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ON THE STRUCTURE AND THE DEVELOPMENT OF THE SUNSPOT GROUPS

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Abstract. An attempt has been made to explain some observed evolutional characteristics of the sunspot groups by the convective model of supergranular motion.

INTRODUCTION

Although since the first telescopic observations of sunspots has past more then three and a half century, we still do not have the complete theory of sunspot to include all observed phenomena. The situation is even more complicated when we search for a theory of the sunspot evolution or the development of sunspot groups. There are many reasons for a such situation. One of them is lack of good, reliable series of sunspot pictures with high resolutions (better than 1.5″), to cover the whole life of sunspot groups. As a matter of fact, we do not have any information from the depth of sunspot formation. That is why in some cases observations and existing theory are in contradiction.

In this paper we should like to emphasize two results from the contribution Bumba et al (1973), and to display present basic ideas for a possible theory of sunspots, heaving in mind papers: Wilson (1968), Weart (1970) and Kubicela (1973).

OBSERVATIONAL MATERIAL

Studying the geometry of sunspot groups during their whole life, Bumba et al· (1973) came to these results:

a) In many groups the distances between the leading and the following parts are „quantized“ approximately at 30 000 km by the first and second day after their birth.

b) The pore and the spots in the sunspot groups show a characteristic elliptical form which can be observed in any phase of evolution of the groups.

Some very convincing examples, as we can see in Fig 1, 2 and 3, show elliptical distribution of sunspot groups.

These pictures are selected from good observational material made by Clark refractor (205/2830 mm) at Ondrejov observatory. About 50 sunspot groups were photographed in about 200 000 pictures by metallic interference filter produced by VEB C. Zeiss Jena, with λ max around 5900 Å in combination with Gevaert Duplo Pan film and with Agfa — Gevaert Copex Pan film. Resolution at the best quality negatives was practically equal to the the theoretical values of objective resolutions, i. e. to 0,5′.

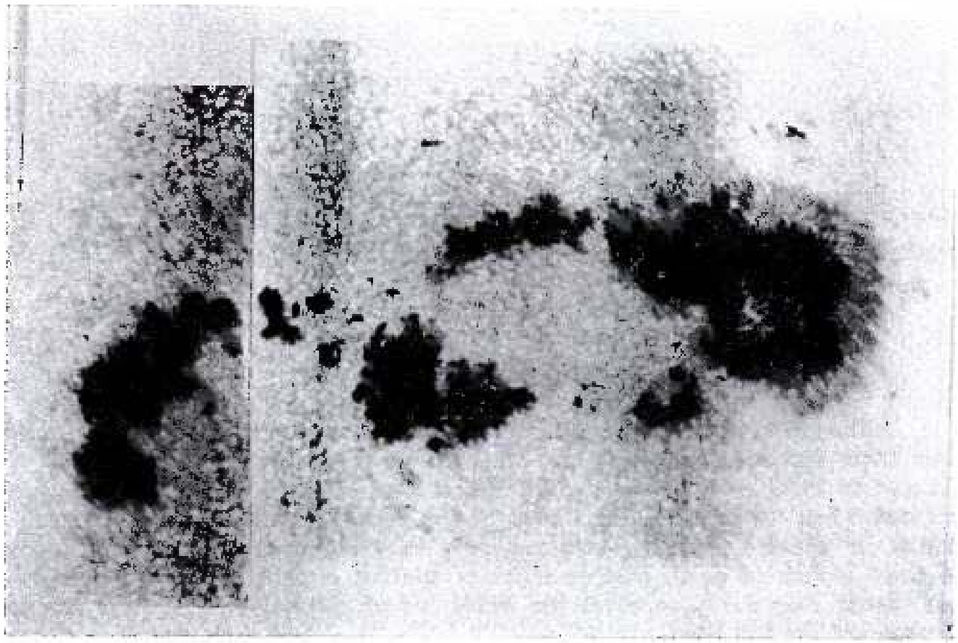

Fig. 1: June 12, 1963, $06^h\ 31^m$

At figure 4 the drawing is given presenting the sunspot groups in series from June 11 till June 14, 1963. Formation and desintegration of elliptical structure is obvious.

