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# VARIATIONS OF THE SEASONAL INEQUALITIES AND IRREGULAR FLUCTUATIONS OF UT2-TAI

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*Summary*. Raw UT2-UTC data, given every 5 days, for the period 1967.0—1978.0 are treated.

Annual and semi-annual terms are identified in UT2-TAI. Their amplitudes are varying within a very wide range: from 0 to 20 ms.

Nonperiodic fluctuations of UT2-TAI with amplitudes of $\pm$ 10 ms also exist.

## 1. INTRODUCTION

UT2-TAI data, given every 5 days, for the period 1967.0—1978.0 will be treated here with the purpose of investigating to what extent the two components of the seasonal inequalities of the Earth's rotation (annual and semiannual) are subject to changes as well as to determine the epochs and amplitudes of the largest irregular fluctuations of the system of Universal Time UT2.

For the transition from UT2-UTC, published by the BIH in its ANNUAL REPORTS (AR), use was made of the relations given in (AR) for 1977 (Table 10, p. B-27).

It has been showed elsewhere (D. Djurović, 1979) that there existed in the UT2-TAI system, in addition to the secular variation, periodical components which are functions of time as well, with periods: 7.4, 3.5, 2.3, 1.64, 1.0 and 0.5 years. By eliminating them the uniformity of UT2 is improved by more than one order of magnitude. If only the secular term, represented by a second degree polynomial is eliminated from UT2-TAI, the standard deviation is $\sigma = \pm 73$ ms.

But after elimination of the term of 7.4 — year period, this error is reduced to $\sigma = \pm 21$ ms, while if the 3.5 — year period term is also eliminated, one obtains $\sigma = \pm 9$ ms.

By solving the system of equations of condition:

$$UT2-TAI = a_0 + a_1 t + a_2 t^2 + \sum_{k=1}^{6} \left[ b_k \sin \frac{2 \pi t}{P_k} + c_k \cos \frac{2 \pi t}{P_k} \right] \qquad (1)$$

by the least square method (LSM), where $P_k = 7.4, 3.5, 2.3, 1.6, 1.0$ and 0.5 years ($k = 1, 2, \ldots, 6$) and $t$ is the time expressed in years reckoned from the middle of the interval covered by UT2-TAI, the following results are obtained (in $10^{-4}$ s):

$$
\begin{array}{lll}
a_0 = -105\,937 \pm 2, & b_1 = -1\,028 \pm 2, & c_1 = -112 \pm 3, \\
a_1 = -10\,321.7 \pm 0.3, & b_2 = -243 \pm 2, & c_2 = 141 \pm 2, \\
a_2 = -0.14\,756 \pm 0.0007, & b_3 = -3 \pm 2, & c_3 = -61 \pm 2, \\
 & b_4 = 73 \pm 2, & c_4 = -10 \pm 2, \\
 & b_5 = -51 \pm 2, & c_5 = 35 \pm 2, \\
 & b_6 = 28 \pm 3, & c_6 = -3 \pm 3.
\end{array} \tag{2}
$$

The standard deviation, calculated from the residuals $R_i$ of the equations of conditions is $\sigma = \pm 4.2$ ms.

The standard deviation of a single UT1-UTC 5 days value (equal to the standard deviation UT2-TAI), as obtained from the residuals of the equations used by the BIH in the calculation of UT1-UTC is $\sigma' = \pm 1.8$ ms. (B. Guinot, 1976). We obtained the same result from the relation:

$$
\sigma'' = \left[ \frac{\sum_{i=2}^{n} (R_i - R_{i-1})^2}{2(n-2)} \right]^{\frac{1}{2}} \tag{3}
$$

As the ratio $\sigma/\sigma'$ is non-randomly greater than unity, $R_i$ contains the components of the neglected periodical and non-periodical (irregular) systematic deviations which will be discussed in more details in the next paragraph.

## 2. THE PERTURBATIONS OF THE SEASONAL INEQUALITIES AND THE IRREGULAR FLUCTUATIONS OF UT2-TAI

By applying the Fourier integral transformations method upon $R_i$ we obtained the spectrum given in Fig. 1. The amplitudes $A$ and the periods $P$ of the possibly neglected terms are:

| | | | | | |
|---|---|---|---|---|---|
| $A$: | 1.4 | 2.2 | 2.6 | 1.9 | 1.9 ms |
| $P$: | 210 | 270 | 400 | 690 | 1820 days |

Because of the time variations of the periods and amplitudes of the eliminated components there remain in the residuals $R_i$ their deviations from the corresponding sine waves. The methods of spectral analysis furnish only a rough picture of the amount and the character of these deviations. The variations of $P$ are reflected in the widths and amplitudes of the peaks, whereas the variations of $A$ affect the accuracy of the determination of amplitudes.

