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PRESENT STATE OF GEODETIC ASTRONOMY

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Introductions, tasks of geodetic astronomy

The main tasks of geodetic astronomy are the determination of geographic latitude, geographic longitude and azimuth of terrestrial direction from observation of celestial bodies, i.e. from astronomic observations.

Closely connected with the determination of geographic longitude is determination of time. It is known that geographic longitude is the difference between local and Greenwich time. Determination of time as an independent task is being performed by time services that connect their quartz and atomic clocks to astronomic observations and emit the obtained time through time-signals.

At an observing station, the direction of the gravitation force is given physically by a plummet or water-level.

The geographic latitude of a point is an acute angle between the direction of the gravitation force in this point and any plane that cuts the axis of rotation of the earth at right angle. The meridian plane of a point is the one that passes through the direction of gravitation force in this point and is parallel to the axis of rotation. Geographic longitude is the angle between the meridian plane in this point and the plane of the Greenwich meridian.

These definitions are necessary as we do not consider the earth a sphere or a rotational ellipsoid but a system of level surfaces that run at right angles to the direction of gravitation force in a given point. The form of earth is geoid. This is a level surface, determined by the surface of the sea assumed calm. The geoid is a very complicated surface and cannot be defined mathematically.

It follows from the above that the direction of gravitation force changes with the altitude above sea-level over the same point. So we come to the notion of the vertical. This is a curve. We can consider it a trajectory that would be described by a heavy mass point, falling in vacuum on Earth that would not be rotating. By the plummet there is given the tangent to the vertical in the point where the plummet hangs. The direction of gravitation force is so the tangent to the vertical.

From the above the important fact follows that the determination of geographic latitude and longitude of a point on Earth does not mean the determination of its position, considering that the Earth is a geoid. The geographic latitude and longitude of this point only determine the position of the tangent to the vertical in this point.

As the vertical is a curve, the geographic co-ordinates of a point on the surface of the Earth differ from the geographic co-ordinates of the intersection of this vertical with the geoid. So geographic co-ordinates of any point have to be referred to the geoid.

Parallels and meridians on the geoid are not curves in a plane and can only be given point by point as geometrical places.

The meridian with the geographic longitude λ on the geoid is the geommetrical place of all points whose meridian planes form the angle λ with the meridian plane of Greenwich. So all meridian planes of points with the same geographic longitude are parallel to each other.

In geodesy different ellipsoids are used as representative of the Earth's form. The direction of gravitation force (the tangent to the vertical in a particular point) generally differs from the normal to the ellipsoid in that point. The difference is called the deviation of the vertical. For this reason the geodetic geographic co-ordinates, B, L (obtained by geodetic transfer on a concrete ellipsoid) differ from the astronomic geographic co-ordinates φ, λ obtained by astronomic measurement.

The deviations of the vertical depend on the kind and orientation of the reference ellipsoid and for this reason they are called relative deviations of the vertical. If a general ellipsoid is taken, which is defined unequivocally, absolute deviations of the vertical are obtained.

In flat and hilly regions the deviations of the vertical amount to some seconds, in high mountains they amount to some tens of seconds.

According to Helmert the differences are called as follows:

$\xi = \varphi - B =$ the deviation of the vertical in latitude
$\zeta = \lambda - L =$ the deviation of the vertical in longitude
$\eta = (\lambda - L) \cos \varphi =$ the deviation in the Ist vertical.

An essential notion in geodesy is the notion of azimuth.

The astronomical azimuth of the direction related to the terrestrial mark is the angle between the plane that runs parallel to the axis of the earth through the direction of the gravitation force in the given point and the plane that runs through the direction that is to be determined and the direction of the gravitation force.

Astronomic azimuth a differs form the geodetic, i.e. ellipsoid azimuth A of the same direction. The difference $(a - A)$ is called the deviation of the vertical in azimuth.