BASIC IDEAS FOR THE STRUCTURE OF SUNSPOTS

All the modern theories of sunspot structure are based on Biermann's suggestion that magnetic fields inhibit the deep convective transport of heat. But none of them provide a clear picture of the sunspot development from its birth as a pore, throughout the whole life. Many outstanding observed facts still remain as unresolved problems.

We are going to start with main results of Wilson's (1968) model because in it there is no other postulate concerning the magnetic field except Babcock's (1961) suggestion of strongly twisted buoyant flux tubes.

In a plasma, deep under the photosphere, magnetic field flux are essentially horizontal and more or less homogeneous.

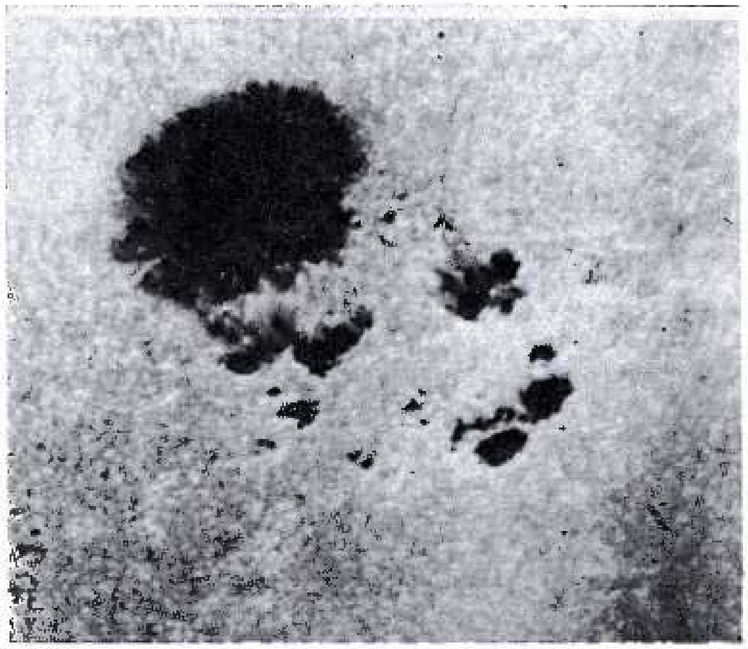

Fig. 2: July 31, 1963, $08^h\ 43^m$

The equation of momentum of such plasma is:

$$\text{grad}\, p = \frac{\text{rot}\,\vec{B}}{4\pi} \times \vec{B} - \rho g \tag{1}$$

which solution in a force — free field is:

$$p_{tot} = p + \frac{B^2}{8\pi} \tag{2}$$

where $p = \dfrac{k \rho T}{\mu}$ is hydrostatic preasure.

Using the above relations we may find the difference in density outside and inside the magnetic tube:

$$\Delta \rho = \frac{\mu}{k T} \cdot \frac{B^2}{8\pi} \tag{3}$$

This magnetic *push* is the unward motion of magnetic tubes. Whether the amount of $\Delta\rho$ is sufficient enough for such motion it depends almost only on the strength of magnetic field.

Although gas pressure and temperature encrease with depth these two parameters are of minor influence, as we can see from extrapolating Allen's tables (Allen, 1963, p. 163)

depth (in km)	temperature (°K)	density (g/cm³)
3500	$2{,}7 \times 10^4$	2×10^{-5}
10000	$5{,}5 \times 10^4$	9×10^{-5}
20000	$9{,}7 \times 10^4$	2×10^{-4}
35000	$1{,}6 \times 10^5$	4×20^{-4}

