MINIMUM DISTANCES OF THE QUASICOMPLANAR ASTEROID ORBITS

J. Lazović, M. Kuzmanoski

Summary. Making use of the orbital elements taken from Ephemerides of Minor Planets for 1977, shortest distances, inferior to 0.0004 AU, have been determined of the asteroid orbits pairs with mutual inclinations not exceeding 0°500. All the numbered minor planets have been examined, out of which 142, forming 77 pairs, have been found to conform to the two previous conditions. Minimum distances of the asteroid orbits have been found which until now were unknown. With some of the pairs these distances are extremely small and might be regarded as curiosities. 13 orbital pairs have been found with minimal distances less than 10000 km, 600 km being the smallest among them.

Quasicomplanarity of asteroid orbi s facilitates the discovery of pairs (j, k) of the numbered minor planets whose orbits might have interesting proximities, i.e. shortest mutual distances. By using orbital elements of the pair of numbered asteroids under consideration j and k we can calculate the inclination I between their orbits by the formula (1)

$$\mathrm{tg}\frac{I}{2} = \mp \sin\frac{u_k - u_j}{2}\,\mathrm{cosec}\frac{u_k + u_j}{2}\,\mathrm{tg}\frac{i_k + i_j}{2}, \tag{1}$$

where u_j and u_k denote arguments of latitude of one among two relative nodes of the orbits investigated, which are determined by formulae

$$\left.\begin{aligned} \mathrm{tg}\frac{u_k + u_j}{2} &= \sin\frac{i_k + i_j}{2}\,\mathrm{cosec}\frac{i_k - i_j}{2}\,\mathrm{tg}\frac{\Omega_k - \Omega_j}{2}, \\ \mathrm{tg}\frac{u_k - u_j}{2} &= -\cos\frac{i_k + i_j}{2}\,\sec\frac{i_k - i_j}{2}\,\mathrm{tg}\frac{\Omega_k - \Omega_j}{2}. \end{aligned}\right\} \tag{2}$$

True anomalies of the relative nodes are determined by using known relation $v = u - \omega$.

Minimum distance ρ of the pair (j, k) of asteroid orbits is determined by the formula

$$\rho = \sqrt{(x_k - x_j)^2 + (y_k - y_j)^2 + (z_k - z_j)^2}, \tag{3}$$

where rectangular heliocentric ecliptic coordinates of two minor planets, as functions of true anomalies v_j and v_k for the proximity positions, are given by the formulae

$$l_i = r_i \cos v_i P_{l_i} + r_i \sin v_i Q_{l_i}, \quad (l = x, y, z; \quad i = j, k), \tag{4}$$

where

$$r_i = p_i (1 + e_i \cos v_i)^{-1}, \quad p_i = a_i (1 - e_i^2) = a_i \cos^2 \varphi_i \tag{5}$$

and

$$\left.\begin{aligned} P_{x_i} &= \cos \omega_i \cos \Omega_i - \sin \omega_i \sin \Omega_i \cos i_i, \\ P_{y_i} &= \cos \omega_i \sin \Omega_i + \sin \omega_i \cos \Omega_i \cos i_i, \\ P_{z_i} &= \sin \omega_i \sin i_i, \\ Q_{x_i} &= -\sin \omega_i \cos \Omega_i - \cos \omega_i \sin \Omega_i \cos i_i, \\ Q_{y_i} &= -\sin \omega_i \sin \Omega_i + \cos \omega_i \cos \Omega_i \cos i_i, \\ Q_{z_i} &= \cos \omega_i \sin i_i, \end{aligned}\right\} \tag{6}$$

the index i taking values j and k, i.e. the numbers of the paii considered of the numbered minor planets.

True anomalies v_j, in the orbit j, and v_k, in the orbit k, corresponding to the shortest distance positions of the two orbits, are determined by the solutions of the equations, (2, 3),

$$\left.\begin{aligned} f(v_j, v_k) &\equiv V_j \sin v_k + W_j \cos v_k + s_j \sin v_j = 0, \\ g(v_j, v_k) &\equiv V_k \sin v_j + W_k \cos v_j + s_k \sin v_k = 0, \end{aligned}\right\} \tag{7}$$

valid for the general case of elliptic motion and not only for quasicomplanar elliptic orbits. For these equations we have

$$s_i = e_i p_i, \quad (i = j, k) \tag{8}$$

and

$$\left.\begin{aligned} V_i &= V_i(v_i) = B_i + D_i \sin v_i + G_i \cos v_i + L_i \sin 2v_i + B_i \cos^2 v_i, \\ & \qquad (i = j, k) \\ W_i &= W_i(v_i) = C_i + A_i \sin v_i + F_i \cos v_i + M_i \sin 2v_i + C_i \cos^2 v_i. \end{aligned}\right\} \tag{9}$$

