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ON THE DETERMINATION OF ANOMALOUS REFRACTION OUT OF ASTROMETRICAL MEASUREMENTS IN THE ZENITH ZONE

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SUMMARY

The method is given for the analyse of the anomalous refraction in the zenith zone. Connected with this the observations of the same zenith stars in the meridian and the prime vertical are used. The moment of passage through the eastern and western part of the prime vertical is registered, while the difference of zenith distances in the meridian passage is measured — always in two instrument position.

This method practically eliminates coordinate errors influences of observed stars. For the succesful employment of this method, we must use the high quality instruments and, especially, correct apparatus for the carreful determination of the inclinations of instrumental parts.

1. In (**1**), it has been analysed the possibility of the determination of the real anomalous refraction influence immediately out of the astrometrical measurements. On the basis of the results of the astrometrical measurement and meteorogical investigations, it is concluded that this determination is practically impossible.

To our mind, the main causes of this fact are: variable nature of the meteorological elements, incorrect formation of air layers around the observational instruments, variable size of the instrumental influences (systematical and accidental) and coordinate errors of the used stars.

According to the above explanation, we would point out that the astrometrical methods for the determination of the size of anomalous refraction should be capable to fulfil the following conditions:

1) the shortest possible time for the measurements of the needed observational values,

2) the use of the high quality instruments, which characteristics and constants are practically unchangeable,

3) the elimination of the star coordinate errors, and

4) the analysis of a narrow zone, only, at the zenith distance.

So, the astrometrical methods cannot give us the interpolational or extrapolational results neither for the time nor the space.

All these conditions cannot achieve at the most, and the suggestion, in (**1**), for a strictly meteorological method, is quite justified, which does not use results of the star observations. The astrometrical methods, however, can also give useful data on anomalous refraction if they are correctly organized.

We came to a conclusion that the observations of the zenith stars in the meridian and the prime vertical might give us very important data about the anomalous refraction in the zenith zone. We would like to present the bases of this method.

This, method, however, may be used for the analysis of instrumental constants too.

2. At the recommended method, the zenith stars are observed, whose upper culmination is south from the zenith (at the maximum zenith distance of 10′). Every star is observed three times: at the both parts of the prime vertical and in the meridian — always in two instrument position. The moment of passage through the prime vertical is registered (as in Struve method), while the difference of zenith distance in the meridian passage is measured (as in Talcott method). We obtain two latitude values in this way. Their difference can be used for the analysis of the instrumental constants, and after that for the analysis of the possible anomalous refraction.

3. When we determine latitudes from the prime vertical observations, we start from the basical formules of the Struve method (**2**):

$$\varphi = \varphi' + \frac{1}{2}(i_E + i_W) \tag{1a}$$

$$\operatorname{tg}\varphi' = \operatorname{tg}\delta \sec s \sec d \cos m \tag{1b}$$

where are:

$$s = \frac{1}{4}[(u_4 - u_1) + (u_3 - u_2)]$$

$$d = \frac{1}{4}[(u_4 - u_1) - (u_3 - u_2)]$$

$$m = \frac{1}{4}(u_1 + u_2 + u_3 + u_4) - \alpha$$

u_1, u_2 — moments of passage through the eastern part of the prime vertical in two instrument position,

u_3, u_4 — moments of passage through the western part of the prime vertical in two instrument position,

i_E,i_W — inclinations of horizontal axis at the eastern and western passage,

δ — the apparent declination of used star, and

α — the apparent right ascension of used star.

The formulas (1) are not suitable for our calculations and therefore we will transform them. Consequently, we use the known formula for the expanding in series this type expression (see (3)):

$$\operatorname{tg} x = \mathrm{p} \operatorname{tg} y$$

where are

$$p > 0 \qquad \text{and} \qquad |y - x| < \frac{\pi}{2}$$

This formula is:

$$x = y - q \sin 2y + \frac{q^2}{2} \sin 4y - \frac{q^3}{3} \sin 6y + \ldots \tag{2}$$

where is

$$q = \frac{1 - p}{1 + p}$$

If the formula (1b) is given in the form

$$\operatorname{tg} \varphi' = p \operatorname{tg} \delta$$

where we put

$$p = \sec s \sec d \cos m$$

which fulfil the condition $p > 0$, then from two expressions (1) we obtain one formula:

$$\varphi = \delta - q \sin 2\delta + \frac{q^2}{2} \sin 4\delta - \frac{q^3}{3} \sin 6\delta + \ldots + \frac{1}{2}(i_E + i_W)$$

or

$$\varphi = \delta + \frac{1}{2}(i_E + i_W) + (- q \sin 2\delta + \frac{q^2}{2} \sin 4\delta - \ldots) \operatorname{arc} 1'' \tag{3}$$

