Publications of the Department of Astronomy — Beograd, № 6, 1976.

UTILIZATION OF SOME TECHNICAL ACHIVEMENTS FOR FUNDAMENTAL DETERMINATION OF RIGHT ASCENSION

J. Bem
Astronomical Observatory of Wrocław University, Poland

Summary. An independent information about the irregularities of the Earth's rotation received by VLBI techniques can be profitable utilized in the determination of the α coordinates of stars. By using the atomic time scale, the reductions of meridian observations become simplified, more objective and homogeneous. The proposed method was tested on a short series of meridian observations.

1. *Introduction*

The task of a fundamental determination of the right ascensions is generally divided into three different operations (Zverev 1950), namely:

— determination of the azimuth of meridian marks and the lateral flexure of the tube,
— evenning of the right ascension system,
— determination of the zero point of the right ascension scale.

The last task is realized by observations of such bodies of the solar system, whose orbital elements and actual positions are best suited for the determination of the correction of the position of the equinox (Duma 1972). This is performed by night and day observations, by meridian and off-meridian instruments. Therefore this point will not be further discussed.

In order to reduce the meridian observations, the chain method (Nemiro 1950) or the Washington method (Baskett 1964) is usually employed. These methods were quite adjusted to the older observations, when the clock rate was unknown and solving as set of linear equations with several hundreds unknowns was impossible. In both methods the stars are divided into three groups — circumpolar, equatorial and other stars. In addition, in the chain method a strict overlapping of series is necessary. Therefore, in this paper a proposition is made for a homogeneous treatment of all observational data. It is similar to the suggestion of Srömgren (1925) for the circumpolar observations. A full objectiveness of the results can be achived under the following conditions:

— easily accessible time base, which can ensure the linearity during whole observational period, 3 — 5 years,
— independently of his own observations, the observer should be able to obtain additional informations about the relative changes of the Earth's rotation,
— to avoid the difficulty with the control of the personal errors the human eye must be excluded from the observational process,
— the observations should be made in such a manner that all instabilities of the instrument could be expressed by certain parameters measured as often as possible,
— and finally, the reduction of the observations should be carried out on a sufficiently fast computer.

The atomic time scale, obtainable at each observatory, enables one to discard the traditional clock correction, usually determined independently every evening. This correction includes mainly the irregularity of the time scale, which arises from the imperfection of the clock mechanism, and to a smaller degree, from the irregularities of the Earth rotation. Therefore, if the latter were obtained independently as by the radio interferometry (Ryle and Elsmore 1973), or VLBI techniques, and the Polar motions by Doppler satelite observations, then in the final calculations where would remain some residue of the conception „clock correction", which in this paper will be denoted by w. In fact, this quantity contains several errors and zero points:

a) Zero point of the sequence of the values describing the difference between the uniform angle and the true angle of the Earth rotation, obtained as mentioned above, with the reference to the atomic time scale.
b) The error of the longitude adopted for the instrument.
c) The systematic error of the instrument.
d) The difference between adopted and the „true" value of the displacement of the zero points of the atomic and efemeris time scales, multiplied by a small factor, equal to 0.002737.
e) The systematic error arising, when the observations are refered to the atomic time scale, from the circumstances such as retardation of the radio signals or the chronograph.

2. *Observational Equations*

If it is postulated that quantity w is constant for entire time span of the data collection for the catalogue, then for the observation of each star the following equations can be written:

$$w + \Delta \alpha_i + a_k A_i \pm g B_i = L_{ik} \tag{1}$$

where w denotes the unknown quantity described above in points *a*) to *e*),

$\Delta \alpha_i$ — correction to the right ascension of the i-th star,
a_k — azimuth of the meridian mark line on the k-th evening,
g — constant of the lateral flexure,
A, B — Mayer's coefficients,
L_{ik} — known data, containing also the changes of the collimation, inclination, azimuth of the instrument referred to the meridian marks, etc.