Coeficients in (9) are dependent on the asteroid orbital elements and are determined by the following expressions

$$\left.\begin{aligned} & d = e_j p_k, \quad h = e_k p_j, \\ & R = P_{x_j} P_{x_k} + P_{y_j} P_{y_k} + P_{z_j} P_{z_k}, \quad S = Q_{x_j} Q_{x_k} + Q_{y_j} Q_{y_k} + Q_{z_j} Q_{z_k}, \\ & T = P_{x_j} Q_{x_k} + P_{y_j} Q_{y_k} + P_{z_j} Q_{z_k}, \quad U = P_{x_k} Q_{x_j} + P_{y_k} Q_{y_j} + P_{z_k} Q_{z_j}; \\ & A_j = h e_j + p_k R, \quad A_k = d e_k + p_j R, \quad B_j = -d S, \quad B_k = -h S, \\ & C_j = -d U, \quad C_k = -h T, \quad D_j = p_k T, \quad D_k = p_j U, \\ & F_j = -p_k (1 + e_j^2) U, \quad F_k = -p_j (1 + e_k^2) T, \\ & G_j = -p_k (1 + e_j^2) S, \quad G_k = -p_j (1 + e_k^2) S, \\ & L_j = \frac{1}{2} e_j D_j, \quad L_k = \frac{1}{2} e_k D_k, \quad M_j = \frac{1}{2} d R, \quad M_k = \frac{1}{2} h R. \end{aligned}\right\} \tag{10}$$

The equations (7) are transcendent and their solution is performed by successive approximations. For this reason it is necessary to know approximate values v_{j0} and v_{k0} of the proximity true anomalies of the orbits j and k. With these values we further find first numerical corrections Δv_{j0} and Δv_{k0} to the true anomalies, which, added to the preceding initial values, yield new values v_{j1} and v_{k1}, which enable the procedure to be reiterated until the accuracy wanted has been reached. The necessary approximate values of the proximity true anomalies of the quasi-complanar asteroid orbits can numerically be determined by the formulae from (4) or (5), if we are concerned with small proximity distances as is just the case here. Upon calculating n numerical corrections to the true anomalies with necessary accuracy, the solutions of the system (7) is obtained in the form

$$v_{jn}=v_{j(n-1)}+\Delta v_{j(n-1)},\quad v_{kn}=v_{k(n-1)}+\Delta v_{k(n-1)},\quad (n=1,\,2,\,3,\,\ldots). \tag{11}$$

Numerical corrections to the true anomalies expressed in radians are found by the formulae, (2, 3),

$$\left.\begin{aligned}\Delta v_{j(n-1)}&=\frac{g_{n-1}\,f'_{v_{k(n-1)}}-f_{n-1}\,g'_{v_{k(n-1)}}}{f'_{v_{j(n-1)}}\,g'_{v_{k(n-1)}}-f'_{v_{k(n-1)}}\,g'_{v_{j(n-1)}}},\\ \Delta v_{k(n-1)}&=\frac{f_{n-1}\,g'_{v_{j(n-1)}}-g_{n-1}\,f'_{v_{j(n-1)}}}{f'_{v_{j(n-1)}}\,g'_{v_{k(n-1)}}-f'_{v_{k(n-1)}}\,g'_{v_{j(n-1)}}}.\end{aligned}\quad (n=1,\,2,\,3,\ldots)\right\} \tag{12}$$

In the expressions for f and g on the right-hand side, and in their partial derivations in v_j and v_k, for true anomalies should be taken for $n = 1$ the initial, i.e. approximate values v_{j0} and v_{k0}. Partial derivations of (12) are expressed by the equations

$$\left.\begin{aligned}f'_{v_j}&=\theta_j\sin v_k+\nu_j\cos v_k+s_j\cos v_j,\\ f'_{v_k}&=V_j\cos v_k-W_j\sin v_k,\\ g'_{v_j}&=V_k\cos v_j-W_k\sin v_j,\\ g'_{v_k}&=\theta_k\sin v_j+\nu_k\cos v_j+s_k\cos v_k,\end{aligned}\right\} \tag{13}$$

where

$$\left.\begin{aligned}\theta_i&=D_i\cos v_i-G_i\sin v_i-B_i\sin 2v_i+K_i\cos 2v_i,\\ \nu_i&=A_i\cos v_i-F_i\sin v_i-C_i\sin 2v_i+H_i\cos 2v_i,\\ K_i&=2L_i,\quad H_i=2M_i,\quad (i=j,\ k).\end{aligned}\right\} \tag{14}$$

