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THE PERTURBED MOTION OF ASTEROID IN PROXIMITY

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Two asteroids are in proximity when they find themselves on their orbits in the positions of their least distance. Here we shall consider the results of our examination of perturbed motion of one asteroid under the influence of other at an interval that includes their proximity. It was interesting because we had found one small distance of asteroids in proximity, so we wanted to see what the estimates of perturbations of orbit elements of a smaller asteroid could be under the influence of a bigger one.

For this we used the diferential equations of osculating elements, (**1**):

$$\left.\begin{aligned} w\frac{d\Omega}{dt} &= (\Omega : W)\, W, & w\frac{d\pi}{dt} &= (\pi : S)\, S + (\pi : T)\, T + (\pi : W)\, W, \\ w\frac{di}{dt} &= (i : W)\, W, & w^2\frac{d\mu}{dt} &= (\mu : S)\, S + (\mu : T)\, T, \\ w\frac{d\varphi}{dt} &= (\varphi : S)\, S + (\varphi : T)\, T, & w\frac{dM_0}{dt} &= (M_0 : S)\, S + (M_0 : T)\, T, \end{aligned}\right\} \tag{1}$$

where are

$$M = M_0 + \mu_0 (t - t_0) + \delta M, \quad \delta M = \int_{t_0}^{t} \frac{dM_0}{dt}\, dt + \int\!\!\int_{t_0}^{t} \frac{d\mu}{dt}\, dt^2. \tag{2}$$

The coordinates of perturbing acceleration are given in terms:

$$S = N (K\, \xi_1 - r\, \rho^{-3}), \quad T = N K \eta_1, \quad W = N K \zeta_1, \tag{3}$$

where the substitutions are taken:

$$K = \rho^{-3} - r_1^{-3}, \quad N = k'' w\, m_1 \sqrt{p}\,, \tag{4}$$

and where are: k — the Gaussian gravitational constant, w — a time interval, m_1 — a mass of perturbing asteroid. The distance ρ between these two asteroids is determined by the equation

$$\rho^2 = (\xi_1 - r)^2 + \eta_1^2 + \zeta_1^2, \tag{5}$$

where are the heliocentric vektors of positions of perturbed and of perturbing asteroid in a moving coordinated system:

$$\mathbf{r} = (r,\ 0,\ 0), \quad \mathbf{r}_1 = (\xi_1,\ \eta_1,\ \zeta_1). \tag{6}$$

Using the standard symbols we have terms:

$$\left.\begin{aligned}
&(\Omega : W) = \frac{r}{p} \sin u \ \operatorname{cosec} i, && (\pi : S) = -\cos v \ \operatorname{cosec} \varphi,\\
&(i : W) = \frac{r}{p} \cos u, && (\pi : T) = (1 + \frac{r}{p}) \sin v \ \operatorname{cosec} \varphi,\\
&(\varphi : S) = \sin v \ \sec \varphi, && (\pi : W) = \frac{r}{p} \sin u \ \operatorname{tg} \frac{i}{2},\\
&(\varphi : T) = (\cos v + \cos E) \sec \varphi, && (M_0 : S) = (\cos v \ \operatorname{cosec}\varphi - 2\frac{r}{p}) \cos \varphi,\\
&(\mu : S) = -\frac{3\,k\,w}{p\sqrt{a}} \sin \varphi \ \sin v, && (M_0 : T) = -(1 + \frac{r}{p}) \operatorname{ctg} \varphi \ \sin v.\\
&(\mu : T) = -\frac{3\,k\,w}{r\sqrt{a}}, &&
\end{aligned}\right\} \tag{7}$$