If the direction towards the terrestrial object runs approximately horisontally, Laplace's equation is valid:

$$(a - A) - (\lambda - L) \sin \varphi = 0.$$

The ellipsoid azimuth A can be obtained from this equation and it is called Laplace's azimuth. Laplace's azimuths are important in big triangulation net--works in geodesy. By using them the accuracy of the transfer of directions can be greatly improved.

There will follow some instances of the application of geodetic astronomy and its methods in geodesy.

1. The orientation of the triangulation net-work on a concrete reference ellipsoid is carried out by astronomic determination of geographic co-ordinates and

by determination of astronomic azimuth in one or more points of the net. If the astronomic observations were carried out in several points, by translation and rotation on the ellipsoid the network is oriented in such a way that the sum of the squares of the deviations of the vertical is minimum.

2. Higher accuracy in transferring the directions in big triangulation networks is reached by determination of Laplace's azimuths in several evenly arranged points of the network.
3. The determination of the dimensions of the earth ellipsoid is carried out by measuring the length of several meridian and other arcs and astronomic determination of the geographic co-ordinates on the ends of these arcs. According to this method the dimensions of the majority of earth ellipsoids used have been determined. It is not used any more nowadays.
4. Astronomic measurements and methods of geodetic astronomy are also used in determining the general earth ellipsoid for the big triangulation network. The dimensions of the ellipsoid and orientation are obtained according to the method of the least squares, so that the sum of the deviations of the vertical is minimum. This so-called „surface method" has served to establish Hayford's ellipsoid.
5. By astronomic-geodetic methods the profiles of the geoid relative to the chosen reference ellipsoid are also determined.
6. Astronomic-geodetic determination of the deviations of the vertical also serves in geodesy for trigonometrical determination of the altitudes and in space triangualtion.
7. The motion of the earth poles and irregularity in its rotation is studied according to the methods of geodetic astronomy.
8. The (ground) control points in photogrammetric and topographic measurements in developing countries are determined astronomically. In practice, this method is of a limited accuracy since the deviations of the vertical are not known.
9. In expeditions the geographic co-ordinates, time and azimuth are determined astronomically.
10. In astronomic navigation the methods of geodetic astronomy are used.
11. Satellite and stellar triangulation is not possible without knowledge of geodetic astronomy.

Basic principles of the determination of geographic co-ordinates and time

By introducing the celestial sphere, the complicated geographic co-ordinate system of level surfaces of the Earth is transferred to a strictly defined spheric co-ordinate system. The celestial sphere has its centre in the centre of the Earth and an infinitely long radius in comparison to the dimensions of the earth. We make use of the basic quality of the celestial sphere that parallel directions intersect in the same point of the celestial sphere. The directions of the gravitation force are lengthened to the intersection with the celestial sphere. The equator on the geoid is not a circle but a closed curve in the space. But the celestial equator is a circle that is obtained by lengthening the directions of the gravitation force in the points of the earth equator to the celestial sphere.

The basis of astronomic determination of geographic co-ordinates are three facts:

1. Geographic co-ordinate system of the Earth is transferred to the celestial sphere by lengthening the direction of the gravitation force and thus a strictly defined spheric co-ordinate system is obtained.
2. In thus defined spheric co-ordinate system on the celestial sphere, the zenith of any observer has the same co-ordinates as the observer on the earth.
3. Apparent co-ordinates of the celestial bodies in geodetic astronomy are known. They can be obtained from the astronomic ephemerides for any moment.

The essence of astronomic determination of geographic co-ordinates is in the determination of the co-ordinates of the observer's zenith relative to the known co-ordinates of stars.

The essence of astronomic determination of time is the fact that by means of astronomic ephemerides for any moment the hour angle of the celestial body at Greenwich can be established. And vice versa: if the Greenwich hour angle has been found, the corresponding moment can be found in astronomic ephemerides. The determination of time for another meridian is carried out through the difference between Greenwich and local hour angle at the same moment.

