DISTRIBUTIONS OF THE JACOBI CONSTANT AND THE KINETIC MOMENT OF THE PALOMAR-LEIDEN SURVEY ASTEROIDS

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1. INTRODUCTION

In a previous paper of ours (Olević D. et al, 1979) we investigated the statistical distributions of the Jacobi constant and the kinetic moment of the numbered minor planets (NA) (Ephemerides of Minor Planets for 1978, Leningrad, 1977). The population NA was divided into two subpopulations according to absolute brightness: the one $B(1,0) < 12^m$, and the second $B(1,0) \geqslant 12^m$. It proved that the distributions of the kinetic moments and Jacobi constant in the two subpopulations were non-randomly different. We deemed it therefore as being of interest to apply similar analysis to the population of the 1836 asteroids listed in the Palomar-Leiden Survey (PLS), (Houten van et al. 1970). We chose thereby the list of asteroids figuring in Table 7 of the publication mentioned, notwithstanding the fact that it comprises also asteroids with less accurately determined osculatory elements. In view of the asteroids' numerosity, the errors in their osculatory elements will practically not affect the results of the statistical investigations.

In the present paper the distribution will be analysed of the Jacobi constant and the kinetic moment in the 1836 PLS asteroids population and its subpopulations I, II, III and IV, formed according to absolute brightness:

Subpopulation	$B(1,0)$	N
I	11.5—15.5	434
II	15.5—16.5	509
III	16.5—17.5	507
IV	17.5—20.5	350

In delimiting $B(1,0)$ we strove the number N of elements in the subpopulations to be nearly equal (in the statistical sense).

2. DISTRIBUTION OF THE JACOBI CONSTANT

The Jacobi constant (in the three-body problem) has the form:

$$h = -k^2 \left[\frac{1}{2a} + \frac{1+m_j}{a_j^3} \sqrt{a} \cos \varphi \cos i' + \delta h'\right]$$

where: k — the gravitational constant, a — major semi-axis of the asteroid orbit, a_J — major semi-axis of the Jupiter's orbit, m_J — the mass of Jupiter, φ — the angle of eccentricity ($e = \sin \varphi$), i — the inclination of the asteroid orbital plane relative to the Jupiter's orbital plane and $\delta h'$ — a variable, with respect to h a small, third order quantity, therefore negligible in all ensuing analyses.

For practical reasons, instead of h, we will use $H = -h \times 10^7$, and instead of the relative frequency f_r — the quantity $10^3 \times f_r$, denoted by F_r.

In Table 1 values F_r are presented in terms of the argument H for the whole PLS population and subpopulations I, II, III and IV.

TABLE 1

The distributions of the Jacobi constant

H	Fr (PLS)	Fr (I)	Fr (II)	Fr (III)	Fr (IV)
830	2	5	0	0	0
840	1	0	4	0	0
850	6	18	4	2	0
860	6	23	2	0	0
870	4	14	2	0	0
880	7	23	6	0	0
890	11	28	16	2	0
900	33	65	28	32	6
910	86	145	110	24	14
920	51	138	47	16	6
930	41	58	59	28	17
940	64	104	67	51	31
950	77	88	96	79	37
960	74	67	79	101	43
970	77	37	102	108	49
980	44	28	41	53	54
990	47	25	41	57	71
1000	83	32	84	99	126
1010	72	28	53	99	120
1020	62	32	51	75	111
1030	65	28	57	81	94
1040	44	14	35	49	91
1050	18	5	12	18	43
1060	13	0	0	14	46
1070	4	0	0	4	14
1080	4	0	0	4	17
1090	3	0	2	4	6
1100	1	0	0	2	3
N:		434	509	507	350

In order to compare the distributions H of the PLS and NA asteroids for equal intervals of H, we calculated the ratios:

$$q_0 = \frac{F_r(PLS)}{F_r'(NA)}$$

$$q_1 = \frac{F_r(PLS)}{F_r''(NA)}$$

$$q_2 = \frac{F_r''(NA)}{F_r'(NA)}$$

where F_r(PLS) relates to the PLS asteroids and F_r(NA) and F''(NA) to the NA subpopulations respectively, formed according to the criteria: $B\,(1{,}0) < 12^m$ and $B\,(1{,}0) \geqslant 12^m$.

Bright PLS asteroids being rare objects, F_r(PLS) and F''(NA) may be regarded as relating to the faint and F_r(NA) to the brighter asteroids.