According to relation (3):

$$2{,}5 \times 10^{-11} B^2 < \Delta\rho < 1{,}8 \times 10^{-10} B^2 \tag{4}$$

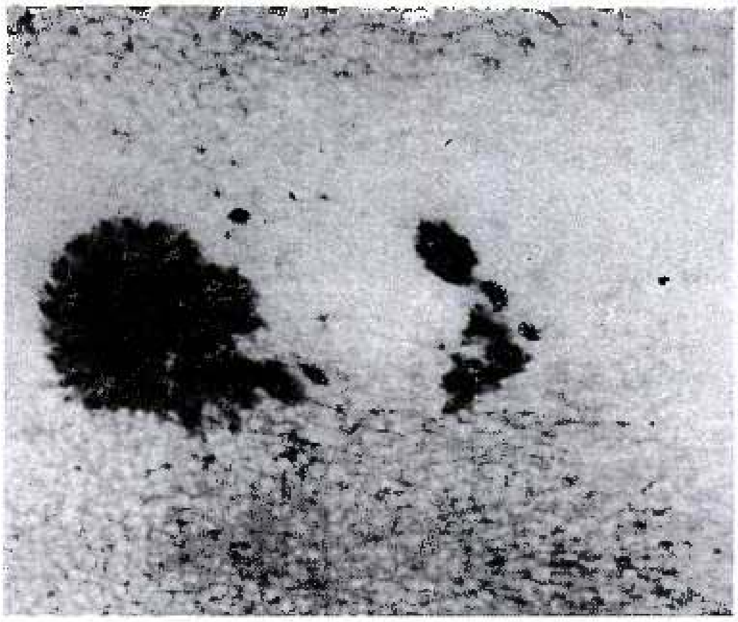

Fig. 3: August 4, 1963, $09^h\ 40^m$

It is a generally accepted opinion, up to now, that sunspot appear at the depth of about 300 km. Characteristics of this layer are (Allen 1963, p. 164): $\tau_{5000} = 1$; $T = 6360°K$; $p = 1.1 \times 10^5$ dyn/cm²; $\rho = 2.7 \times 10^{-7}$ g/cm³. It is obvious from such consideration that sunspot must be formed much deeper somewhere in convective zone, as the field strength of 100 G is not sufficient for upward movement of magnetic flux tube.

Accepting the idea that a sunspot is a deep phenomenon, convection as a mechanism of energy transport, may well explain deformation of magnetic

