SECULAR AND PERIODICAL VARIATIONS OF UT2-TAI

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Summary. Basic data used in this paper are raw differences UT2-UTC given every 5 days for the period 1967.0—1978.0.

By Fourier integral transformations and Π (*T*) type transformations of the residuals UT2--TAI, periodical inequalities were identified, whose periods (*P*) and amplitudes (*A*) are:

P:	7.4	3.5	2.3	1.6	1.0	0.5 years
A:	100	24	4	12	7	3 ms.

Relative variations of *P* are particulary great for 2.3 and 1.6 — year terms.

1. INTRODUCTION

The methods currently used in the investigation of the inequalities of the Earth's rotation during the last twenty years were those of spectral analysis of random functions. This led to the discovery of several periodical inequalities for which no clear explanation has always been found.

An overview of some harmonic terms of periods from 1 to 7.5 years, considered, or at least assumed, by different authors is given in the Table I.

TABLE I

Authors	Periods of harmonic terms in years	Data	Period of Observation
Proverbio E., Carta F. (1968):	2.0,2.2,2.9,3.5,5.3	UT2-A3	1955.5—1967.5
Korsun A. A., Sidorenkov N. S. (1970):	1.4,2.2	UT2-TAI	1955.5—1968.4
De Prins J., Gillard R. (1971):	1.2,2.5	UTO-AT UTO-AT	1957.2—1970.0[1] 1956.0—1969.9[2]
Iijima S., Okazaki S. (1972):	1.36,1.58,2.14,3.4	UT2-A3	1955.5—1969.0
Lambeck K., Casenave A. (1973):	1.29, 1.56,2.0,2.32, 2.92,7-7.5	UT2-TAI	1955.5—1972.0
Djurović D. (1976):	1.70, 1.76	UT1-UTC Polar coord.	1967.0—1971.0 » »

[1] Neuchâtel [2] Ottawa

However well known hypothesis which, strictly speaking, do not fit the natu of the analysed data are underlying the methods used by the quoted authors, i most cases Blackman-Turkey method or Fourier method.

If the period of a given component is variable, peaks appear in the spectru at various places, whereby one might erroneously believe that they belong to diff rent harmonic terms.

The selectivity functions display so called secondary peaks, whose amplitud are often not negligible. An example will appear later when parasitic peaks, gen rated by the term of period $P = 3.5$ years, will be analysed.

The amplitudes of most of the newly discovered terms are small which mak it necessary to check the reality of their identification, on the condition that ne data and new methods are used.

Basic data used in this paper are raw differences UT2-UTC given eve five days for the period 1967.0—1978.0, which are published by the BIH in i ANNUAL REPORTS (AR). To avoid the difficulties resulting from the jumpli changes of UT2-UTC as well as from changes due to the adjustment of UTC, transition to the UT2-TAI system was made by using the relations between UT and TAI (RAPPORT ANNUEL, 1977, Table 10, p. B-27).

2. SECULAR TERM IN UT2-TAI

There are various ways by which the secular term may be approximate either by a set of parabolas having the same tangent line (Markowitz, 1970), eith by combining the polynomials and the nutation terms (Iijima S., Okazaki S., 1972 but most often by using orthogonal or ordinary polynomials P_n. The coefficien P_n are calculated by the least square method (LSM).

The degree of the polynomial P_n being not definitely determined, it happe that different authors take different values for n, usually $n = 2, 3, \ldots, 6$. Howeve the choice of n is not without influence upon the content and the amplitudes the residuals $R = \mathrm{UT2} - \mathrm{TAI} - P_n$.

The standard deviation σ_n, computed from the residuals R of the equatio of condition can profitably be used as a magnitude indicator of the amplitude the residuals R of the equations of condition. The results we obtained are the foll wing:

n:	1	2	4	6	8	10
σ_n:	± 80	73	35	26	20	15 ms.

As well known, the residuals of course decrease when n increases in such a way th σ_n tends asymptotically to $\pm$ 10 ms. (Fig. 1).

The contents of the residuals R is based by the choice of n as it appears b comparing the curves in Fig. 2., which give R for $n = 1$ and $n = 2$ (the latter den ted by R') and the curves in Fig. 3 giving R for $n = 6$ (curve a) and $n = 10$ (cur b). By analysing these curves we observe that the shape of the spectrum of the res duals R is also dependent on n. Therefore, not every nonrandom peak in the spe trum is to be taken as ensuring the existence of corresponding periodical inequaliti in the Earth's rotation.