So there are two tasks to be solved:

1. The determination of the co-ordinates of the zenith of the observer relative to stars.
2. The determination of the Greenwich hour angle of the celestial body.

Both tasks can be solved by measuring horisontal angles or by measuring zenith distances of celestial bodies relative to the direction of the gravitation force in the observing station. The principle could be explained geometrically on a globe representing the celestial sphere.

Determination of geographic co-ordinates by measuring zenith distances of two celestial bodies

From observations there are obtained z_1', T_1, z_2' T_2, where z_1' is *app.* zenith distance, at the moment T_1, of the first celestial body and z_2' the *app.* zenith distance, at the moment T_2, of the record celestial body. The apparent zenith distances are corrected for the influence of astronomic refraction and parallax and the real zenith distances z_1 and z_2 are obtained.

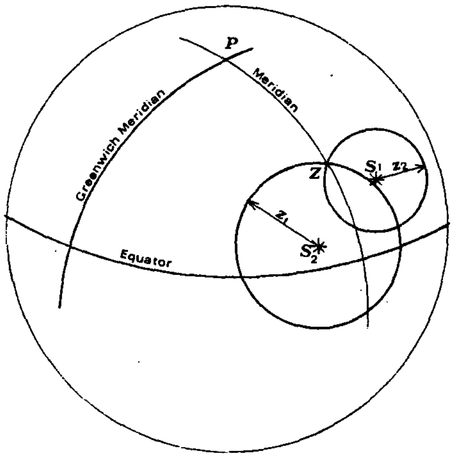

Fig. 1.

For the moments T_1 and T_2 by means of the Star Almanach the co-ordinates of both celestial bodies are obtained: α_1, δ_1, α_2, δ_2, and both Greenwich hour angles are established. Now the positions of both celestial bodies can be referred to the celestial sphere.

The procedure for determining the geographic co-ordinates is as follows: Any point on the surface of the sphere is chosen as celestial north pole P and a polar circle which corresponds to this points is constructed. This is the celestial equator. Any great circle through P could be initial, i.e. Greenwich meridian. The positions S_1 and S_2 can be transferred. Also two small circles with the centres in S_1 and S_2 and spheric radii z_1 and z_2 can be drawn. The observer's zenith is in one of the two dissections of the two small circles. As the wanted co-ordinates are always approximately known, it is clear which dissection corresponds to the observer's zenith.

Determination of local time from measured zenith distance of one celestial body

From observations the apparent zenith distance z' is obtained and corresponding notation U' was made on the clock that shows approximate Greenwich time. It is also known whether the star was observed on the eastern or the western side of the sky and the geographic co-ordinates of the observing station.

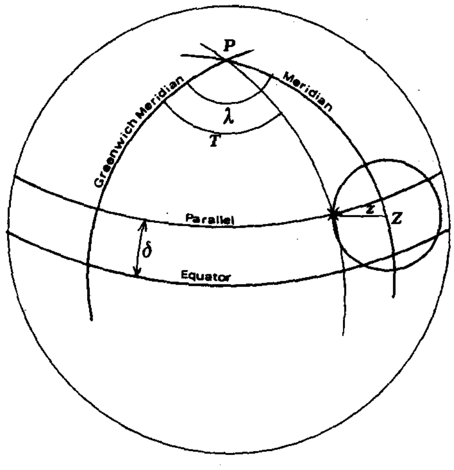

Fig. 2.

Let point P be chosen as celestial north pole and its polar circle is constructed, which is the celestial equator. Any great circle through P should denote the initial meridian.

By means of known geographic co-ordinates the position of the observer's zenith Z is drawn. The great circle through P and Z is then the meridian of the observation station. For the moment T' the declination of the celestial body is obtained from the Star Almanach and the corresponding parallel of the star is drawn. The star is somewhere on this parallel, i.e. in one of the dissections with a small circle, whose centre is in the zenith and whose radius is equal to the zenith distance z.