If observations are grouped in a series of near nights, and if on the successive nights the instrument is set up in opposite clamp positions, then the lateral flexure of such a group of nights (at least two nights), can be assumed constant. The difference of two equations, relating to the same i-th star on successive nights, give a new set of equations:

$$2g\ B_i - \Delta a\ A_i = \Delta L_i. \tag{2}$$

As can be seen from this equations, the constant of the lateral flexure can be calculated without the knowledge of the correct right ascensions. Therefore, the principle of the absoluteness is preserved. It must be pointed out that in such a calculation of the lateral flexure nearly all stars spread along the whole accesible meridian are taken into account, instead of one or several circumpolar stars. The by-product, Δa, characterizes the invariability of the meridian marks position during these successive nights. It is evident that, if the differences ΔL in eqs. (2) were described by the trigonometric series limited to only two terms — $\sin z$ and $\cos z$, then they do not correspond to the real instrument. Analyzing the residuals from eqs. (2), it is possible to decide whether the constant B can be treated as the Mayer coefficient or if the lateral flexure must be calculated in other way (Petrov and Bem 1973).

If the lateral flexure is put into the right-hand side of eqs. (1), then the fundamental set of equations, containing all observations of the catalogue, can be written as:

$$w\ \cos \delta_i + \Delta \alpha'_i + a_k\ \sin z_i = L'_{ik}, \tag{3}$$

where $\Delta \alpha' = \Delta \alpha\ \cos \delta$ and $z = \varphi - \delta$ or $z = \varphi + \delta - 180°$ for the upper and lower culminations, recpectively. To avoid simultaneous solving this set of equations with many thousands of unknowns, it is better to seperate the equations with a distinguishing feature (Vegt and Ebner 1972). In such a case, from the set (3) the equations concerning the stars observed at both culminations can be excluded. The solution of this indeterminate system of equations can be written as:

$$\begin{aligned} a_k &= f_{1k} - q_{1k}\, w, \quad k = 1,\ 2,\ \ldots\ m, \\ \Delta\alpha'_j &= f_{2j} - q_{2j}\, w, \quad j = 1,\ 2,\ \ldots\ n. \end{aligned} \tag{4}$$

By m and n there are denoted the number of nights and the number of stars observed at both culminations, respectively. If the invaniability of the zero point of the initial catalogue is postulated, then the additional equation can be written as follows:

$$\Sigma\ \Delta\ \alpha' = \Sigma\ \Delta\ \alpha \cos \delta = 0. \tag{5}$$

It is valid for the whole sphere, as well as for the zone limited in declination, especially for the circumpolar zone. This last equation allows the calculation of the unknown w for the circumpolar zone, and then also the other unknowns from the eqs. (4).

If observations are made only during two nights, and if the „catalogue" contains only two circumpolar stars and two equatorial stars (this gives 3 observations on each night), then the system of equations (3) consist only of six equations and of the additional condition (5). In such simple case it is evident, that the unknown azimuths are independent of the constant w, and of the equatorial stars. This consideration may be extended to all observational data, with the conclusion, that all coefficiets q_{1k} in eqs. (4) are equal to zero. Such a result, of course, was obtained in the practical example quoted below, in paragraph 4. Therefore, when solving the eqs. (3) it is sufficient to find the coefficients $f_{1k} = a_k$. In contrast to the unknown quantities $\Delta\,\alpha'_i$, the azimuths a_k get high weights in this system of equations (Nemiro 1950), so that determination of $\Delta\,\alpha$ from the circumpolar set becomes unnecessary.