It results from these remarks that we have to determine how to choose the value of n, that could be justified in the light of the dynamics of the Earth's rotatiton

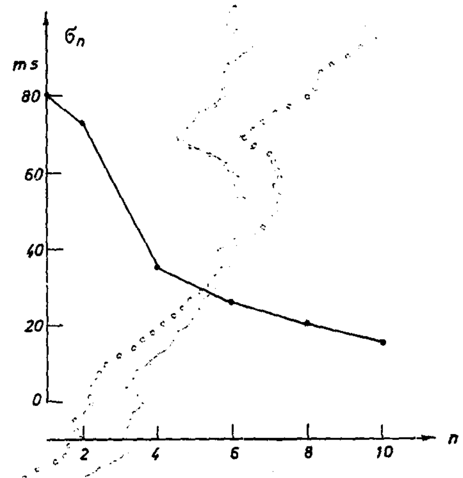

Fig. 1. Standard deviation σ_n as function of the polinomial degree n.

It is well known that the linear term in P_n corresponds to the different duration of the seconds of the UT2 and TAI systems while the quadratic term can be explained by the secular deceleration of the Earth's rotation. We consider, therefore, that at 10 to 20-year intervals, covered at present by UT2-TAI, the secular term should be approximated by a second degree polynomial. As seen in Fig. 2, the residuals of the equation of condition for linear or parabolic polynomial display the same periodical fluctuations.

Using LSM, we indeed obtain for the P_n the following expressions:

for $n=1$: $P_n = -10^s.6414 - 0^s.002\ 83105\ (t-t_0)$
$\pm$ 29 252 (1)

and for $n=2$: $P_n = -10^s.5945 - 0^s.002\ 82421\ (t-t_0) - 0^s.000\ 000\ 03871\ (t-t_0)^2$
$\pm$ 38 221 214 (2)

(t is the time in days, reckoned from the central Julian date $t_0 = 2441\ 491.5$, of the interval analysed here).

The second order coefficient

$$a = -3.871 \times 10^{-8}\ \text{s/day} \tag{3}$$

allows to calculate the secular variation of the length of the day:

$$Ld = 36525 \frac{d^2 Pn}{dt^2} = -2.8 \text{ ms per julian century} \tag{4}$$

which fairly well fits with the results derived from the antique eclipses of the Sun ($Ld = -2$ ms according to Melchior, 1973).

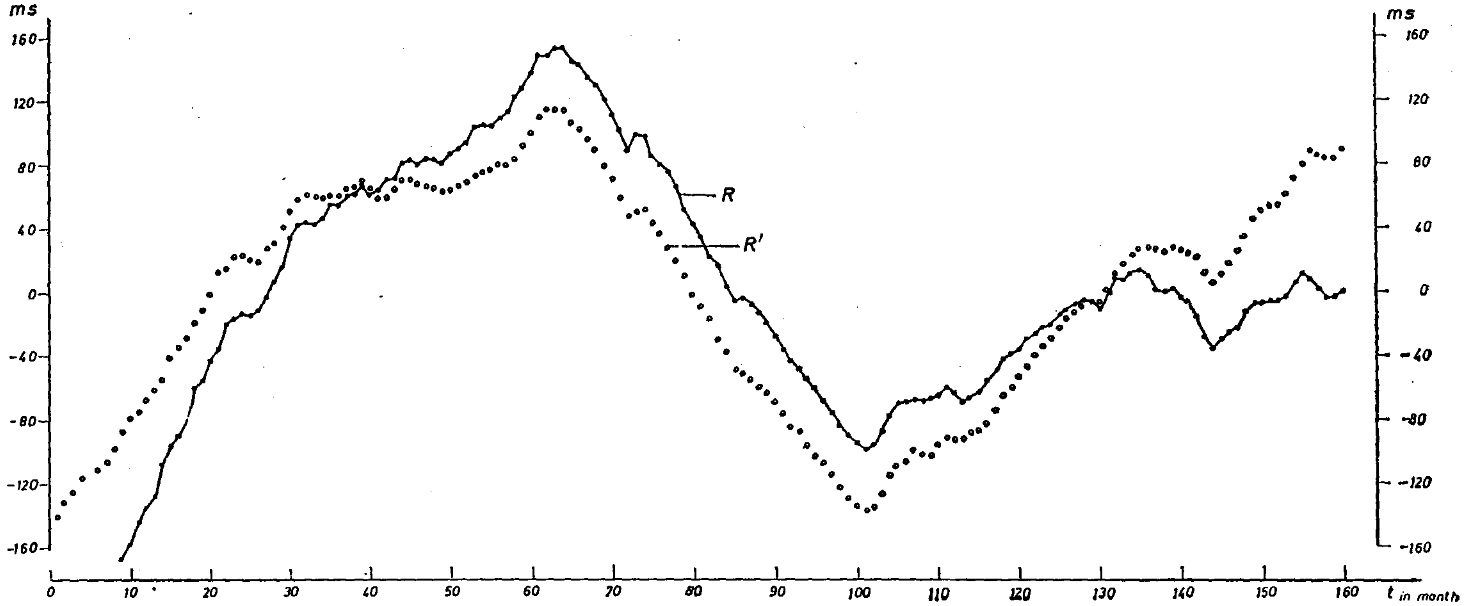

Fig. 2. Reziduals R corresponding to P_1 and to P_2.