The equations (1) we solved by numerical integration using Encke's integration scheme by:

$$\left.\begin{aligned}
\frac{1}{w}\int_a^{a+iw} f(t)\,dt &= {}^{I}f(a + i\,w) - \frac{1}{12} f^{I}(a + iw) + \frac{11}{720} f^{III}(a + iw) -\\
&- \frac{191}{60480} f^{V}(a + iw) + \frac{2497}{3628800} f^{VII}(a + iw) - \ldots,\\
{}^{I}f\left(a - \frac{w}{2}\right) &= -\frac{1}{2} f(a) + \frac{1}{12} f^{I}(a) - \frac{11}{720} f^{III}(a) +\\
&+ \frac{191}{60480} f^{V}(a) - \frac{2497}{3628800} f^{VII}(a) + \ldots
\end{aligned}\right\} \tag{8}$$

and

$$\left.\begin{aligned}
\frac{1}{w^2}\int_a^{a+iw}\!\!\int f(t)\,dt^2 &= {}^{II}f(a + iw) + \frac{1}{12} f(a + iw) - \frac{1}{240} f^{II}(a + iw) +\\
&+ \frac{31}{60480} f^{IV}(a + iw) - \ldots,\\
{}^{II}f(a) &= -\frac{1}{12} f(a) + \frac{1}{240} f^{II}(a) - \frac{31}{60480} f^{IV}(a) + \ldots
\end{aligned}\right\} \tag{9}$$

The calculations we applied to the pair of quasicomplanar orbits of asteroids 589 Croatia and 1564 Srbija. For this pair we found that their shortest distance is $\rho = 0.000498$ AU, (2). Using this result we took that the moment of proximity of these two asteroids is the mean moment of time interval on which the previous formulas will be applied. This moment we marked as 0 and from it we took the intervals of 26 days earlier (—) and later (+). The very integrals, i.e. the perturbations of orbital elements for 1564 under the influence of 589 (which mass is m_1), we calculated for the intervals: from —20 to +20 days, from —20 to 0 and from 0 to +20 days. For time interval in the scheme for integration we took $w = 2$ mean days.

We cosider though the 0 moment is not exactly known to us and the osculating elements of orbits of these asteroids constantly change their values — this doesn't essentially lessen for us the importance of estimate of perturbing influence of one asteroid to other in their proximity.

From the rates of major semi-axis of the observed asteroids, as well as from the calculated rates of their heliocentric distances, we can see that the observed asteroids in general move through the middle of the ring of asteroids. So, they are far enough both from Jupiter and Mars; they are further from Jupiter than from Mars. That enabled us in examination of perturbations in motion of the asteroid 1564 (in the quoted interval about found proximity) to restrict ourselves only to disturbing influence of the asteroid 589 as the bigger one. Thus, we neglected the perturbations apart from before mentioned great planets in the interval round the proximity.

The derivatives of the orbital elements of the disturbed asteroid (Srbija) we calculated starting with its given elements as the constants. And then by integrating we got slight perturbations so we concluded that we should be able to please ourselves by special perturbations of the first order.

Here are the tables surveys of the main worths that we calculated. Because of the simplification of the calculation for the mass of the asteroid Croatia, that produces the perturbations, first we took the value of the mass unit.

The Table I gives, for the observed interval, the values (in the units 10^{-6}): of heliocentric distances of Srbija (r) and Croatia (r_1) and of their reciprocal distances (ρ). We see that r and r_1 round the proximity rise; that r is less before the proximity and after that bigger than r_1.

TABLE I (*In the units* 10^{-6})

t	r	r_1	ρ	t	r	r_1	ρ
	3100000 +	3180000 +			3100000 +	3180000 +	
—20^d	48997	2814	33862	0^d	89734	9739	498
—18	53097	3516	30469	+ 2	93773	10419	3425
—16	57191	4216	27078	+ 4	97805	11098	6789
—14	61280	4914	23690	+ 6	101813	11774	10166
—12	65363	5609	20304	+ 8	105850	12448	13547
—10	69440	6302	16918	+10	109862	13120	16930
— 8	73511	6994	13535	+12	113866	13789	20314
— 6	77576	7683	10154	+14	117863	14456	23701
— 4	81635	8370	6777	+16	121853	15121	27090
— 2	85688	9056	3413	+18	125835	15783	30480
				+20	129809	16443	33873