Let us suppose that the period of some particular term is variable, its amplitude being constant. The transformation of the function of time $R$ (whose discrete values $R_i$ are analysed) into a function of frequency, angular velocity or period, results in a supperposition of ordinates which are in different phases. This, in turn, causes a decrease of the amplitude $A$ relative to its true value $A'$. In the case of

Fourier transformations the amplitude multiplier (under known conditions, fulfilled in the present instance) is defined by the relation:

$$\rho = \frac{\sin 2\pi L(f - f_0)}{2\pi L(f - f_0)} \tag{4}$$

where 2L (4000 days) is the interval, $f_0$ is the frequency of the considered term and $f$ is a tentative (step) frequency. For $f = f_0$, corresponding to the central line of the peak, $\rho = 1$.

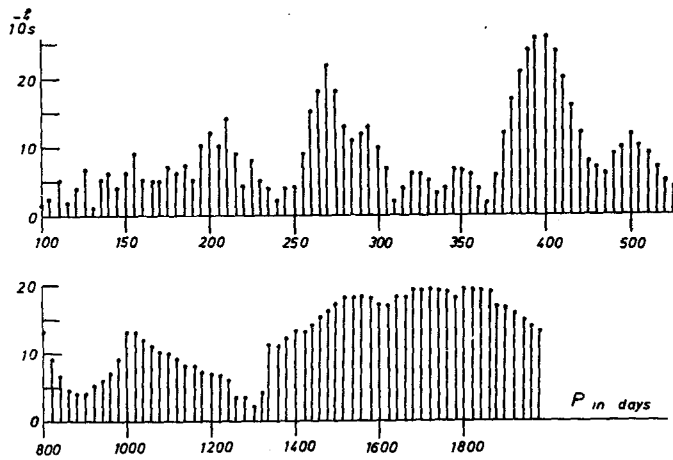

Fig. 1. The spectrum of the residuals $R_t$ = UT2-TAI — (secular and periodic terms).

In order to make a quantitative analysis of the variationso f $P$ and of their reflection in the shape of the spectrum, let us suppose that for $t \in (0, L)$ all waves of some particular term cover equal subintervals $P = P_0 + \Delta P (P_0 = 1/f_0)$, and that for $t \in (L, 2L)$ they also cover equal subintervals $P' = P_0 - \Delta P$. In both parts of the interval (thus over the whole of the interval) $\rho$ will be the same but lower than unity. For instance, if $P = 600$ days (1.6 years) and if the mean variation of the period is $\Delta P = 30$ days (5%), $\rho = 0.8$. In other words we will have $A = 0.8$ A'.

$\Delta P$ can be estimated by comparing the theoretical ($k$) and empirical ($k'$) peak widths. The theoretical width is obtained from the condition $\rho = 0$. From this condition it follows:

$$k = \frac{P_0^2}{L} \tag{5}$$

Provide the peak is well defined, one is able to determine

$$k - k' = 2\,\Delta P \tag{6}$$

and to get information on the stability of the period and on the effects of its possible fluctuations on the calculated amplitude $A$.

By using $\Pi$ (T) type transformations for the isolation of the periodical inequalities (Labrouste, 1943), a more complete picture of their nature is obtained. This is due to the fact that these transformations make it possible to pursue the inequalities in the course of time. Furthermore, by a suitable choice of transformation a selectivity can be attained, which corresponds to the fixed task (Djurović, 1979).

Before proceeding to the presentation of the present results of the application of the mentioned transformations we will shortly remind the theoretical foundations of Π (T).

By transforming the series $R_i$ into series $Z_i(m)$ by the relation:

$$Z_i(m) = R_{i+m} - R_{i-m} \quad (m = \text{const}) \tag{7}$$

a selectivity filter is obtained, and its amplitude multiplier $\gamma_m$

$$\gamma_m = 2 \sin\left(\frac{2m\pi}{P}\right) \tag{8}$$

depends upon the period $P$.