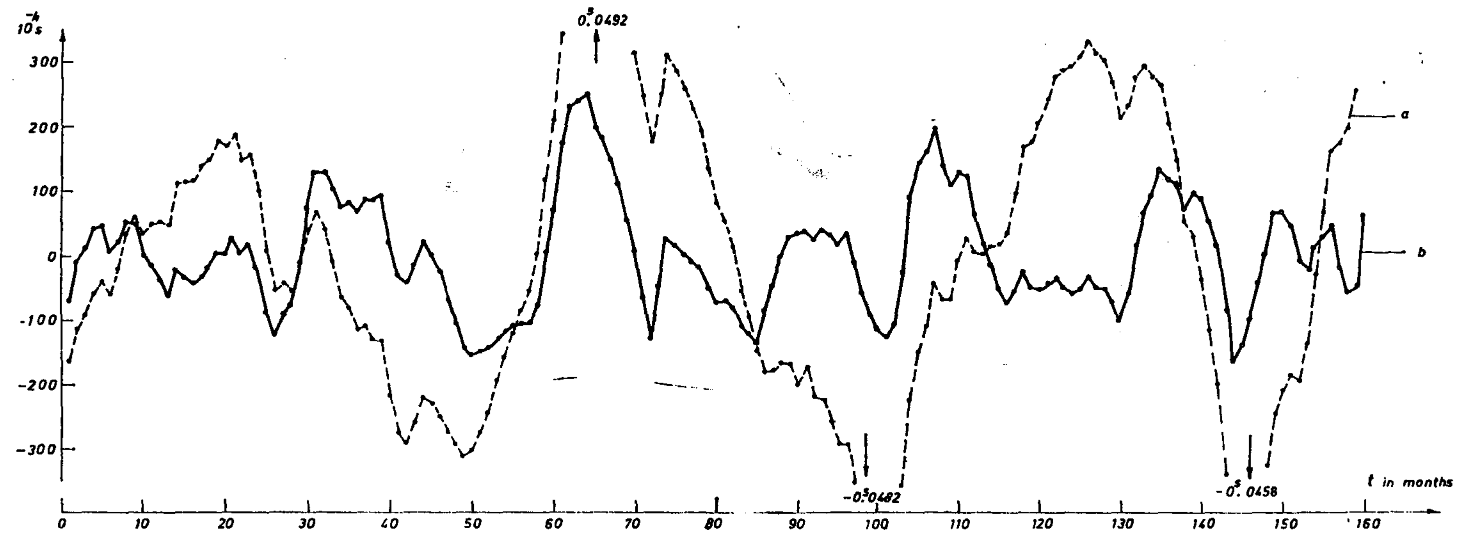

Fig. 3. Reziduals R corresponding to P_6 and to P_{10}.

3. PERIODICAL INEQUALITIES IN UT2-TAI

As seen in Fig. 2, the largest amplitude (approximately 100 ms) is exhibited by the 7.5 — year period term. This term cannot be accounted for by the theory of tidal effects. The nearest tidal term with the argument $2\,\Omega_{☾}$ has a period of 9.3 years. However, the amplitude of this 9.3 year term is about 1 ms but up to now no other phenomenon is known which could be responsible of such a large inequality.

However this term has also been identified by Lambeck and Cazenave (1973). In order to search for other periodical inequalities, we first eliminated this term at least approximately. To achieve such a goal we solved by LSM 6 systems of the equations of condition:

$$UT2-TAI = a_0 + a_1(t-t_0) + a_2(t-t_0)^2 + a_3 \sin\frac{2\pi}{P}(t-t_0) + a_4 \cos\frac{2\pi}{P}(t-t_0) \quad (5)$$

where we used for P the following values: 7.0, 7.2, ..., 8.0×365.2422. The concluded value was $P = 7.4 \times 365.24$, as it gave the minimum standard error ($\sigma = \pm 21.2$ ms) and the following a_k values were obtained:

$$\begin{aligned} a_0 &= -\,10.5894\ \mathrm{s} \pm 0.0012 \\ a_1 &= -\,(28.2895 \pm 0.0065) \times 10^{-4}\ \mathrm{s} \\ a_2 &= (45.55 \pm 0.82) \times 10^{-9}\ \mathrm{s} \\ a_3 &= -\,(975 \pm 11) \times 10^{-4}\ \mathrm{s} \\ a_4 &= -\,(188 \pm 14) \times 10^{-4}\ \mathrm{s}. \end{aligned} \quad (6)$$

Let F (t) represent the right side of the last equation computed with the given values of a_k.