Introducing the azimuth corrections $a \sin z$ into the right-hand side of eqs. (3), we get for i-star n_i reduced observations: $w \cos\delta_i + \Delta\alpha'_i = L''_{ij}$, with the weights $p(z)$, or the formulae:

$$\Delta\alpha'_i = \frac{\sum_{j=1}^{n_i} L''_{ij}\, p(z_j)}{\sum_{j=1}^{n_i} p(z_j)} - w\cos\delta_i\,. \tag{6}$$

The condition (5), weighted in the same way as the above formulae, can be written as a sum for all N stars:

$$\sum_{i=1}^{N} \Delta\,\alpha'_i\, p(z_i)\, n_i = 0. \tag{7}$$

From eqs. (6) and (7) we get

$$w = \sum_{i=1}^{N} n_i\, p(z_i) \left(\sum_{j=1}^{n_i} L''_{ij}\, p(z_j) / \sum_{j=1}^{ni} p(z_j) \right) / \sum_{i=1}^{N} n_i\, p(z_i) \cos\delta_i. \tag{8}$$

and going back to eqs. (6), we get all unknown quatities $\Delta\alpha'$.

3. *The alternatives for the requirement $w = const.$*

The weights $p(z)$ assigned to each observation represent some empirical function of the zenith distance of the observed star. This function can be given a priori, i.e., from the analysis of the residuals of eqs. (2). After getting all unknown quantities w, $\Delta\,\alpha'_i$, a_k, we get also a new set of residuals of eqs. (3), and finally a second version of the $p(z)$ function by investigation of the local dispersions (Bielicki 1972). With the new weights for eqs. (3), we get a new set of values w, $\Delta\,\alpha'_i$, a_k, through first eqs. (4) and then from the eqs. (6), (7), and (8). This iterative cycle can be continued so long, as only one of the unknown quantities, $\Delta\,\alpha'_i$, differs in two successive iterations by more than a given criterion.

It seems reasonable to represent the weights as a function of the zenith distance. If some of eqs. (3), grouped about the zenith distance $z1$, give residuals v_i slightly greater than in the other regions, then the final errors of $\Delta\, \alpha'_i$ are practically independent of this residuals. The mean error of the whole system increases insignificantly. Because the coefficients on the left-hand side are unchanged, it is impossible to see the decrease in accuracy of $\Delta\, \alpha'$ about $z1$. For the meridian observations an increase of the residuals of eqs. (3) can be caused by the seeing (which is a function of z), the pivots irregularities, local refraction phenomena (building, trees, shape of the earth surface) and a false mathematic model of the lateral flexure of the tube.

The simplicity of this method results from the assumption of the condition $w =$ const. during the whole observational period. From the elements included in the constant w, and mentioned in the Introduction, only the point relating to the invariability of the systematic errors of the instrument during such a long time can be problematic. But if points three and four, as listed in this Introduction, apply, then this postulate seems to be acceptable. The verification of the postulate $w =$ const. can be made after final iterative solution by analyzing the residuals ordered according to the time. If the dependance $w(t)$ is noticeable, then the solution of eqs. (3) must be acquired in a different way e.g., after solving eqs. (3) under condition $w =$ const. and with $p(z) = 1$, (the observations are cleaned from the rough errors by Chauvenet's or other criterion), we get a field of residuals v_i, which describes the random and systematic errors of the observational process. To this field a function of two variables, zenith distance z and epoch of observation t can be fitted. The systematic part includes chiefly the errors arising from the false mathematical model of the lateral flexure and from the unrealistic condition $w =$ = const. A simultaneous consideration of this two effects is possible, if the field of the residuals v_i, were described by a system of orthogonal functions.

Transforming the epoch of observation t from the interval (t_0, t_k) into variable t' from the interval $(0,2\,\pi)$, the field of residuals will be extended over the whole sphere. In order to describe such field we can use the spherical functions:

$$v_i = \sum_{j=0}^{g} b_j\, K_j(t'_i, z_i) + \varepsilon_i\,, \qquad i = 1, 2, \ldots N. \tag{9}$$

Here N denotes the number of all observations. Functions K_j must be subjected to the orthogonalization process, because on the sphere there are zones not covered by observations, like zones $|z| > 75°$, and circumpolar zone, and because the observational points are not distributed uniformally in the remaining regions. A complete algorithm of such treatment was given by Brosche (1966), who employed it to make a comparison between two catalogues. The choice of the highest order g of the spherical functions is equivalent to the seperation of the systematic part from the residuals v_i. The remaining, ε_i, contain now the random part. It can be done by the use of the χ^2 test, simulteneously with the analysis of the corelation function of the deviations ε_i, (Yatskiv and Kuryanova 1975).