TABLE II

t	$w\frac{d\Omega}{dt}$	$\int f$	$w\frac{di}{dt}$	$\int f$	$w\frac{d\pi}{dt}$	$\int f$
	″	″	″	″	″	″
		0		0		−16
−26[d]	0		0		+4	
		0		0		−12
−24	0		0		+4	
		0		0		−8
−22	0		0		+5	
		0		0		−3
−20	0		0		+6	
		0		0		+3
−18	0		0		+6	
		0		0		+9
−16	0		0		+8	
		0		0		+17
−14	0		0		+9	
		0		0		+26
−12	0		0		+12	
		0		0		+38
−10	+1		0		+16	
		+1		0		+54
−8	+2		−1		+23	
		+3		−1		+77
−6	+7		−3		+37	
		+10		−4		+114
−4	+32		−13		+75	
		+42		−17		+189
−2	+299		−126		+273	
		+341		−143		+462
0	+112602		−48111		+1940	
		+112943		−48254		+2402
+2	+394		−171		−204	
		+113337		−48425		+2198
+4	+57		−25		−46	
		+113394		−48450		+2152
+6	+19		−8		−17	
		+113413		−48458		+2135
+8	+9		−4		−8	
		+113422		−48462		+2127
+10	+5		−2		−4	
		+113427		−48464		+2123
+12	+3		−1		−2	
		+113430		−48465		+2121
+14	+2		−1		−1	
		+113432		−48466		+2120
+16	+1		−1		0	
		+113433		−48467		+2120
+18	+1		−1		0	
		+113434		−48468		+2120
+20	+1		0		0	
		+113435		−48468		+2120
+22	+1		0		+1	
		+113436		−48468		+2121
+24	0		0		+1	
		+113436		−48468		+2122
+26	0		0		+1	
		+113436		−48468		+2123

(*For* $m_1 = 1$; *in the units* 10^6)

$w\frac{d\varphi}{dt}$	^If	$w\frac{dM_0}{dt}$	^If	$w^2\frac{d\mu}{dt}$ (In the units 10^3)	^If	^{II}f
″	″	″	″	″	″	″
	− 29		+ 72		+ 90	
+ 6		− 17		− 22		− 125
	− 23		+ 55		+ 68	
+ 8		− 19		− 24		− 57
	− 15		+ 36		+ 44	
+ 9		− 23		− 28		− 13
	− 6		+ 13		+ 16	
+ 11		− 27		− 33		+ 3
	+ 5		− 14		− 17	
+ 13		− 33		− 39		− 14
	+ 18		− 47		− 56	
+ 17		− 42		− 47		− 70
	+ 35		− 89		− 103	
+ 22		− 54		− 59		− 173
	+ 57		− 143		− 162	
+ 30		− 73		− 77		− 335
	+ 87		− 216		− 239	
+ 44		− 104		− 106		− 574
	+ 131		− 320		− 345	
+ 68		− 160		− 159		− 919
	+ 199		− 480		− 504	
+ 121		− 282		− 268		− 1423
	+ 320		− 762		− 772	
+ 272		− 623		− 567		− 2195
	+ 592		−1385		−1339	
+1065		−2419		−2111		− 3534
	+1657		−3804		−3450	
+ 512		− 900		− 484		− 6984
	+2169		−4704		−3934	
−1053		+2339		+1862		−10918
	+1116		−2365		−2072	
− 269		+ 594		+ 449		−12990
	+ 847		−1771		−1623	
− 121		+ 262		+ 187		−14613
	+ 726		−1509		−1436	
− 68		+ 146		+ 98		−16049
	+ 658		−1363		−1338	
− 44		+ 92		+ 58		−17387
	+ 614		−1271		−1280	
− 31		+ 64		+ 37		−18667
	+ 583		−1207		−1243	
− 22		+ 46		+ 25		−19910
	+ 561		−1161		−1218	
− 17		+ 35		+ 17		−21128
	+ 544		−1126		−1201	
− 14		+ 27		+ 12		−22329
	+ 530		−1099		−1189	
− 11		+ 22		+ 9		−23518
	+ 519		−1077		−1180	
− 9		+ 18		+ 6		−24698
	+ 510		−1059		−1174	
− 8		+ 15		+ 4		−25872
	+ 502		−1044		−1170	
− 7		+ 12		+ 3		−27042
	+ 495		−1032		−1167	