The residuals $R'' = \mathrm{UT2} - \mathrm{TAI} - \mathrm{F(t)}$ still contain a remaining contribution of the 7.4 — year term relative to the sine curve as defined by the coefficients a_3 and a_4. However as their order of magnitude is only a few ms, there are practically no subharmonics of this term in the spectrum of R''.

The spectrum shown on the figure 4 was obtained by integral Fourier transform and only two maxima appear which correspond to $P = 570$ and $P = 1250$ days.

From previous researches (Lambeck K. and Cazenave A., 1973, Djurović D., 1976) we know that the period of the bi-annual term looks quite variable. Therefore we have assumed that both maxima take their origin in the bi-annual term. To verify this assumption we have applied to the mean 30. day R'' the transformation $\Pi(T) = T_8\,T_{10}$ (Labrouste H. and Labrouste Y., 1943, Djurović D., 1979), which do not exhibit a great selectivity but which amplifies the signals over a wide period range around the bi-annual term. This is evident from Fig. 5, illustrating the selectivity curve of this transformation: all the terms of periods $P \leqslant 12$ units (30 days being taken as the unit) are well damped; well damped are also the remaining contribution of the 7.4 — year term (90 units). On the contrary signals, with periods running from 20 to 40 months are amplified, the curve's maximum being at $P = 26.0$ units.

The output signal is represented in Fig. 6. The zeros of the function represented by it, take place for $t = 29, 50, 70, 94, 114$ (t being the time, reckoned in 30 — days units from the Julian epoch $t_1 = 2439\,491.5$). Hence, the half periods of the isolated harmonic terms are respectively: 21, 20, 24 and 20 months (the mean value is around 21 months) and it becomes clear that we actually are not dealing with a bi-annual term but with a 3.5 — year period term. In our view, it can be

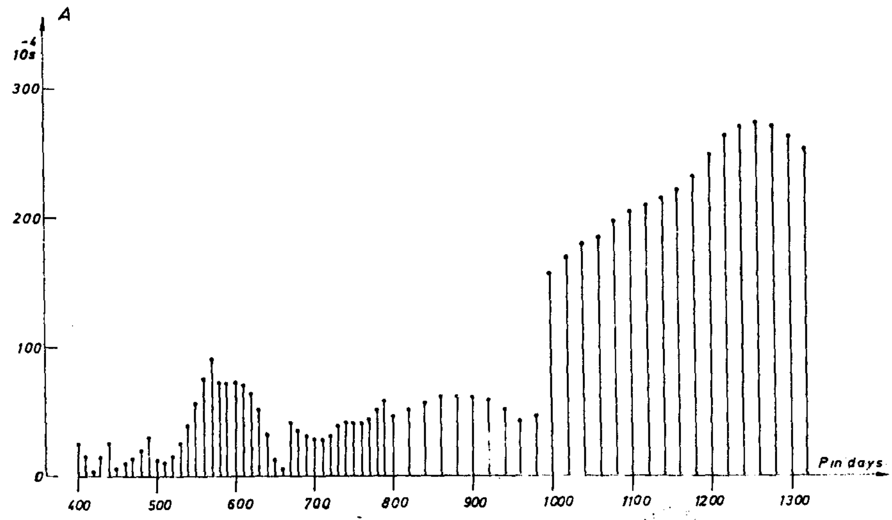

Fig. 4. The spectrum of the residuals R'' = UT2-TAI — (secular and 7.4 year terms).