The next step cosists in the introduction of the correction given by formula (9) in all free quantities L'_{ik} of eqs. (3). After new solution is performed we get a new sequence of the unknown quantities w, $\Delta\, \alpha'_i$, a_k, and a new field of residuals v_i. The whole cycle can be repeated with the same or some other aproximation function. If no term from the relevant part of the result $\Delta \alpha'_1, \Delta \alpha'_2, \ldots$, differs from the coresponding term of the former solution by more than the required final accuracy of the catalogue (about 1 ms), then the iterative cycle can be closed.

4. *Test on a Short Series of Observations*

In February 1972 a short series of visual observations was made with the transit circle of the Wrocław Astronomical Observatory. Part of the stars were observed in both culminations, in the evening and morning, during five successive days. Within such a short time the instrument-observer system can be assumed constant, and instead of the quantities describing the irregularities of the Earth rotation, which should be given by the radiointerferometric or other techniques, the quantities $UT1 - UTC$ were used. Unfortunately, the lack of the sufficiently numerous observational data which could satisfy the conditions mentioned in the Introduction, permitted to test only the variant with the weights. The final values of the unknowns $\Delta \alpha_i$ and a_k, as inessential here, will not be given.

From 172 observational equations, the following 44 unknown quantities are calculated: corrections to the right ascension of 35 stars, azimuths of each series, and the constant w. The constant of the lateral flexure and the a priori system of weights, were assumed on the basis of about one thousand observations, which were made several month earlier. The right-hand side of equations (3) was calculated from the formulae:

$$L'_{ik} \sec \delta_i = \alpha_{app}, i \pm g B_i + \lambda + \Delta \lambda - TE_i - (UT1 - UTC)_k + \\ + (TAJ - UTC) - 6^h38^m458^s.36 - 236^s.55536049\,(d + TE_i) - \\ - 0.0929\, T^2 + 31^s.868 - \text{nut}. \tag{10}$$

$$TE_i = T_i + (\text{radio signal} - \text{clock}) + (UTC - \text{radio signal}) + \\ + (TAJ - UTC) + 31^s.868. \tag{11}$$

The moment of transit of each star on the ephemeries time scale was calculated indirectly, by means of the atomic time from the formula: $TE = TAI + 31.868$ s, given by A. Stoyko (1970). Here T_i denotes a moment of transit (with the readings on the clock in UT), corrected for all known instrumental errors and constants. It must be pointed out, that the ephemeris time scale is only formaly included in the argument of the Newcomb formula in equation (10). In fact, the atomic time scale corrected by the above mentioned value of 31.868 s was used. In this way we avoided any systematic errors arising from the accepted in our epoch, reciprocally unparallel ephemeris time scales $E0$, $E1$, and $E2$. If we assumed some other difference between the atomic and ephemeris time scales (Balmino 1974), we get only a change in the calculated constant w. It was verified in this practical case by omitting from eq. (10) the quantity $UT1 - UTC = 0.1434$ s for the beginning of February 1972. All the rest of results remain the same within the accuracy of 5×10^{-5}s.

In each iterative cycle, for rejecting the wrong observations, instead 3σ criterion the Chauvenet's criterion was used, modified by Bielicki (1972). However for $N = 172$ all three criteria were nearly equal: 60, 62 and 60 ms, respectively.