By addition of $Z_i(m)$ from $m = 1$ to $m = k$ a complex transformation $T_i(k)$ is obtained, whose amplitude factor is denoted by $\tau_k$:

$$T_i(k) = Z_i(1) + Z_i(2) + \ldots + Z_i(k) = (R_{i+1} + R_{i+2} + \ldots + R_{i+k}) - \\ -(R_{i-1} + R_{i-2} + \ldots + R_{i-k}), \tag{9}$$

$$\tau_k = \gamma_1 + \gamma_2 + \ldots + \gamma_k = 2 \frac{\sin\frac{k\pi}{P} \sin\frac{(k+1)\pi}{P}}{\sin\frac{\pi}{P}} \tag{10}$$

The series $T_i(k)$ can be transformed again in the same way effecting thereby a double transformation

$$\Pi(T) = T_i(k) T_i(l) \tag{11}$$

hereafter denoted $\Pi(T) = T_k T_l$. The amplitude multiplier of the manifold transformation

$$\Pi(T) = T_k T_l \ldots T_r \tag{12}$$

is

$$\rho = \tau_k \cdot \tau_l \ldots \tau_r. \tag{13}$$

The seasonal inequalities in the Earth's rotation were detected some 40 years ago (Stoyko, 1936, 1937). Many attempts have since been made to explain them for from a geophysical viewpoint, however, as is well known, a great many unknowns are involved there. The study of the perturbations of the annual and semiannual term contributes to a better comprehension of their nature and facilitates the finding out of their explanation.

The perturbations of other terms mentioned above have been the subject of one of our previous papers (Djurović, 1979), hence our attention will be centered here only on the investigation of the fluctuations of the period and amplitude of the annual and semi-annual terms.

Let $R_i'$ be the residuals of the equations of condition, analogous to those foregoing, from which the annual and semi-annual terms have been omitted. Thus,

$R'_t$ are remainders of UT2-TAI after eliminating the secular and periodical terms whose periods $P_k > 1$ year. The spectrum $R'_t$ obtained by the Fourier integral transformations is given in the Fig. 2.

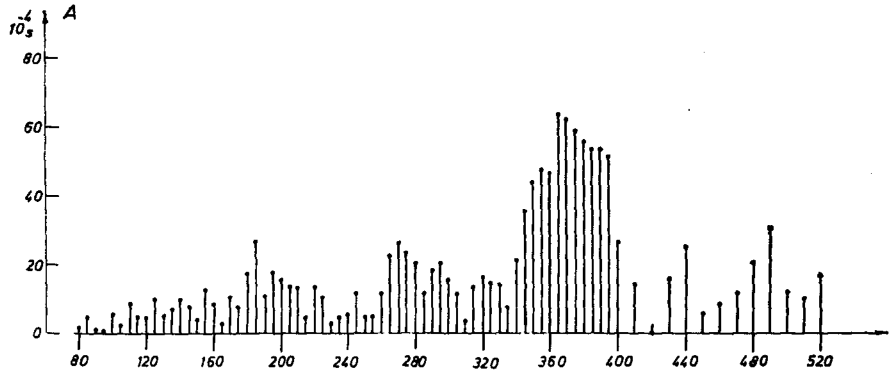


Fig. 2. The spectrum of the residuals $R'_t$ = UT2-TAI — (secular and periodic terms with periods 7.4, 3.5, 2.3 nad 1.6 years).

The peaks, associated with the annual and semi-annual terms, are interpreted as a consequence of the fact that seasonal variations of the Earth's rotation are calculated according to a formula which is not consistent with the true nature of the phenomenon. However we are not able to give an explanation of the central peak ($P = 270$ days) even if one might argue that it is not a subharmonic of the annual term, an inference we are led to have considering its amplitude and position in the spectrum. This is easily proved by the given expression for $\rho$. For 2L = 4000 days we find that the first subharmonic of the annual term has a period $P = 323$ days and an amplitude $A = 0.22\,A$ ($A$ being the amplitude of the annual term).

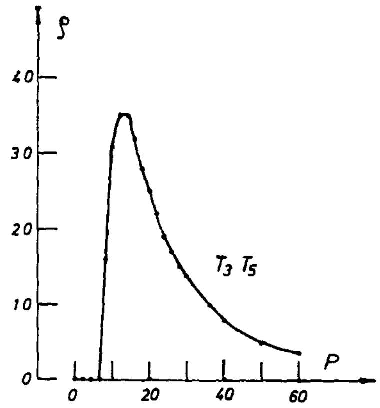


Fig. 3. The selectivity curve of $\Pi(T) = T_3T_5$.