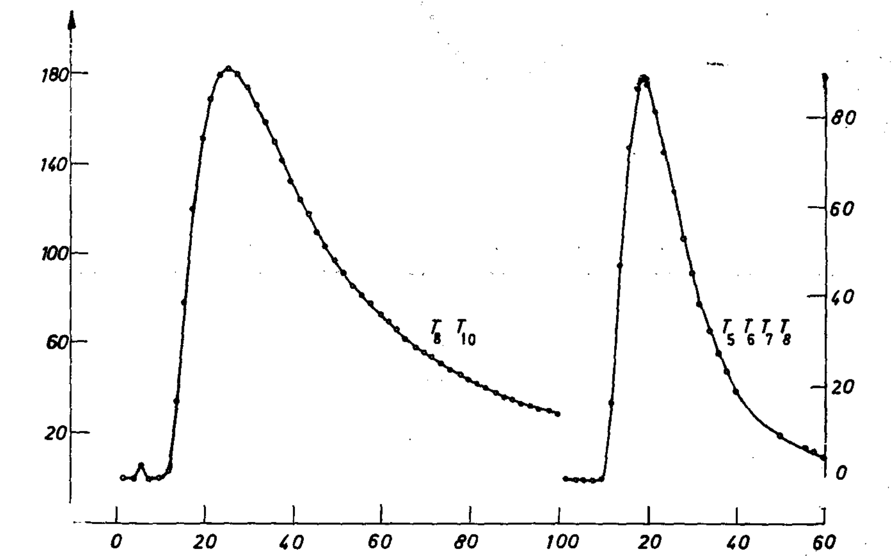

Fig. 5. The selectivity curves of $\Pi(T) = T_8T_{10}$ and $\Pi(T) = T_5T_6T_7T_8$ transformations.

asserted that this term has a regular sine shape with an amplitude $A \approx 24$ ms, the period variations being of the order of 10%.

From the above analysis it follows that the advantage of the Π (T) type transformations over the spectral analysis lies above all in that it offers the possibility of investigating the time variations of the amplitude and of the period of the concerned term.

Hence a more complete information on its nature might be obtained. Moreover, a wide choice of transformations is presented and consequently a suitable filter selectivity might be secured. From the example of the identification of the bi-annual term we may realize that rigorous selectivity may sometimes be undesirable. These are two principal reasons for which we are going to use the Π (T) type transformations along with Fourier transformations.

Owing to the existence of the 3.5 — year period term we had, in singling out the term of the period $P = 570$ days, to resort to a more selective transformation that the previous one. Our choice was the transformation $\Pi(T) = T_5T_6T_7T_8$, by which a maximum amplification is obtained at $P = 19.2$, whereas the terms with periods shorter or equal to that of the annual term, as well as those of periods greater or equal to $P = 3.5$ years, are properly damped (cf selectivity curve on Fig. 5). For instance, if the amplitude factor is unity for $P = 19.2$ it is reduced to only 0.05 for $P = 3.5$.

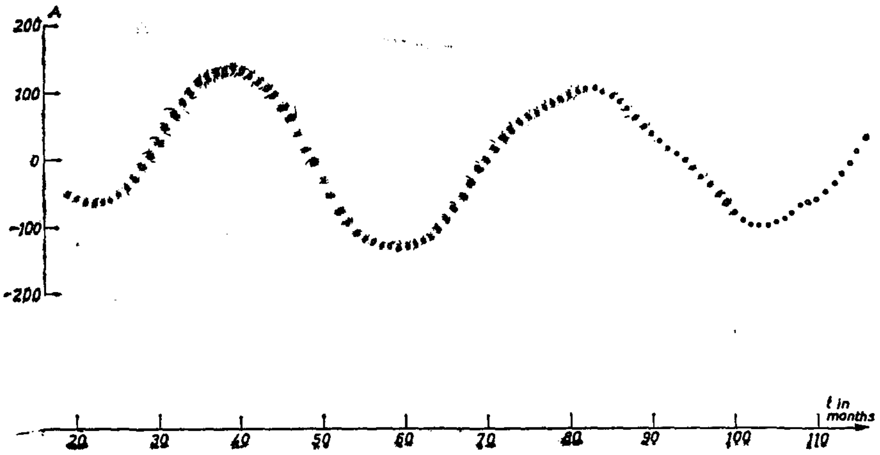

Fig. 6. The transformation of the mean 30 days R'' by $\Pi(T) = T_8T_{10}$.

The output signal of this last filter is represented on Fig. 7. As evident, it is generated by the 3.5 — year term. By these results the conclusion is suggested that within the narrower vicinity of $P = 19.2$ there were no periodical terms, that is, the respective peak in the R'' spectrum is a parasitic one. However, as it will be seen later on, a mistake would have been committed if such a conclusion was adopted.

From the example of the identification of this term it will also be seen that the application of the Fourier transformation results in a very erroneous amplitude value and that it even vanishes if a more selective filter $\Pi(T) = T_5T_6T_7T_8$ is used. These are the consequences of period variations. Owing to the summing of the ordinates in different phases the amplitude is reduced while the corresponding peak is widened.

In view of the number of spectral lines forming the peak at $P = 570$ days the probability of its appearance being due to a by chance resonance of errors is extremely small.

It is well known from theory that the selectivity function of the Fourier transformations $f(\lambda) = \sin\lambda/\lambda$ exhibits a principal maximum (for $\lambda = 0$) and a set of symetrical secondary maxima, whose amplitudes are decreasing as the absolute value of λ increases. The first secondary maximum appears at $\lambda = 4.49$ radians.