From the results of the above relations we obviously expected information on the mutual accordance of the distributions H of the faint asteroids of the two surveys (q_1) and the distributions H of the faint and bright asteroids (q_0 and q_2)

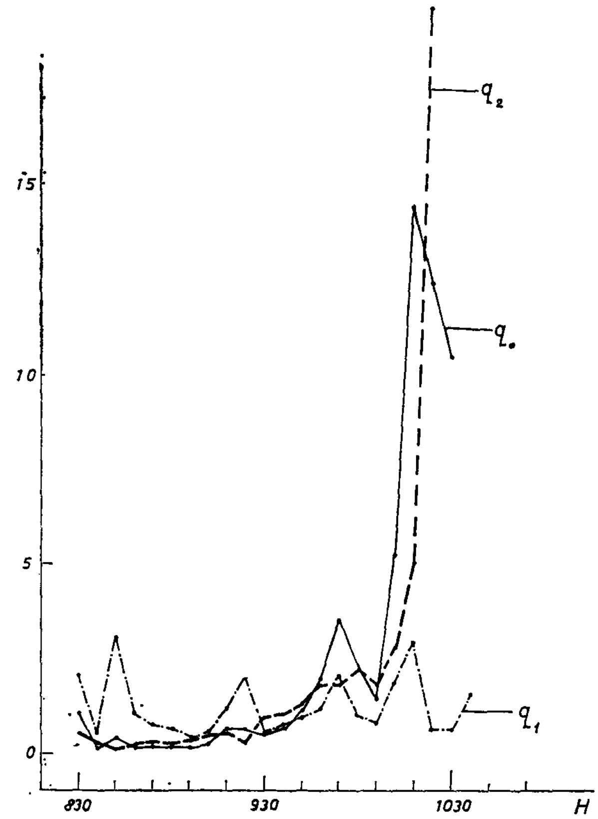


FIG. 1

The functions q_0, q_1 and q_2 are graphically illustrated in Fig. 1. A similarity of q_0 and q_2 is stated with very pronounced peaks for $H = 1010$, whereas q_1 does not vary with H. From the similarity of q_0 and q_2 and the fact that q_1 remains constant the conclusion follows that there is an agreement of the distributions H of the faint NA and PLS asteroids. The existence of peaks is an indicator that within a given interval H the distributions of the bright and faint asteroids are systematically divergent. More thorough information on the measure of dependence of the distribution H on absolute brightness is yielded by the comparison of the function $q_0(i)$ with the functions $q_1(i)$ (i = I, II, III and IV), calculated by using in the expressions for q_0 and q_1 instead of F_r(PLS), the corresonding F_r for the subpopulations I, II, III and IV.

It appears from Fig. 2, illustrating the functions $q_0(i)$, that the divergence of the distributions H of the faint and bright asteroids begins at $H \approx 990$ and that it becomes the more pronounced, the greater the brightness difference is. Besides, all four curves display a peak at $H = 1010$, corresponding to $a = 2.25$ a.u. This fact may be of importance, for it has theoretically been proved that the probability of destructive collisions (and disintegration) is the greatest in the interval of a from 2.0 to 2.5 a.u. (for inclinations from 0.05° to 20° and eccentricities from 0.05 to 0.30) (W-H IP, 1977). Moreover, it has experimentally been shown (Fujiwara A., 1977) that in the event of the rocks breaking up, the number of splinters grows exponentially with their size decreasing.

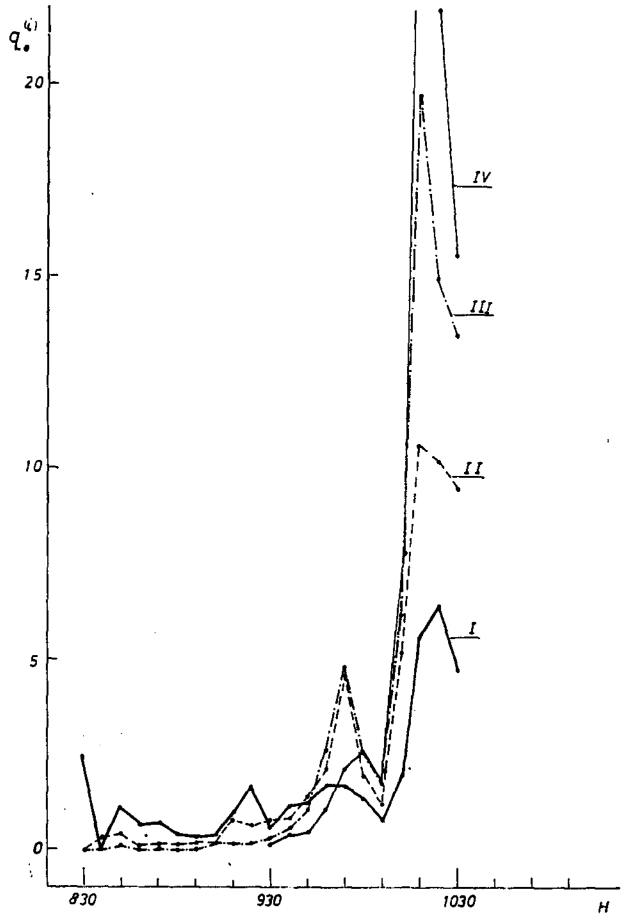


FIG. 2

The functions $q_1(i)$ are illustrated in Fig. 3. In analogy to q_1, no tendency of some systematic variations of them as a function of H is to be observed either, convincing us still more that earlier established accordance of the distributions is real one.