ρ min, i.e. the shortest distance between two elliptic orbits exists provided ρ^2 min exists also, with regard to $\rho > 0$. From (3), (4) and (5) we have $\rho^2 = F(v_j, v_k)$. The equations (7) are obtained by way of the conditions for the extreme

$$\frac{\partial}{\partial v_j}\rho^2=\frac{\partial F}{\partial v_j}=0,\quad \frac{\partial}{\partial v_k}\rho^2=\frac{\partial F}{\partial v_k}=0.$$

Consequently, out of the solutions obtained for the equations (7) we single out those values of true anomalies v_j and v_k which correspond to the proximity, that is those satisfying conditions for the minimum of the function F

$$\frac{\partial^2F}{\partial v_j^2}\cdot\frac{\partial^2F}{\partial v_k^2}-\left(\frac{\partial^2F}{\partial v_j\,\partial v_k}\right)^2>0\ \text{and}\ \frac{\partial^2F}{\partial v_j^2}\left(\text{or}\ \frac{\partial^2F}{\partial v_k^2}\right)>0.$$

Another way of choosing true anomalies of the proximity consists in calculation, by using all the solutions of the sistem of equations (7), corresponding values for ρ by means of (3). The required values of the true anomalies of proximity are those corresponding to the minimum value for ρ obtained. This second method, in application with modern electronic computers, yields a prompt solution.

We have used asteroid orbital elements found in Ephemerides of Minor Planets for 1977, (6). There are 1940 minor planets numbered. Our computation did not include circular orbit of the minor planet 330. By checking up these orbits we stated that there were 2592 pairs of quasicomplanar asteroids with inclinations I between their orbits not exceeding $0^\circ\!.500$. What we were next interested in was whether among them were such that minimum distances between their respective orbits were inferior to 0.0004 AU (60000 km). This distance limit was choosen on account of the fact that in (7) the perturbation effect was studied of the larger asteroid upon the smaller one within an interval comprising minimum distance of 0.000498 AU known at that time. The perturbations obtained in (7) were on the very limit of measurements accuracy with two orbital elements, whereas for the remaining elements perturbing effects were still less. For this reason we choose the lesser value for the upper limit of the proximity distances, below which mutual perturbations of asteroids might perhaps have a perceptible effect. We indicate that in (8) attention has been drawn to this limiting distance as being of interest. Thus, our investigation of the asteroids numbered were restricted by the conditions of having simultaneously $I \leqslant 0^\circ\!.500$ and $\rho < 0.0004$ AU. For completing the task, as set above, of the determination of the minimum distances of quasicomplanar asteroid orbits we made use of the methods from (4) and (5), obtaining, as a result, approximate values of the possible true anomalies of proximity. The method (4) yielded in each case four pairs, while the method (5) two pairs of true anomalies with which the computation could be started according to formulae indicated above. With the initial solutions obtained according to both methods (4) and (5), the same final solutions for proximity have been found. However, the formulae from (5) are simpler and contain less parasitic solutions, accordingly they are more suitable for calculation. It appeared otherwise from our investigation that in many cases the computation of only first corrections to the true anomalies was sufficient, while only some the pairs of orbits needed three corresponding corrections (12) at the most.

We found that, among all 1939 numbered minor planets, there were 142 whose orbits could be grouped into 77 pairs with mutual inclinations $I \leqslant 0^\circ\!.500$ and minimum distances $\rho < 0.0004$ AU (60000 km). In Table I first two columns give the numbers j and k of those quasicomplanar asteroid pairs, in the third and forth are true anomalies v_j and v_k, corresponding to the proximities of their respective orbits, in the fifth column are values of the minimum distances ρ of these orbits in astronomical units, while the last column gives these distances in kilometers, their values being rounded off to 100 km.

With five pairs of orbits of the numbered asteroids (215, 1851), (452, 534), (461, 1782), (954, 1898) and (993, 1635) we found two values of distances below our accepted upper limit, but only the lesser one is corresponding to the proximity.