In analysing the deviations of the 41 individual series UT1-UTC (Djurović, 1978) this term was also identified which is an argument in favor of its actual existence.

The width of the peak, corresponding to the annual term is relatively well defined (it covers an interval $P$ from 340 to 410 days) and approximately equals its theoretical value of 67 days. It follows from this fact that the perturbations of the period of the annual term cannot be significant. Nearly the same inference can be drawn from the results obtained by the transformation $\Pi(T) = T_3 T_5$ of the mean 30-day $R'_t$. The selectivity curve of this transformation is represented in Fig. 3 and the results in Fig 4. (curves a).

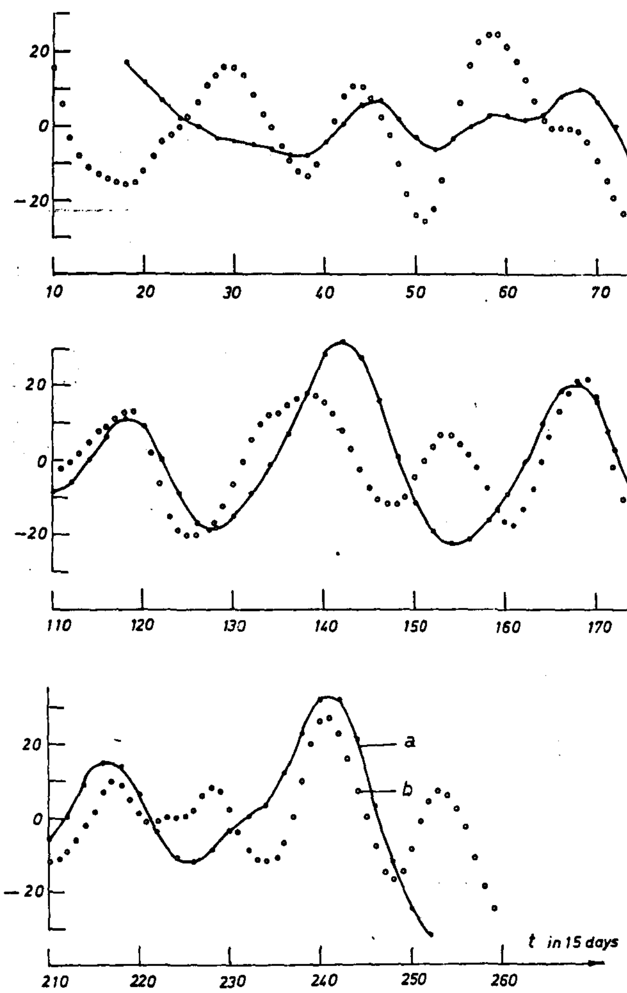


Fig. 4. The transformation of the mean 30 days $R'_t$ (curves a) and the mean 15 days $R_t''$ (curves b) by $\Pi(T) = T_3 T_5$.

From the results of Fig. 4 it follows that the amplitude of the annual term is varying within a very wide range: from 5 to 30 units or 3 to 18 ms. (0.6 ms being the unit).