All observations fulfiled this criterion, so after four iterations, when differences between $\Delta\alpha'_1, \ldots\ldots \Delta\alpha'_n$, in two successive cycles were less than 1 ms, the calculation was closed. In table 2 the differences of the results obtained after k-th cycle and last cycle, where $k = 1, 2, 3$, are given for all unknown quantities. The constant w, containing mainly the systematic error observer-instrument and the error of adopted longitude (see Introduction, points a to e), assumed finally value $w = -0.037$ s.

After each cycle a new system of weights was calculated from the residuals of eqs. (3) earlier sorted according to the zenith distance, by the following formulae:

$$z_k = \frac{1}{20}\sum_{j=k+1}^{k+20} z_j, \qquad p(z_k) = \frac{20}{N}\sum_{i=1}^{N} v_i^2 : \sum_{j=k+1}^{k+20} v_j^2,$$

where N is the total number of observations and $k = 0, 5, 10, \ldots$ The difference between the initial weights given a priori from the earlier observations and the weights a posteriori calculated after fourth cycle, are given in table 1.

Finally, it must be pointed out, that there are no rigorous postulates in this method concerning the duration of an observational series, as well as the number of observations and its arrangement. As in the chain method (Nemiro 1950), daily observations are unnecessary and the strict overlapping of series is not required. It is evident, that if during night a series of many observations is obtained, then there is an adventegous decrease in the number of the unknown azimuths. In addition, the system of eqs. (3) containing the stars attainable at both culminations, is gathered up in one block system not only on the basis of the atomic time scale, but also by a system of corrections of the right ascensions of these stars. Hence this method becomes more similar to the block adjustment systems, as introduced in the photographic astrometry (Ebner 1970, Vegt and Ebner 1974). A serious inconvenience is brought in by the requirement of the uniformity of distribution of the observations of each star during the whole observational period of the catalogue.

5. *Appendix*

The equations (1) and (10) are obtained as follows: the moment of transit on ephemeris time scale is calculated by formula (11), and then with this argument from the Newcomb formula, we get the right ascension of the ephemeris sun $\alpha_\odot$. The angular distance of the meridian marks line from the zero ephemeris meridian is equal to $1 = \lambda + \Delta\lambda + w_0 + w_k$. The sum $w_0 + w_k$ is a difference between the uniform and true angles of the Earth rotation, which must be given in principle by the radio-interferometric measurements. This quantity is similar (but not the same) to the data published in the *BIH* Circulars, assumed from the Time Service observations. Therefore, in the above numerical example, for the date k we assumed $w_k \approx -(UT1 - UTC)_k$, and the above sum was substituted by expression

$$w_0 + w_k \approx w'_0 + (TAI - UTC) + 31^s\!.868 - (UT1 - UTC)_k.$$

Subtracting the above angular distance 1 from the Greenwich ephemeris sidereal time $TE - 12^h + \alpha\odot_{ef} + nut.$, we get the angular distance of the equinox from the meridian marks line, or the mean local sidereal time:

$$\alpha = TE - 12^h + \alpha\odot_{ef} + nut. - \lambda - \Delta\lambda - w_k - w_0 = L' - w_0.$$

Comparing this formula with the apparent right ascension α_{app} of the just observed star (calculated on the basis of the initial catalogue), we get $\alpha_{app} - \alpha = {}_{app} - L' + + w_0$. Denoting $\Delta\,\alpha = \alpha_{app} - \alpha$, $L = \alpha_{app} - L'$, $w = - w_0$, and introducing the azimuth aA and the lateral flexure bB corrections, we get the observational equation (1).