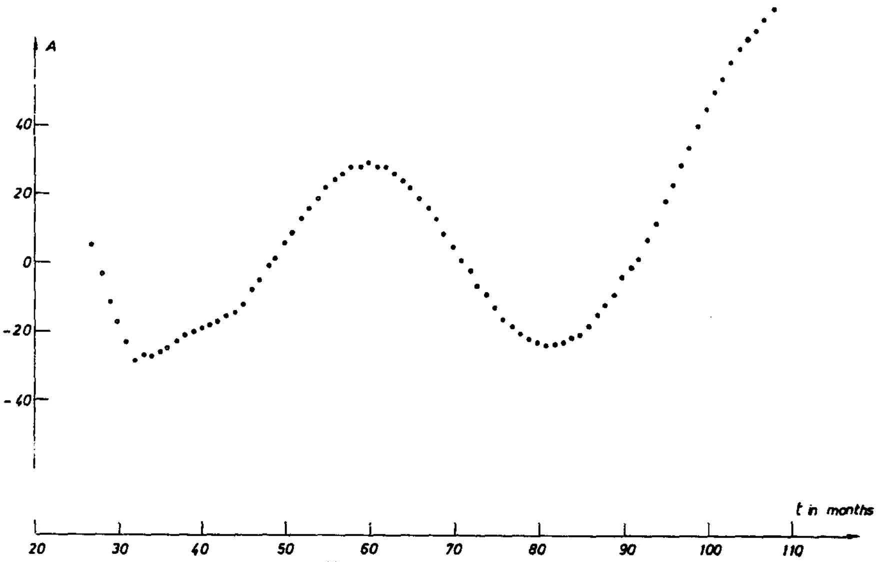

Fig. 7. The transformation of the mean 30 days R'' by $\Pi(T) = T_5T_6T_7T_8$.

Thus, denoting by f_0 the frequency of the 3.5 — year term, the first secondary maximum takes place at the frequency f, determined by the relation:

$$2\pi L(f - f_0) = 4.49 \qquad (7)$$

where 2L is the interval length, which in the present case is 4000 days. It follows that the first secondary maximum of the 3.5 — years term has a period $P = 840$ days and an amplitude $\Delta A = 0.22\,A$ (A being the amplitude of the primary peak, which, as already stated, is 24 ms). ΔA is nearly equal to the amplitude of the peak at $P = 570$ days. However, owing to the theoretical value of first sub-harmonic of the 3.5 — year term which is not close to the specified abscisse, there is no causal connection between them. To verify this, we also eliminated from UT2-TAI, the 3.5 — year term in addition to the secular and 7.4 — year terms. Let R''' be the residuals of the equations of condition enclosing these three terms.

Two peaks in the spectrum R''' may be observed (Fig. 8). The first is situated at $P = 600$ days and has an amplitude $A = 6.5$ ms, while the second peak covers a very wide range of periods, from $P = 700$ days to $P = 1200$ days. The last peak is barely discernible. One is rather inclined to say that within the zone P from 650 to 1000 days a white noise spectrum is existing. This apparent effect is a consequence of the predominance of the peak at $P = 1250$ days. It is obvious that the first peak in the spectrum R''' and the peak at $P = 570$ days in the spectrum R'' are due to the same cause.

From the evidence that the amplitude of the second peak ($A = 5.5$ ms) in Fig. 8 is even smaller than the one of the first peak we infer that the first peak is not a subharmonic of some of the terms of periods within the 700 to 1200 days diapason.

Considering the amount of the departure of the 7.4 — year term from the approximating sine wave and in view of the distance of its primary peak from the peak at $P = 600$ days, it is quite clear that the latter cannot be explained by the former.

By analysing R''' (Fig. 9) we noticed that they were in fact a quasiperiodical function of time with an amplitude of about 20 ms. Considering this fact and bearing in mind the conclusions reached earlier, we deemed it as the most probable explanation that the peak at $P = 600$ days was responsible for the inequality which, being not rigorously periodical, is damped by the selectivity filter $\Pi(T) = T_5T_6T_7T_8$. The supposition that it represented a subharmonic of some term of period $P < 600$ days was rejected, since within this interval only semi-annual and annual terms were identified respectively with amplitudes of 2.5 and 6.6 ms, which cannot have subharmonics of amplitudes of a few ms.