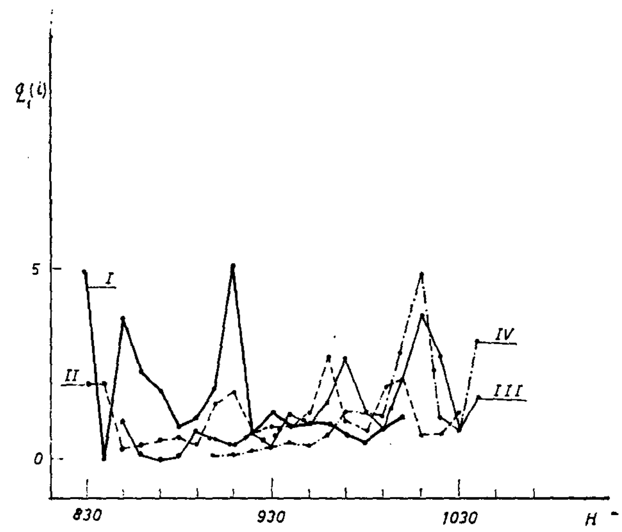


FIG. 3

3. DISTRIBUTION OF THE KINETIC MOMENT

The kinetic moment Q_m is defined by the relation:

$$Q_m = \sqrt{a} \cos \varphi \cos i'$$

Table 2 presents the distribution Qm of the PLS asteroids and their subpopulations I, II, III and IV.

Let r_0, r_1 and r_2 be ratios of the relative frequences Q_m, defined as q_0, q_1 and q_2. The functions r_0, r_1 and r_2 are illustrated in Fig. 4 through 6.

The curve r_1 (Fig. 4) runs nearly parallelly to abscissa, therefore we consider that the distributions Q_m of the faint PLS and NA asteroids are in good agreement too. The curves r_0 and r_2 display a pronounced peak for $Q_m = 1.540$. To this value of the argument corresponds the major semi-axis $a = 2.3$ a.u., that is a for which the probability of destructive collisions at inclination $i = 10°$ and eccentricity $e = 0.15$ is maximum.

By analysing the curves in Fig. 5 we state: a) The distribution Q_m of the bright PLS and NA asteroids is practically the same. The greater the brightness difference, the more pronounced is the extremum, located within the interval Q_m 1.4 to 1.5. b) After $Q_m \approx 1.6$ the distributions Q_m of the faint and bright asteroids are in good agreement.

From the results in Fig. 6 the same conclusion follows as that drawn from the results in Fig. 3.

TABLE 2

The distributions of the kinetic moment

Qm	*Fr* (PLS)	*Fr* (I)	*Fr* (II)	*Fr* (III)	*Fr* (IV)
1.250	1				
1.280	1				
1.310	4				
1.340	1				
1.370	1				
1.400	3	0	0	2	12
1.430	13	0	0	6	61
1.460	47	7	26	50	128
1.490	123	48	91	146	239
1.520	140	53	119	176	239
1.550	134	90	121	192	134
1.580	94	74	125	98	73
1.610	76	51	83	106	55
1.640	78	97	83	96	29
1.670	52	62	79	50	9
1.700	65	129	69	40	12
1.730	67	134	97	22	9
1.760	62	162	75	10	0
1.790	28	79	26	8	0
1.820	2	5	4	0	0
1.850	2	9	0	0	0
1.880	0	0	0	0	0
1.910	1				
1.940	3				
1.970	2				
2.000	0				