Table 1. The weights a priori and a posteriori as a function of zenith distance.

z_n	p1	p2	z_s	p1	p2
1°0	1.0	1.1	1°0	1.1	0.8
2.8	1.0	0.9	5.0	1.0	1.0
4.6	1.1	0.9	8.8	1.0	0.8
6.2	1.0	1.0	12.6	0.9	0.8
8.7	1.2	1.1	15.7	1.2	0.9
12.5	1.2	1.2	18.6	1.3	1.2
16.5	1.2	1.6	22.3	1.5	1.6
22.8	1.1	1.5	25.9	1.4	2.0
60.4	0.6	0.6	30.2	1.1	1.1
64.1	0.7	0.8	35.3	0.9	0.8
66.9	0.7	0.8	40.1	1.0	0.7
68.5	0.8	0.9	45.1	1.1	0.7
70.1	0.9	1.1	51.6	1.6	0.8

Table 2. The convergence of the iterative cycles demonstrated by differences between the results obtained after the three first iterations and the last, for the unknown quantities $\Delta\alpha'$, a, and w, in 0.0001 s.

FK 4	Differences		
Number	I—IV	II—IV	III—IV
103	+ 1	+ 4	— 7
108	— 1	+ 5	— 5
1397	—12	+ 4	— 6
595	—18	+ 5	— 4
99	+ 6	+ 4	— 5
598	+14	+ 5	— 1
571	0	0	— 3
122	+ 1	+ 7	— 4
1096	+16	+ 1	— 3
554	+ 5	— 1	— 2
138	—20	+ 7	— 3
569	—57	+10	— 3
550	—23	+ 5	— 3
590	—10	+ 2	0
112	— 7	+ 5	— 4
131	—11	+ 5	— 4
1416	— 5	+ 6	— 4
573	+ 5	+ 6	— 4
555	— 6	+ 6	— 5
374j	— 1	+ 6	— 5
2232	—14	+ 7	— 5
563	— 4	+ 7	— 5

FK 4 Number	Differences I—IV	II—IV	III—IV
144	—17	+ 7	— 5
576	+ 4	+ 7	— 6
578	— 4	+ 7	— 6
136	—21	+ 8	— 6
375j	— 3	+ 8	— 7
114	—27	+10	— 6
591	— 2	+ 8	— 7
150	—25	+ 9	— 7
582	— 1	+ 8	— 8
588	+11	+ 7	— 7
547	0	+ 9	— 8
104	—30	+10	— 7
127	—32	+ 9	— 7
azimuths			
a1	—80	— 2	+ 4
a2	+33	— 7	+ 5
a3	+57	— 1	+ 1
a4	—63	+ 4	0
a5	+30	— 4	+ 2
a6	+29	— 2	0
a7	+40	— 5	+ 5
a8	— 2	+ 2	— 2
constant			
w	+10	— 8	+ 6

Remark: stars 374j and 375j are taken from Astr. Ezhegodnik.

LITERATURA

Balmino G., 1974 Cel. Mech. vol. 10, 4, p. 425.

Baskett T., 1964 Publ. US Nav. Obs. vol. XIX, part I.

Bielicki, M., 1972 IAU Symp. № 45, p. 112.

Brosche P., 1966 Veröfi. Astr. Rechen Inst. Heidelberg, № 17.

Duma D., 1972 Astrometria i Astrofizika, t. 15, Kiev.

Ebner H., 1970 A. N. B. 292, H. 2.

Nemiro A., 1950 Izv. Glav. Astr. Obs. Pulkove, t. XVIII.

Petrov G. M., Bem J., 1973 Acta Astr., 23, 49.

Ryle M., Elsmore B., 1973 M. N. R. A. S., vol. 164, 3.

Stoyko A., 1969 Bull. cl. sci. Acad. Roy. Belg. 55, 4.

Strömgren B., 1925 A. N. B. 225, 5378.

Vegt Chr., Ebner H., 1972 Astr. Astrophys. 17, 2.

Vegt Chr., Ebner H., 1974 M. N. R. A. S., 167, 1.

Yatskiv Ya., Kuryanova A., 1975 Preprint Gl. Astr. Obs. Kiev.

Zverev M., 1950 Usp. Astr. Nauk, t. V.