The Table II gives the values of the derivatives of the orbital elements of the asteroid Srbija multiplied by time interval ($w = 2$ days) for the unit mass of the asteroid Croatia ($m_1 = 1$); we also quoted the values of the first sums, and at the mean motion (μ) also of the second sums.

In the Table III up to now we have given fictive values of the perturbations of the orbital elements of the asteroid 1564 (in order of their magnitudes) under the influence of the asteroid 589 with the unit mass.

In order to estimate the probable rates of the perturbations round the proximity we ought to multiply the values from the Table III by the mass of the asteroid 589.

Though we do not know exact values of asteroid masses we know that they are insignificant. However, we can estimate the probable values of their masses and the probable values of the corresponding perturbations in the neighbourhood of their proximity. We suppose that asteroids are spherical though they may be not spherical in general, (**3**). We suppose that their structure is similar to those of meteorites. The density of asteroids is usualy supposed to be between 3.0 and 3.5 g cm^{-3}, (**4**, **5**). We shall take for the average value of the density of asteroids $\sigma = 3.3$ g cm^{-3}. The probable values of the asteroid radius R shall follow from the empirical formula

$$lg\, R = 3.3135 - 0.2\, g, \tag{10}$$

where g stands for the stellar magnitude of an asteroid if it is at the distance of one astronomical unit from the Sun and the Earth. For the cosidered pair these values are: $g_1 = 9.98$ for the asteroid 589, $g = 12.05$ for the asteroid 1564, (**6**). By means of (10) we have the corresponding radius: $R_1 = 20.8$ km and $R = 8.0$ km.

TABLE III (*For* $m_1 = 1$; *in the units* 10^{10})

Interval / *Perturbation*	-20^d *to* $+20^d$	-20^d *to* 0	0 *to* $+20^d$
$\delta\Omega$	$+11''.3434$	$+5''.6636$	$+5''.6798$
$\delta\omega$	-11.1314	-5.5175	-5.6139
δi	-4.8468	-2.4200	-2.4268
$\delta\pi$	$+0.2120$	$+0.1461$	$+0.0659$
δM	-0.1111	-0.4547	$+0.3436$
$\delta\varphi$	$+0.0524$	$+0.2042$	-0.1518
$\delta\mu$	-0.0000592	-0.0001716	$+0.0001124$

The probable value of the mass m_1 of the bigger asteroid (589), given by units of the solar mass, we can estimate by

$$m_1 = \left(\frac{R_1}{R_s}\right)^3 \frac{\sigma}{\sigma_s}, \tag{11}$$

where R_s and σ_s are radius and the average density of the Sun. So we have

$$m_1 = 6 \times 10^{-14}. \tag{12}$$

From such an insignificant mass we could not expect noticeable perturbations. An exeption could happen if two asteroids were sufficiently close to each other. The proximity we found is very small, but not sufficient enough to give measurable perturbations. That is seen from Table IV, which we got by Table III and the rate (12). The worths of the perturbations of the orbital elements of the asteroid 1564 are given with one more decimal place than it could be usualy found in practice.

TABLE IV

Interval / *Perturbation*	-20^d *to* $+20^d$	-20^d *to* 0	0 *to* $+20^d$
$\delta\Omega$	$+0''.007$	$+0''.003$	$+0''.003$
δi	-0.003	-0.001	-0.001
$\delta\omega$	-0.007	-0.003	-0.003
$\delta\pi$	0.000	0.000	0.000
$\delta\varphi$	0.000	0.000	0.000
δM	0.000	0.000	0.000
$\delta\mu$	0.00000	0.00000	0.00000

The conclusion: the perturbations of the considered asteroids in their proximity may be neglected.

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