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NUMERICAL DETERMINATION OF THE APPROXIMATE TRUE ANOMALIES OF THE QUASICOMPLANAR ASTEROIDS PROXIMITY — A NEW VARIANT

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Summary. A simpler method, as compared with the previous known ones, of the determination of the approximate true anomalies of proximity of two elliptic orbits is laid out for the case their minimum distance is small. These anomalies are the starting point of the solution of our equations for the proximity. The method is very efficient in the serial investigations of large number of pairs of minor planets orbits.

The determination of the minimum distance of a pair (j, k) of orbits of two asteroids implies the solution of two transcendent equations (1)

$$f(v_j,\ v_k) \equiv V_j \sin v_k + W_j \cos v_k + s_j \sin v_j = 0,$$

$$g(v_j,\ v_k) \equiv V_k \sin v_j + W_k \cos v_j + s_k \sin v_k = 0,$$

where V_i and W_i $(i = j, k)$ are definite functions of the unknown true anomalies v_j and v_k, corresponding to the proximity positions, i.e. to the minimum distance of the two orbits. These equations are valid for the elliptic motion in general and their solution is performed by the method of successive approximations. Consequently, their solution can be started provided we know some approximate solutions of them v_{j0} and v_{k0}.

For the pairs of quasicomplanar asteroids, with very small shortest distances of their orbits, the only ones which we are interested in, simple formulae can be derived by means of which the required approximate values are determined v_{j0} and v_{k0} of the true anomalies in the proximity. For reasons of simplicity we denote them herein by v_j and v_k. Thus, the index $_0$ will be omitted as we find out here only approximate values and there therefore cannot be any ambiguity. For the proximity positions with small shortest distance of the orbits of two quasicomplanar asteroids j and k the advantage can be taken of the fact their heliocentric radii vectors, as well as their longitudes (longitude $\lambda = \Pi + v = \Omega + \omega + v$) are then nearly equal: $r_j \approx r_k$, $\lambda_j \approx \lambda_k$. By putting in equality instead of approximation, we obtain equations

$$\frac{p_j}{1 + e_j \cos v_j} = \frac{p_k}{1 + e_k \cos v_k}, \tag{1}$$

$$\Pi_j + v_j = \Pi_k + v_k, \tag{2}$$

where $p_i = a_i(1 - e_i^2)$, $\Pi_i = \Omega_i + \omega_i$ and idex $i = j, k$. Upon introducing the substitution

$$\Pi = \Pi_k - \Pi_j, \tag{3}$$

we have from (2)

$$v_k = v_j - \Pi. \tag{4}$$

The expression (4) is significant as it is by it that we are able to find the approximate value v_k of the true anomaly of the proximity position in the orbit k by means of the corresponding approximate value v_j in the orbit j. Consequently there remains to find the expression for the determination of the true anomaly v_j. After the elimination v_k from (1), by means of (4), we obtain the equation for v_j

$$t \sin v_j + q \cos v_j = p, \tag{5}$$

where

$$p = p_k - p_j, \quad t = e_k p_j \sin \Pi, \quad q = e_k p_j \cos \Pi - e_j p_k. \tag{6}$$

Let us introduce two new parametric quantities x and y, dependent on the orbital elements of the asteroids j and k,

$$x = \frac{q}{t}, \quad y = \frac{p}{t}, \tag{7}$$

assuming thereby that $t \neq 0$, that is $\Pi \neq 0$ or $180°$, as was to be expected for asteroid orbits in view of the equation (3). Now from (5) and (7) there follows the equation

$$y = x \cos v_j + \sin v_j \tag{8}$$

as in (2) and (3), which we will solve here in a different, simpler way. It consists in the introduction of a new angle α, depending on certain orbital elements of the asteroids under consideration j and k. This angle is determined by means of the simple expression

$$\operatorname{tg} \alpha = x, \tag{9}$$

whence an unique value for α within the interval $-90° < \alpha < +90°$, in view of our assumption concerning the quantity t. Finally, from (8) and (9) we obtain the expression

$$\sin(v_j + \alpha) = y \cos \alpha, \tag{10}$$

by way of which the approximate value is determined v_j of the true anomaly of the proximity position in the orbit j. The second corresponding approximate value of the true anomaly v_k in the orbit of asteroid k is then found by means of (4). The left-hand side of the equation (10) exists if the condition is satisfied $|y \cos \alpha| \leqslant 1$, and it is satisfied if small distances are required at the proximities of the quasi-complanar asteroid orbits, as demonstrated in the paper (4). The expression $\sin(v_j + \alpha)$ has the sign of y, considering the limits introduced for α, in consequence of which it is always $0 < \cos \alpha \leqslant 1$. For each one of the calculated values of the left-hand side of the equation (10), different from ± 1, we obtain two corresponding values for v_j: v_{j1} and v_{j2}. Now with these two values we calculate, by means of (4), two corresponding values of v_k: v_{k1} and v_{k2}. Thus, by means of the expressions (10) and (4) we obtain two pairs of the true anomalies (v_{j1}, v_{k1}) and (v_{j2}, v_{k2}), of which only one pair represents the required approximate values of the true anomalies of the proximity of orbits j and k. This multiplicity of signs is a consequence of the fact that for one calculated value of $\sin(v_j + \alpha) \neq \pm 1$ no definite quadrant for the angle $v_j + \alpha$ is known, as two values of it could follow. In case only when $\sin(v_j + \alpha) = +1$ or -1 we would obtain one value only for v_j, and then, through (4), one value only for v_k. Thus, by means of the procedure exposed we were able to obtain a formula simpler than that in (3) and only one pair at the most of the parasitic anomalies, not corresponding to the minimum distance between the orbits of the considered asteroid pair. Thus this method, taken as a variant of the numerical determination of the approximate true anomalies of proximity, appears more suited to the application than the previous one from (3). It is more suitable than those in (1) and (2) as well.