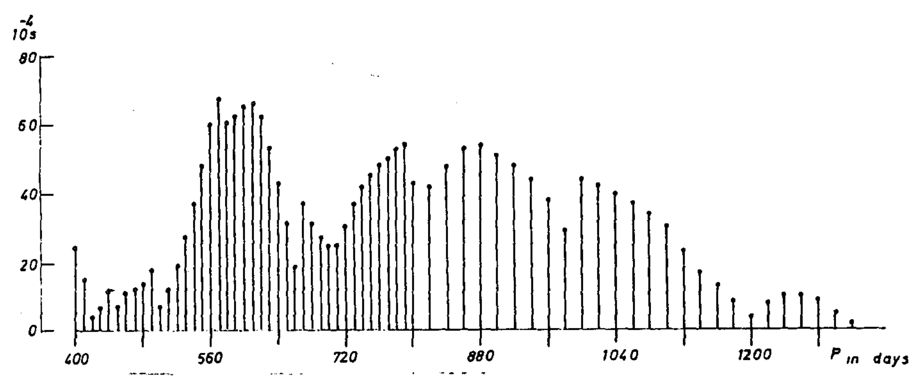

Fig. 8. The spectrum of the residuals $R''' =$ UT2-TAI — (secular, 7.4 and 3.5 year terms).

The term of period $P_5 = 2.3$ years, corresponding to the second peak in the spectrum R''', was identified through the transformation $\Pi(T) = T_8T_{10}$. The output signal of this filter is represented in Fig. 10. The zeros of the function have the following values: $t = 18, 32, 48, 59, 74, 92, 104$ and 116. We found, accordingly, that the function period varied from 22 to 36 months. The mean values of the periods and amplitudes are $P = 28$ months (2.3 years) and $A = 3.5$ ms. (the ordinates in Fig. 10 are to be multiplied by the coefficient $q = 0.178$).

In order to facilitate the study of the term of period $P = 600$ days, the secular as well as all the identified harmonic terms were subtracted from UT2-TAI. Then

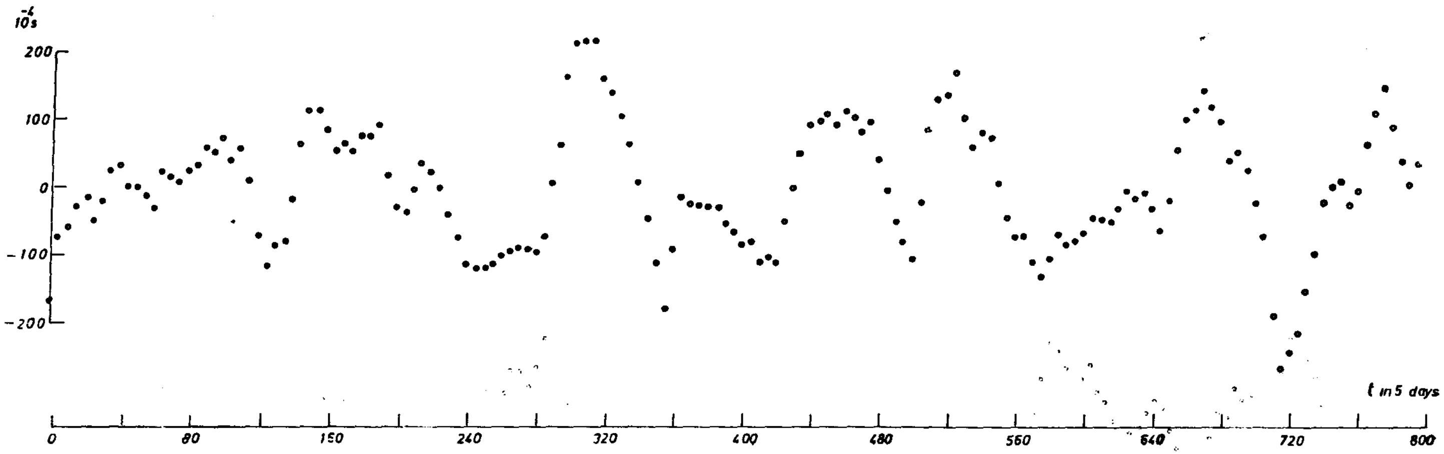

Fig. 9. The residuals R'''.