From Table I we gather that there are 13 orbital pairs whose minimum distances are under 10000 km. We see also that there are 7 pairs: (215, 1851), (227, 1737), (389, 972), (703, 1130), (763, 985), (960, 1818) and (1736, 1759) with minimum distances lesser than 5000 km. From all pairs the least distance between orbits is found with the minor planets 215 Oenone and 1851 ≡ 1950 *VA*, amounting to only 0.000004 AU or 600 km. The inclination between these two orbits is $I = 0^\circ.007$, the least of all inclinations found for the 77 orbital pairs, as is demonstrated in Table II. For the proximity of orbital pair (215, 1851) it was proved that absolute differences of their longitudes (longitude $\lambda = \Pi + v = \Omega + \omega + v$) as well as of their heliocentric radii vectors are the least of all, $|\lambda_{1851} - \lambda_{215}| = 0^\circ.000$, $|r_{1851} - r_{215}| = 0.00000$ AU. The proximity positions of their orbits are $0^\circ.651$ apart from one (nearer) relative node of their orbits. Concerning this relative node of orbits of asteroids (215, 1851) it might be added that absolute difference of heliocentric radii vectors related to it is 0.00378 AU.

Out of 77 founded pairs there are 60 with proximities less than 2° apart from one (nearer) relative node of the pair of orbits. Maximum angular distance of the proximity position from the nearer of the relative node, among all 77 orbital pairs, is $19^\circ.469$, and minimum $0^\circ.034$. Consequently, by using minimum differences of true anomalies of both relative nodes (calculated according to formulae (2)) and all approximate and possible true anomalies of proximity (found according to the procedure from (4) or (5)), only one choice is left of the pair of corresponding approximate true anomalies of proximity, with which further calculations should proceed, i.e. with which the determination of the numerical corrections (12) to the true anomalies could be started. Our investigation shows that, for the 77 orbital pairs from Table I, absolute values of differences of the asteroid heliocentric radii vectors at the relative node (nearer to the proximity position) maximum is 0.22391 AU and minimum 0.00016 AU. Moreover, our investigations have shown that for the 77 asteroid pairs quoted maximum absolute differences of their longitudes and radii vectors for the proximity positions are $|\lambda_k - \lambda_j|_{\max} = 0^\circ.030$ and $|r_k - r_j|_{\max} = 0.00016$ AU. This is a proof that longitudes, as well as radii vectors, are approximately equal at proximities with small distances, thus $\lambda_j \approx \lambda_k$ and $r_j \approx r_k$, which has been used in (4) and (5).

In this way minimum values of proximity distances of asteroid orbits have been found not known to date. For some of asteroid pairs very small distances were obtained which might be considered as curiosities. The results acquired may contribute to the better understanding of the structure of the asteroid ring and to further research into the minor planets motions.

TABLE I

j	k	v_j	v_k	ρ AU	ρ km
16	1245	287°.62796	306°.64487	0.000181	27100
21	367	67.74485	259.52543	0.000265	39600
24	1462	211.84576	157.08553	0.000391	58500
39	251	107.59110	27.58321	0.000218	32600
43	211	160.44300	4.00326	0.000250	37400
47	1541	100.30959	226.09830	0.000178	26600
50	1335	349.60812	351.90199	0.000158	23600
76	1692	69.37264	220.59650	0.000089	13300
79	1200	222.73182	20.44765	0.000273	40800
84	227	167.91930	278.42529	0.000254	38000
110	1393	83.56425	182.54042	0.000063	9400
111	1092	217.74901	67.87884	0.000196	29300
143	469	327.65561	8.66986	0.000099	14800
163	1874	172.72566	275.88308	0.000288	43100
171	1581	49.09445	12.08206	0.000112	16800

TABLE I (continued)