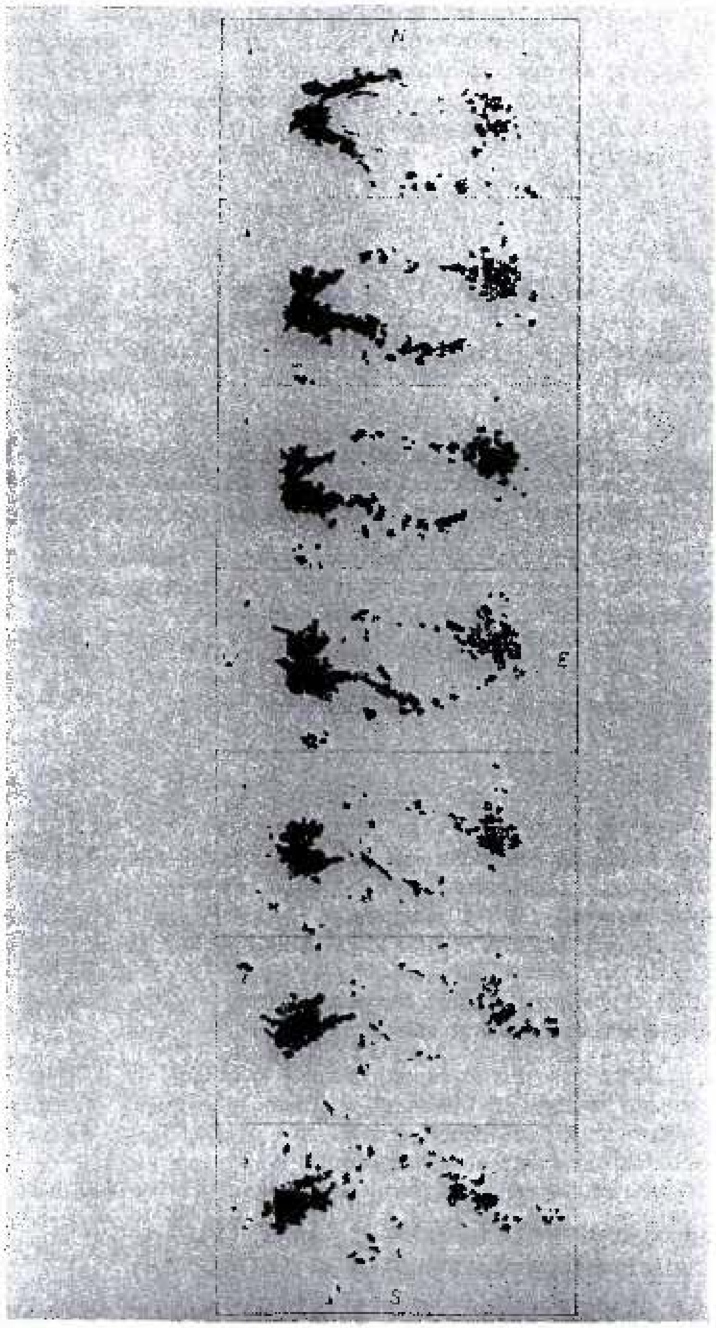

Fig. 4: Drawings of spot nuclei in sunspot groups of June 11 — 14, 1963

tubes, and formation of pore at the surface. According to Wilson's (1968) model un upwards convective movement create a cool cone of plasma with the strong magnetic field which inhibit the transport of energy. Outside the cone, the magnetic field is essentially „frozen — in“ to the material, but inside the cone the field may partly leak through the plasma. In such a way magnetic tubes in the cone become strong and deformed and in a form of an arch appears at the surface as a pore. At that spot the visible granules were cut off from their main source of thermal energy. Magnetic field inside and outside the cool cone is given by:

$$\frac{\partial \vec{B}}{\partial t} = \mathrm{rot}\ (\vec{v} \times \vec{B}) + \frac{C}{\varphi \pi \sigma} \nabla^2 B \tag{5}$$

Using Allen (1963) and Schröter (1966) date at unit optical depth outside the cone ($\sigma = 7.3 \times 10^{11}, v = 2 \times 10^5$ cm/s) the field is definitely „frosen – in“, but inside the cone ($\sigma = 5 \times 10^9, v = 2 \times 10^3$ cm/s) magnetic diffusion plays an important part in the transport of the field. This is in well aggremant with Wilson's model.

Althought this model gives a good explanation for energy deficit in umbra, as well as some other observed effects, the main problems remain: the magnetic field profile B (τ), and the mechanism for twisting the magnetic tubes.

Schwarcshild's and Rayleigh's conditions show only that in convective zone definitely occurs convective transport of energy. According to Ledoux et al (1961) Rayleigh number for Solar convective zone is:

$$R = 2 \times 10^{11} \tag{6}$$

which is far greater then nececery to start the convection.

DISCUSSION

The fine structure of Solar magnetic fields being strong in certain small regions is well known characteristic found by many observers even inside the sunspot umbra Severny (1959). Livingston and Harvey (1969) and Sheeley (1969) gave some quantitative results aboat this fine structure: in a region of 1500 km the field strenght is about 150 G. Weart (1970) shows, by a simple calculation, that the supergranular convection may produce twisted tubes of magnetic field observed size and strenght.

We would like to use the same convective model of supergranular cell (Fig 5) in the aim to explain observed features of sunspot groups: „quantization“ and elliptical forms. Starting with two assumptions: that a latitudinal subsurface field exists, and that supergranular convective velocity decreases with depth, an obvions consequence is that field lines will be twisted by convection.