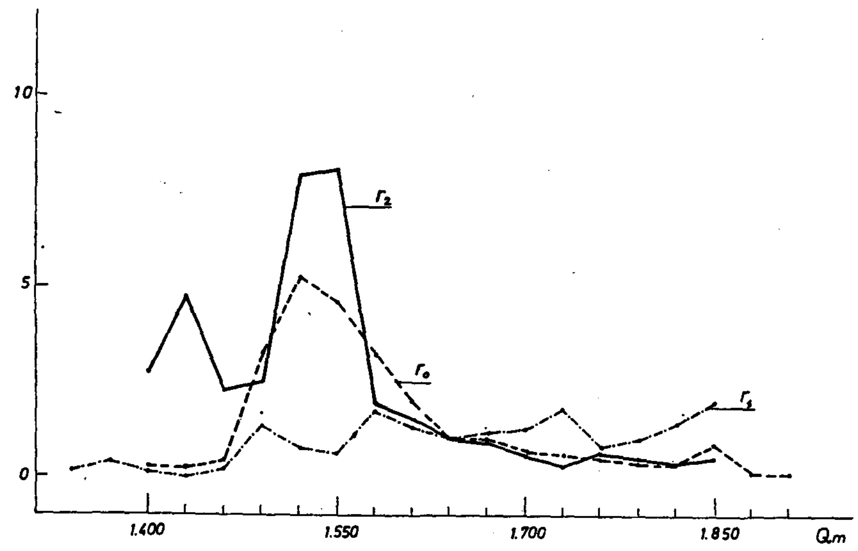
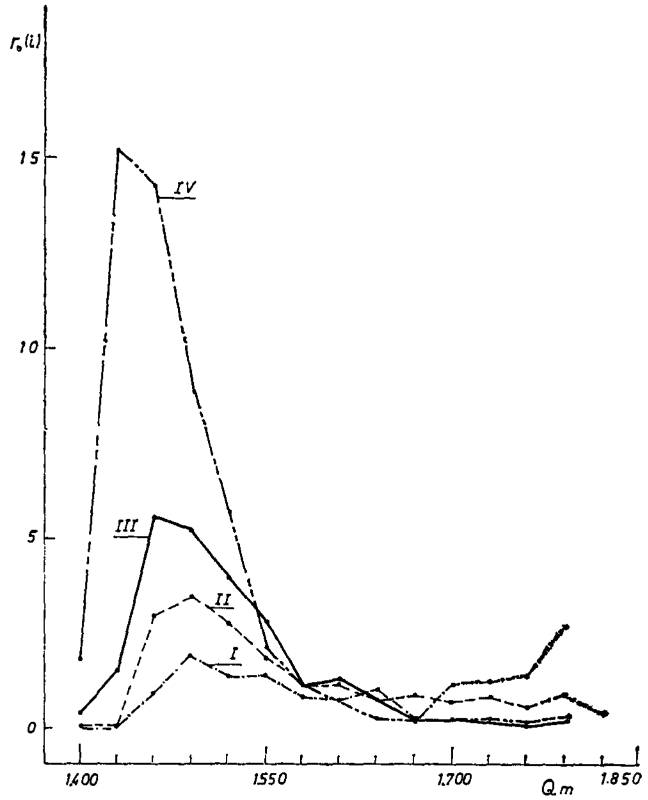
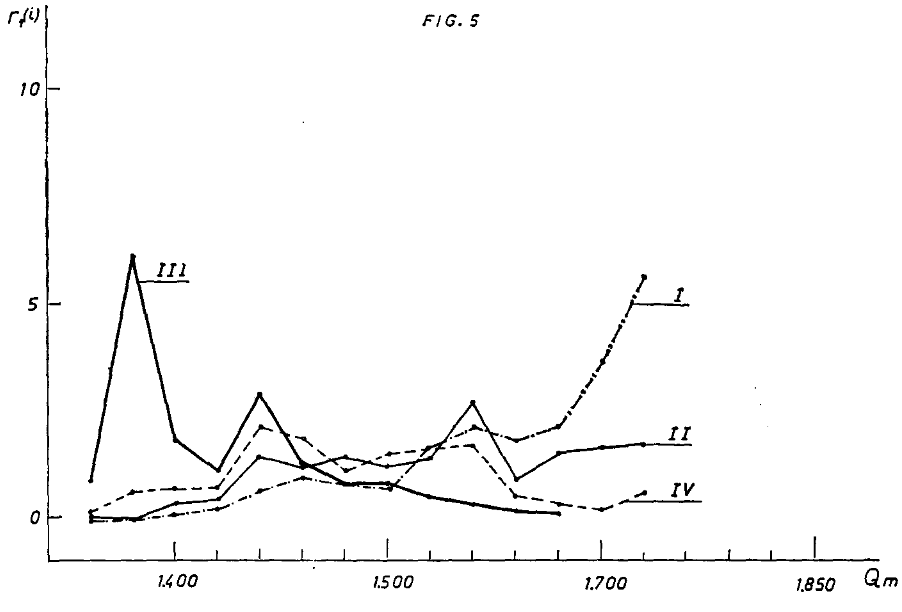


FIG. 4



$r_o(i)$
15
10
5
0
IV
III
II
I
1,400
1550
1.700
1.850
Qm

FIG. 5



$r_f(i)$
10
5
0
III
I
II
IV
1,400
1,500
1,700
1,850
Qm

FIG. 6

*

This work is a part of the research project of the Basic Organization of Associated Labour for Mathematics, Mechanics and Astronomy of the Belgrade Faculty of Sciences, funded by the Republic Community of Sciences of Serbia.

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