j	k	v_j	v_k	ρ AU	ρ km
205	992	219°.65970	51°.28628	0.000035	5200
212	406	21.76786	88.12420	0.000359	53700
215	1851	327.56118	306.90024	0.000282	42200
215	1851	83.09412	62.43319	0.000004	600
227	1737	279.46945	319.48135	0.000007	1000
243	1848	280.48391	74.84068	0.000303	45300
263	848	232.47876	271.77483	0.000345	51600
277	586	218.09333	106.66858	0.000291	43500
280	1325	7.78597	112.54814	0.000311	46500
311	1397	248.34106	122.18496	0.000066	9900
335	873	269.39551	298.49932	0.000149	22300
355	1700	135.20738	220.15767	0.000373	55800
376	1374	30.25708	285.31912	0.000394	58900
379	461	57.91706	308.60447	0.000369	55200
379	1635	71.04394	104.22916	0.000328	49100
384	722	66.32166	206.04756	0.000280	41900
389	972	116.67807	290.86408	0.000009	1300
400	1187	325.73505	124.06061	0.000175	26200
412	891	130.28246	283.27849	0.000065	9700
452	534	347.33623	55.52453	0.000364	54500
452	534	175.86183	244.05013	0.000168	25100
452	1131	46.48910	198.78498	0.000381	57000
460	1200	160.75821	277.39576	0.000254	38000
461	1782	79.98817	282.21957	0.000360	53900
461	1782	258.29803	100.52943	0.000238	35600
548	1551	265.30252	353.90335	0.000292	43700
554	557	67.32449	1.49396	0.000356	53300
577	765	346.69547	246.55519	0.000245	36700
650	1130	5.43865	69.20157	0.000207	31000
685	1307	80.96919	315.35742	0.000268	40100
703	1130	347.15430	45.74047	0.000014	2100
750	1850	259.47332	141.79536	0.000227	34000
753	1534	118.03826	278.25153	0.000369	55200
763	985	201.40629	230.58391	0.000014	2100
794	799	294.81511	183.47045	0.000344	51500
813	1676	341.63021	91.41892	0.000328	49100
848	1363	310.11490	323.05521	0.000210	31400
938	1331	156.85415	193.31457	0.000140	20900
938	1815	55.05279	292.98345	0.000043	6400
954	1898	44.63678	313.00164	0.000051	7600
954	1898	226.69343	135.05830	0.000291	43500
960	1818	95.88576	108.95271	0.000010	1500
962	1802	254.93451	182.85379	0.000294	44000
991	1305	107.68683	205.67714	0.000327	48900
993	1635	310.78987	61.34895	0.000342	51200
993	1635	125.85873	236.41781	0.000183	27400
1037	1791	232.15446	337.48418	0.000387	57900
1044	1630	258.95018	36.57779	0.000380	56800
1060	1203	184.45908	88.85490	0.000262	39200
1078	1634	253.84839	110.01125	0.000147	22000
1079	1100	136.82421	244.33272	0.000098	14700
1082	1782	316.70102	27.81620	0.000112	16800
1135	1381	203.88874	176.56846	0.000228	34100
1137	1667	117.17875	196.56229	0.000232	34700
1142	1539	58.16334	274.64358	0.000360	53900
1169	1810	257.58366	231.12576	0.000325	48600
1251	1492	53.25505	191.76226	0.000070	10500

TABLE I (continued)

j	k	v_j	v_k	ρ AU	ρ km
1289	1635	10°.03757	358°.04769	0.000370	55400
1443	1774	26.51935	295.81216	0.000346	51800
1446	1527	228.55678	120.85455	0.000399	59700
1487	1581	166.90727	174.95300	0.000168	25100
1560	1829	344.08138	291.62300	0.000325	48600
1590	1827	236.69278	55.04312	0.000192	28700
1644	1836	254.00077	79.91455	0.000205	30700
1651	1856	47.12304	332.04404	0.000125	18700
1736	1759	216.41884	261.38227	0.000022	3300
1740	1821	130.74980	218.35347	0.000237	35500

TABLE II

j	k	I	j	k	I	j	k	I
16	1245	0°.253	376	1374	0°.129	960	1818	0°.041
21	367	0.187	379	461	0.464	962	1802	0.187
24	1462	0.303	379	1635	0.392	991	1305	0.234
39	251	0.225	384	722	0.229	993	1635	0.041
43	211	0.408	389	972	0.251	1037	1791	0.029
47	1541	0.156	400	1187	0.145	1044	1630	0.470
50	1335	0.292	412	891	0.312	1060	1203	0.364
76	1692	0.387	452	534	0.072	1078	1634	0.488
79	1200	0.064	452	1131	0.422	1079	1100	0.496
84	227	0.184	460	1200	0.055	1082	1782	0.413
110	1393	0.141	461	1782	0.089	1135	1381	0.143
111	1092	0.496	548	1551	0.142	1137	1667	0.354
143	469	0.335	554	557	0.453	1142	1539	0.410
163	1874	0.018	577	765	0.393	1169	1810	0.084
171	1581	0.208	650	1130	0.389	1251	1492	0.318
205	992	0.163	685	1307	0.318	1289	1635	0.332
212	406	0.137	703	1130	0.310	1443	1774	0.054
215	1851	0.007	750	1850	0.126	1446	1527	0.158
227	1737	0.246	753	1534	0.312	1487	1581	0.361
243	1848	0.345	763	985	0.048	1560	1829	0.390
263	848	0.318	794	799	0.041	1590	1827	0.488
277	586	0.461	813	1676	0.347	1644	1836	0.143
280	1325	0.463	848	1363	0.151	1651	1856	0.367
311	1397	0.360	938	1331	0.443	1736	1759	0.041
335	873	0.261	938	1815	0.362	1740	1821	0.092
355	1700	0.421	954	1898	0.105			

The computation has been performed on IBM 360/44 in the Computing Centre of the Institute for Mathematics in Beograd.

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