Magnetic energy of a flux tube of uniform field strenght $B = B_x$, lenght L, and circular cross-section of radius R is:

$$\frac{1}{8\pi} \int B_x^2\, d\, V = \frac{1}{8\pi} B_x^2 R^2 \pi L \tag{7}$$

A gas in simple circular motion would twist the field lines into spirals. It will create an azimutal field component:

$$B\varphi = \frac{2\pi r}{L} \cdot B_z \tag{8}$$

that increase the energy for:

$$\frac{1}{8\pi}\int B^2 \varphi \, dV = \frac{1}{8\pi}\int_0^R\int_0^{2\pi}\int_0^L B^2 \varphi \, r \, dr \, d\varphi \, dx \tag{9}$$

To produce n pairs of such twists, as a convective cell must, it requires the energy:

$$E_n - \frac{B_z^2 \, \pi^2 \, n^2 \, R^4}{L} \tag{10}$$

As the motion in supergranular cell is rapid only near the surface flux tube will be flattened with elliptical cross-section as shown in Fig 5. The energy of twisting n pairs of such profile is:

$$E'_n \approx \frac{B_z^2 \, \pi^2 \, n^2 \, a^4}{L}\left(\frac{b}{a}\right) \tag{11}$$

where a is the semimajor axis and b is the semiminor axis.

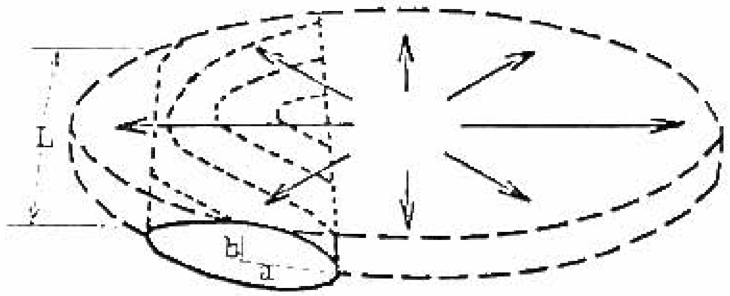

Fig. 5: Velocity field in the upper part of supergranular cell.

If its ratio is:

$$\frac{a}{b} \geqslant 5 \tag{12}$$

the result is correct to about 25%. A volume of gas V, density ρ, and plasma velocity v will have the kinetic energy:

$$E_k = \frac{1}{2} V \rho \, v^2. \tag{13}$$

If one part of this convective energy, say $K = 10\%$ will go into twisting the field, we shall have:

$$K \cdot \frac{1}{2} V \rho v^2 = \frac{B_x^2 \pi^2 n^2 a^4}{L} \left(\frac{b}{a}\right) \tag{14}$$

So, putting $V = d\,L\,b$, we are putting for the semimajor axis:

$$a = \sqrt[3]{\frac{k\,dL^2 \rho v^2}{2\pi^2 B_x^2 n^2}} \tag{15}$$

If we take $v = 0.5\,\mathrm{km\ s^{-1}}$, and accept the life time of supergranular flow as 0.5 day we obtain $d = 2.1 \times 10^4\,\mathrm{km}$. Using the values: $\rho = 3 \times 10^{-7}\,\mathrm{g\,cm^{-3}}$; $B_x = 100\,G$; $K = 10\%$ $n = 1$ and for $L = 1.6 \times 10^4\,\mathrm{km}$ (about a half of supergranular dimension) we have obtained:

$$a \approx 1300\,\mathrm{km}$$

When the tube emerges to the photosphere it will tend to take a circular cross-section. Comparing (10) and (11) we may replace a $\sqrt[4]{\frac{6}{a}}$ with R. If for b we accept one scale height, ie 200 km, $R = 800$ km ie for full diameter of magnetic tube:

$$2R = 1600\,\mathrm{km}$$

what is in good agreement with observed data of magnetic field fine structure It is important to emphasize that calculated values for $2R$ is not very sensitive on some changes in accepted amount for K or B. The convective energy which is enough to produce twisting of flux tube could push them towards the edges of supergranular cells.

The configuration of sunspot groups, and its dimensions (Fig. 1, 2, 3, 4,), together with above obtained results for supergranular convection, lead us to a suggestion of their possibly deep mutual connection. In the same time, we are well aware of the fact that supergranular convection is definitely more complicated than it is shown here. Nevertheless, we are under the impression that one complex, and may be „open", supergranular model would help us in a better understanding of sunspot groups.

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