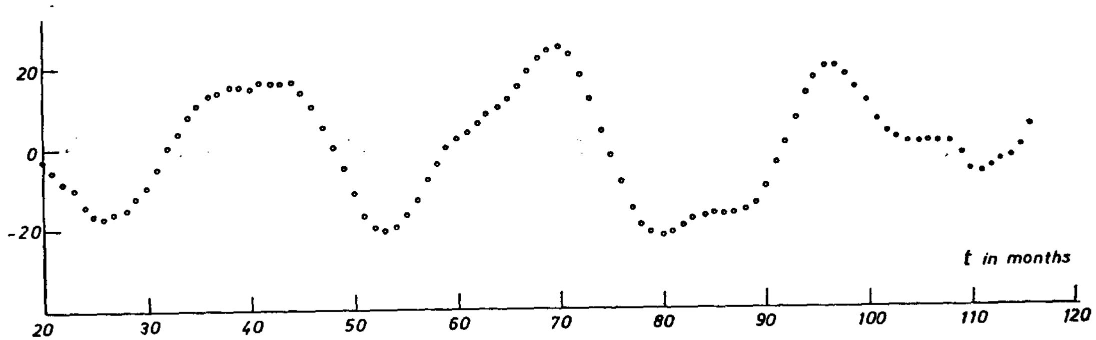

Fig. 10. The transformation of the R''' by $\Pi(T) = T_8 T_{10}$.

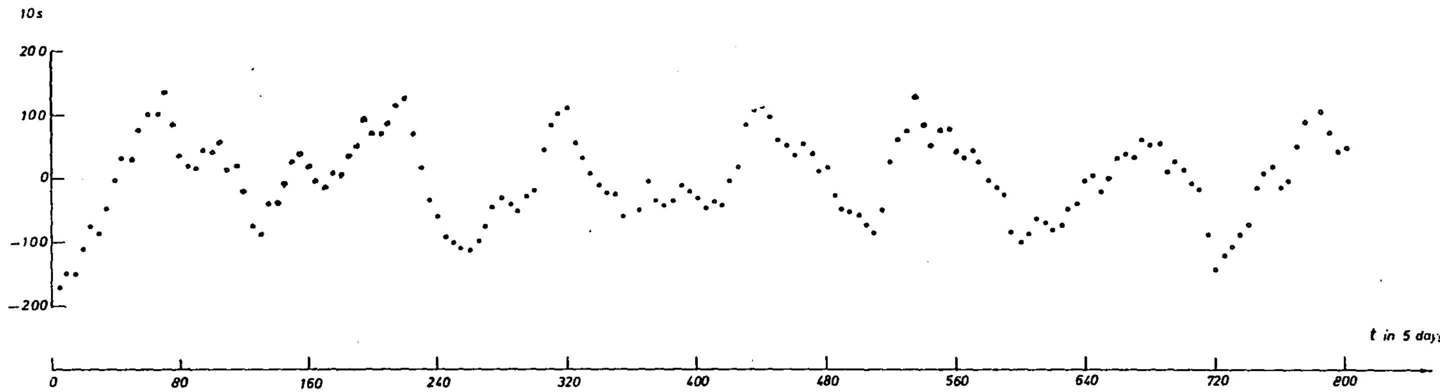

Fig. 11. The residuals R^{IV} = UT2-TAI — (secular, 7.4, 3.5, 2.3, 1.0 and 0.5 year terms).

we solved the system of the equations of condition by LSM:

$$UT2-TAI = a_0 + a_1 t + a_2 t^2 + \sum_{k=3}^{7} a_k \sin \frac{2\pi t}{P_k} + b_k \cos \frac{2\pi t}{P_k} \tag{8}$$

and we analysed the residuals R^{IV} of these equations.

The periods P_k (in years) have the following values: $P_3 = 7.4$, $P_4 = 3.5$, $P_5 = 2.3$, $P_6 = 1.0$ and $P_7 = 0.5$.

In the Fig. 11 illustration is presented of the residuals R^{IV}. The periodical inequality, with which the peaks at $P \approx 600$ days in the spectra R'' and R''' are associated, is obviously isolated. The zeros of the function R^{IV} take place at the following values of t (in 5-day units): 40, 120, 180, 230, 300, 340, 420, 480, 520, 580, 650, 700 and 760. The mean values of the period and amplitude (the latter being relatively more stable than the former) are, respectively: $P = 600$ days (1.64 years) and $A = 12$ ms.

4. CONCLUSIONS

By means of Fourier integral transformations and the Π (T) type transformations of the residuals UT2-TAI, periodical inequalities were identified, whose periods and amplitudes are:

P:	7.4	3.5	2.3	1.6	1.0	0.5	years
A:	100	24	4	12	7	3	ms

However the periods P are not constant, their relative variations being particularly great for the 2.3 and the 1.6 — year terms.

The amplitude of the residuals and the shape of their spectrum are based by the choice of the degree of the polynomial by which the secular term is approximated. At intervals of the order of 10 years the approximations by the third or higher degree polynomials cannot be explained.

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This work is a part of the research project of the Basic Organization of Associated Labour for Mathematics, Mechanics and Astronomy of the Belgrade Faculty of Sciences, funded by the Republic Community of Sciences of Serbia.

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