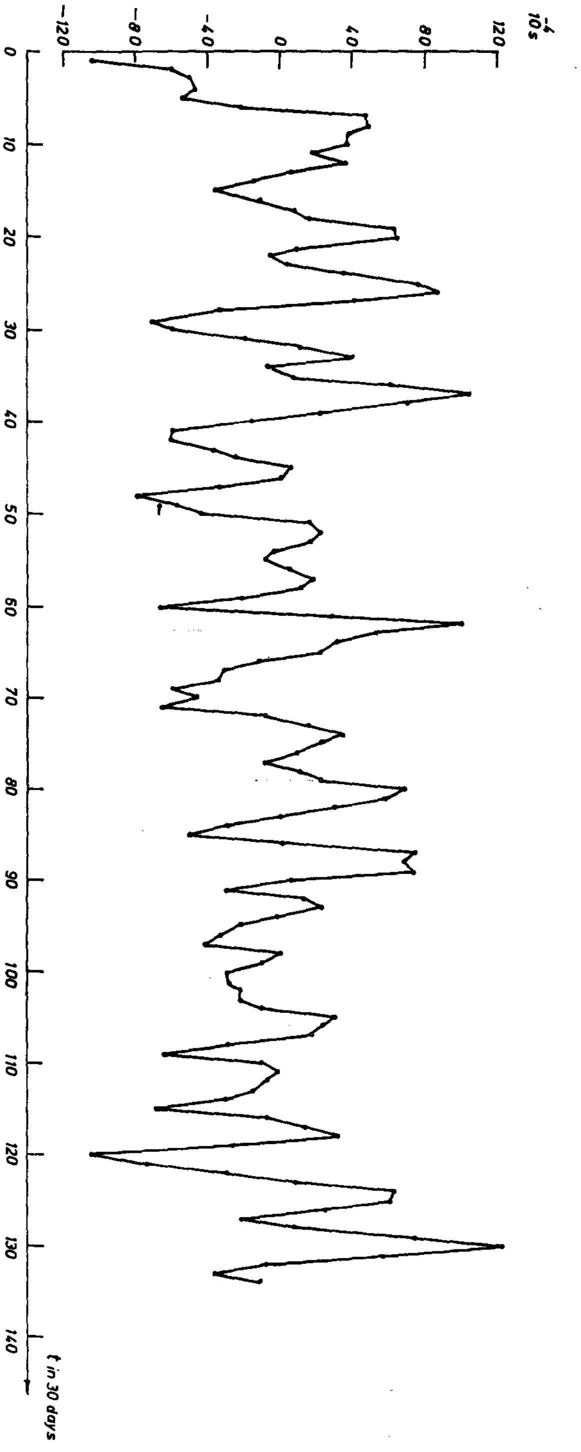


Fig. 5. The nonperiodic fluctuations of UT2-TAI.

The perturbations of the semi-annual term are also great. While there are periods when it almost disappears, there are also periods when its amplitude reaches nearly 15 ms (Fig. 4, curves *b*).

The results illustrated in Fig. 4b are obtained by the transformation $\Pi(T) = T_3 T_5$ of the mean 15-day values of $R'' = R'$ — annual term.

From the results presented above it follows:

1. The formula used for the calculation of $\Delta T$ does not correspond rigourously to the phenomenon of the seasonal inequalities in the Earth's rotation. This is evidenced by the existence of the annual and semi-annual terms in UT2-TAI.
2. The seasonal inequality displays significant deviations in amplitude.
3. Residuals $R_i$ obtained after elimination from UT2-TAI of secular and all known periodical terms cannot be accounted for from the viewpoint of the neglected periodical terms (Fig. 1) nor from the viewpoint of the perturbations of the periodical terms.

Let $G$ and $G_{min}$ be parameters defined by the relations:

$$G = \frac{\sum_{i=2}^{n} (R_i - R_{i-1})^2}{2 \sum_{i=1}^{n} R_i^2} \tag{14}$$

$$G_{min} = 1 + \frac{\mu}{\sqrt{n + 0.5(1+\mu^2)}}, \tag{15}$$

where $\mu = -1.645$ is the quantil of the order of 0.05 ($u_q$ is a quantil of the order $q$ if the probability of the randomly variable $X > u_q$ is $q$) of the Gaussian probability distribution. The application of the above relations yields the following results: $G = 0.177$ and $G_{min} = 0.942$. According to Abbe's criterion (Linnik, 1958), if $G < G_{min}$, $R_i$ cannot be accounted for in terms of the accidental errors.

On the basis of the comparison of $\sigma$ with $\sigma'$ (or $\sigma''$), the comparison of $G$ with $G_{min}$ and the analysis of the curve in Fig. 5, illustrating $R_i$, we state that nonrandom, nonperiodic fluctuations of UT2-TAI do exist. The epochs and amplitudes of the largest among them are given in the Table 1.

The answer to the question whether these fluctuations were in fact disguised systematic errors of UT2 or of the acceleration of the Earth's rotation, remains unsolved for the time being.

TABLE 1

Extreme irregular fluctuations of UT2-TAI

| *JD* | $R_i$ | *JD* | $R_i$ |
|---|---|---|---|
| 2439 000.5+ | | | |
| 494 | — 13.6 ms | 1564 | 11.0 |
| 499 | — 11.7 | 1589 | 11.1 |
| 504 | — 13.3 | 1594 | 11.9 |

| | | | |
|---|---|---|---|
| 509 | — 11.0 | 1604 | 11.0 |
| 529 | — 13.0 | 2554 | — 11.7 |
| 534 | — 12.1 | 3159 | 11.6 |
| 559 | — 12.0 | 3164 | 11.2 |
| 924 | — 10.4 | 4379 | 10.8 |
| 1554 | 10.6 | 4394 | 11.6 |

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