The angle T in the Fig. 2. is the hour angle of the star at Greenwich. By means of the hour angle T of a star with known co-ordinates, from the Star Almanach Greenwich time U can be obtained and the state c_p of the clock by comparison with Greenwich time:

$$c_p = U - U'.$$

Local time is obtained from Greenwich time by means of known geographic longitude.

Instruments in geodetic astronomy

1. If an accuracy of $\pm$ 1″ or less is desired, usual glass-circle, one-second theodolites can be used, e.g. Wild's, Zeiss etc.
   For astronomic observations the theodolites must have auxiliary equipment:
   — eyepiece prisms
   — telescope

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— illuminating gear for reading the micrometers and making the cross-wires visible
— striding level
— Horrebow level
— a plate to make possible the attachment of the instrument to a pole
— equipment for observations of the sun
  a) glass filters
  b) Roelofs solar prism
— various accessories to determine the direction of the meridian (meridian prisms).

2. For more accurate determination in geodetic astronomy universal instruments can be used, e.g.
   — Wild *T* 4
   — astronomic theodolite *DKM3A* Kern with double circles
   — Gigas precise theodolite Tpr with photographic registration
   — geodetic-astronomic universal theodolite Theo 003 Zeiss-Jena.
3. For mesurements of the highest accuracy special instruments are used:
   — transit instrument
   — astrolabes
     a) prisms and mercury horizon
     b) „impersonal“ Danjon astrolabe
     c) astrolabe Ni2.
4. The clocks used in geodetical astronomy must be transportable. The following clocks can be used:
   — ordinary pocket or wrist-watches
   — stop watches with double finger
   — chronometers with second finger, e.g. type Nardin
   — quartz clocks
     a) small quartz clock CAQ Rohde—Schwarz
     b) transportable quartz chronometer Chronotom CP Patek Philipp (weight 3.8 kg).

Geodetical astronomy as subject at technical faculties in Yugoslavia

Term	V	VI	VII	VIII	
Belgrade			2+2	2+4	geodetical astronomy
Ljubljana	3+0	0+3			geodesy
			2+1	1+2	geodetical astronomy
Sarajevo				4+4	geodetical astronomy
Zagreb		4+0	0+2		spherical astronomy
			3+0	0+3	practical astronomy

Besides, the are 15 hours of practical exercise in the field in Zagreb.

Importance of geodetic astronomy

The subject geodetic astronomy must be an essential part in the education of every engineer of geodesy, if it is desired this engineer to be capable of working in all fields of his profession, including research.

The foundation of this standpoint is the fact that geodetic astronomy, in spite of introduction of new measuring methods including satelite geodesy, has not lost its importance. For decades it will still be necessary to measure astronomic azimuths and geographic co-ordinates with different accuracy for practical and scientific tasks.

Besides, geodetic astronomy is of high professional importance considering the use of artificial earth satelites in geodesy.

Undoubtedly, geodetic astronomy is of great paedagogical importance in the education of a future engineer of geodesy. This subject does not only give knowledge about close connections between geodesy and astronomy but it gives also valuable experience in working with precise instruments and clocks.

Last but not least, the great importance of geodetic astronomy for formation of man must be stressed. The thinking man has always tried to understand the happenings in the sky and to make use of them for technical and scientific development. So geodetic astronomy offers the future engineer of geodesy the possibility of orientation in the widest sense of word in his relation to cosmos.

Perspectives of the development of geodetic astronomy

The development of geodetic astronomy can be seen in the development of instruments and of instrumental technique. Instruments are getting smaller, more compact and less sensitive for outside influences. The persepctive is also in the manner of traating the results of observations (Computers), but less in inventing new measuring methods. As is known, there exists a series of astronomic-geodetic methods that can be applied according to different criteria.

In 1951 Embacher explained the simultaneous method for determination of the azimuth and latitude by observing two stars in their greatest digression. In 1961 this method was supplemented by Otto, in 1974 G. Hemmleb in Dresden developed a modified Embacher's method that also makes possible the determination of geographic longitude.

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