With two pairs of the true anomalies obtained with indices 1 and 2 we can start the solution of the above transcendent equations $f(v_j, v_k) = 0$ and $g(v_j, v_k) = 0$, and next, with their solutions, we calculate corresponding distances ρ between the orbits j and k. According to the lesser of two values obtained for ρ, corresponding to the proximity, respective pair of true anomalies is being chosen for the proximity positions of the orbital pair investigated. Another way for choosing the proximity true anomalies among those found possible is that they must satisfy conditions of the minimum distance ρ, i.e. for the function $\rho^2 = F(v_j, v_k)$, in other words with the values of true anomalies of proximity, conditions

$$\frac{\partial^2 F}{\partial v_j^2} \cdot \frac{\partial^2 F}{\partial v_k^2} - \left(\frac{\partial^2 F}{\partial v_j \, \partial v_k}\right)^2 > 0 \quad \text{and} \quad \frac{\partial^2 F}{\partial v_j^2} > 0 \; \left(\text{or} \; \frac{\partial^2 F}{\partial v_k^2} > 0\right)$$

must be fulfilled. A third approach to the question of selecting the pair of true anomalies, now approximate ones, corresponding to the proximity of orbits of asteroids j and k consists of selecting among possible pairs the one giving minimum absolute value of differences of true anomalies of the relative nodes of the two orbits under consideration and all possible approximate values found earlier of true anomalies for proximity. This third approach, which we will illustrate by an example, has found its justification through additional investigations made in (4), for it has been demonstrated that the proximity positions, in their majority, are at small angular distances from a relative node. However, the first two ways of selecting the pair of proximity true anomalies are more reliable then the third. The calculation they involve has to be carried out completely, whereas with the third the selection is performed with the initial, i.e. with approximate values.

As an example of the application of the new procedure exposed herein, we take the already investigated quasicomplanar minor planets $j \equiv 589$ Croatia and $k \equiv 1564$ Srbija, with the same elements as used in (3), in order to make possible the comparison of the results of the two methods. By using formulae (3), (6), (7), (9) and (10) we have obtained two values in the first orbit j

$$(v_{j0})_1 = 118^\circ\!.288, \quad (v_{j0})_2 = 272^\circ\!.607, \tag{11}$$

of which only one represents approximate value for the proximity position. Now we have introduced the index $_0$ with true anomalies, indicaring thereby that their values are approximate. With the values (11) and by means of the expression (4) we determine the next two values of the true anomalies, of which one only corresponds to the proximity position, in the second orbit k

$$(v_{k0})_1 = 105^\circ\!.602, \quad (v_{k0})_2 = 259^\circ\!.921. \tag{12}$$

The same values (11) and (12) have been found in (3) also; but there four pairs of values have been obtained and not two as here. Thus, three parasitic solutions follow from the method (3), not corresponding to the proximity, whereas one only pair of parasitic values of the true anomalies is obtained when this method is used. Let us compare the values (11) and (12) with the values of true anomalies of relative nodes of orbits of the pair considered

$$v_j = 296^\circ\!.047, \quad v_k = 283^\circ\!.352, \quad v_j' = 116^\circ\!.047, \quad v_k' = 103^\circ\!.352,$$

where the first pair of values corresponds to the ascending node of the orbit j with reference to the orbit k, while the second pair corresponds to the descending relative node. Minimum differences obtained being

$$(v_{j0})_1 - v_j' = 2^\circ\!.241, \quad (v_{k0})_1 - v_k' = 2^\circ\!.250,$$

it is inferred that the proximity is nearer to the descending relative node and that the approximate values of the true anomalies of proximity are

$$v_{j0} = (v_{j0})_1 = 118^\circ\!.288, \quad v_{k0} = (v_{k0})_1 = 105^\circ\!.602. \tag{13}$$

It is to be noted that the same solutions would have followed had we instead of the substitution (9) applied the substitution $x = \mathrm{ctg}\,\beta$ whereby the equation is obtained $\cos(v_j - \beta) = y \sin\beta$ whence we could determine $(v_{j0})_1$ and $(v_{j0})_2$.

By comparing the approximate values (13) with the already known exact values v_j and v_k of the true anomalies of proximity obtained previously for the pair of asteroids under study, (1), we obtain for their differences

$$v_j - v_{j0} = 0^\circ\!.010, \qquad v_k - v_{k0} = 0^\circ\!.001,$$
$$0^\circ\!.0097, \qquad 0^\circ\!.0005,$$

while the sum of the squares of these differences is

$$0.000, \quad \text{i. e. } 0.0001$$

the same as follow by applying the method (3).

It appears that with the solutions (13) it is sufficient to calculate only the first corrections of these true anomalies of the asteroids considered in order to obtain minimum distance of their orbits. Accordingly we can draw the conclusion that the method is very efficient and suited for the investigation of large number of pairs of asteroid orbits, leading to the same solution as that in (3) but in a simpler way.

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