stated with the pair (1446 Sillanpää, 1527 Malmquista), its value being 0.0200 mean days. Kinematic duration longer than half a day is found with 24 pairs, whereas with 8 of them this duration exceeds one full day. All the angular widths Δv, as defined above, about the proximity, are less than 1°, the largest being those of the pair (960 Birgit, 1818 Brahms), which atain the values $\Delta v_{960} = 0\overset{\circ}{.}92032$ and $\Delta v_{1818} = 0\overset{\circ}{.}90094$. In table I 11 pairs are found with $\Delta v > 0\overset{\circ}{.}2$, and 2 pairs only with $\Delta v > 0\overset{\circ}{.}5$. Minimum Δv are with the pair (1446, 1527), amounting to $\Delta v_{1446} = 0\overset{\circ}{.}00518$ and $\Delta v_{1527} = 0\overset{\circ}{.}00508$; it has just been stated above that this pair had the smallest duration of proximity. Otherwise, maximum absolute value of the difference of the angular width about of proximity is $\text{Max}\,|\Delta v_k - \Delta v_j| = 0\overset{\circ}{.}02$, roundedoff, both being related to the asteroid pairs (50, 1335) and (960, 1818).

The shortest distance between the orbits of all the numbered minor planets has the pair (215 Oenone, 1851 ≡ 1950 *VA*), being only 0.000004 AU = 600 km, (2). The kinematic duration of proximity of this pair is $\Delta t = 0.5726$ mean days, and the corresponding angular widths about of the proximity are $\Delta v_{215} = 0\overset{\circ}{.}12390$ and $\Delta v_{1851} = 0\overset{\circ}{.}12897$.

Table II summarizes the values obtained for 13 pairs, found in (2), with minimum distances between their orbits under 10000 km. The first two columns give the numbers j and k of the numbered asteroids in the respective pairs, the third gives the minimum distances of their orbits in kilometers, in the fourth and the fifth are the times at which the asteroids are at the distance 0.000400 AU, before (marked by t_-) and after (marked by t_+) the moment of proximity, being taken, as already indicated, as the zero time. Further columns present values of the true anomalies of both limiting positions (0.000400 AU) lying astride the proximity of the two asteroids, the index $_-$ being attached to those preceding and the index $_+$ to those following the proximity.

It is seen from Tables I and II that there are pairs with larger minimum distances between their orbits, yet having more prolonged duration of proximity and wider angular widths Δv, than pairs with closer orbits, including the asteroid pair (215, 1851) with the closest proximity. This might be accounted for by the fact that some orbital pairs are mutually placed in such a way that the asteroids concerned take longer time to pass the closest sections of their respective orbits, that is, those enclosing the proximity, moving nearly parallelly, whereas the „parallelity" of other asteroid pairs is a short-lived occurence.

TABLE II

j	k	ρ_{min} km	t_-	t_+	v_{j-}	v_{j+}	v_{k-}	v_{k+}
110	1393	9400	$-0^{d}.3724$	$0^{d}.3726$	$83^{\circ}.48084$	$83^{\circ}.64769$	$182^{\circ}.46192$	$182^{\circ}.61897$
205	992	5200	—0.4002	0.4004	219.57905	219.74039	51.20229	51.37029
215	1851	600	—0.2864	0.2862	83.03215	83.15605	62.36867	62.49764
227	1737	1000	—0.2268	0.2268	279.42317	279.51574	319.43499	319.52772
311	1397	9900	—0.1604	0.1602	248.30911	248.37297	122.15520	122.21468
389	972	1300	—0.1390	0.1390	116.64724	116.70889	290.83153	290.89664
412	891	9700	—0.5864	0.5864	130.16328	130.40163	283.15717	283.39983
703	1130	2100	—0.1972	0.1972	347.07414	347.23447	45.66014	45.82077
763	985	2100	—0.2190	0.2190	201.35831	201.45427	230.53655	230.63129
938	1815	6400	—0.1246	0.1246	55.02541	55.08016	292.95633	293.01057
954	1898	7600	—0.1656	0.1656	44.59816	44.67539	312.96311	313.04018
960	1818	1500	—1.5626	1.5618	95.42486	96.34518	108.50149	109.40243
1736	1759	3300	—0.1706	0.1706	216.37944	216.45824	261.34090	261.42366

The results obtained make it possible to acquire better knowledge of an interesting asteroid group and may prove useful in our further researches into their motion. These results may also be of practical consequence in the observations of these asteroids.

The calculations have been performed at IBM 360/44 computer of the Computation Centre of the Institute for Mathematics in Beograd.

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