If we have registrations of passage through the n threads, then the mean latitude is obtained from

$$\varphi = \delta + \frac{1}{2}(i_E + i_W) + \frac{1}{n} \sum_{k=1}^{n} \left(- q_k \sin 2\delta + \frac{p_k^2}{2} \sin 4\delta - \ldots \right) \operatorname{arc} 1'' \tag{4}$$

or, shortly,

$$\varphi - \delta = \sigma \tag{5}$$

where we mark the sum of the known, measured sizes (at the right side of the formula (4)) with σ. At the following analysis, the mark of this latitude will be φ_v.

From (1) we can see, that for zenith stars p is not far from one, and therefore q is a small value. For this reason the above-mentioned series converges quickly. It is enough to use terms with q and q^2 only.

We note, that in (2), p. 500, there is a formula which is similar to our expression (4). It is, however, more complicated than our, its use is harder, and moreover it demands the knowlegde of the zenith distances too.

4. We will analyse the influence of the systematical and accidental errors on the latitude from the prime vertical observations.

We mark with $\Delta(\varphi_v - \delta)$, Δi etc. systematical errors of the corresponding values $(\varphi_v - \delta)$, i etc.

After the differentiation of the relation (3), for the series of observations on the k^{th} thread, we obtain

$$\Delta(\varphi_v - \delta)_k = \frac{1}{2}(\Delta i_E + \Delta i_W) + (-2\, q_k \cos 2\delta + 2\, q_k^2 \cos 4\delta - \ldots)\, \Delta\delta + \\ + (-\sin 2\delta + q_k \sin 4\delta - \ldots)\, \Delta q_k \qquad (6)$$

or, shortly,

$$\Delta(\varphi_v - \delta)_k = \frac{1}{2}(\Delta i_E + \Delta i_W) + a_k\, \Delta\delta + b_k\, \Delta q_k \qquad (7)$$

where a_k and b_k mark coefficients in front of $\Delta\delta$ and Δq_k at (6). This relation (7) gives the size of the systematical errors influence on the value $(\varphi_v - \delta)$.

The influence of the accidental errors we obtain from

$$\varepsilon^2_{(\varphi_v - \delta)_k} = \frac{1}{4}\left(\varepsilon^2_{iE} + \varepsilon^2_{iW}\right) + a_k^2\, \varepsilon_\delta^2 + b_k^2\, \varepsilon_q^2 \qquad (8)$$

where we mark with $\varepsilon(\varphi_v - \delta)$, ε_{iE}, ε_{iW} etc. the mean square errors of the corresponding elements.

We will analyse the expression (7) and (8) in detail.

Coefficients a_k and b_k are functions of δ and q. Value a_k is always sensibly smaller than one. Value b_k primarly depends on the $(-\sin 2\delta)$, one might, say as well, that it is practically smaller than one. The weight of inclination error is always the same.

After the differentiation of the expression

$$q = \frac{1-p}{1+p} = \frac{1 - \sec s \sec d \cos m}{1 + \sec s \sec d \cos m}$$

we can express the formulas (7) and (8) at the function of the registered moment errors:

$$\Delta(\varphi_v - \delta)_k = \frac{1}{2}(\Delta i_E + \Delta i_W) + a_k\, \Delta\delta + b_k c_{1k}\, \Delta u_{1k} + b_k c_{2k}\, \Delta u_{2k} + \\ + b_k c_{3k}\, \Delta u_{3k} + b_k c_{4k}\, \Delta u_{4k} + b_k c_{5k} \Delta\alpha \qquad (9)$$

$$\varepsilon^2_{(\varphi_v-\delta)k} = \frac{1}{4}\left(\varepsilon^2_{iE} + \varepsilon^2_{iW}\right) + a^2_k\,\varepsilon^2_\delta + b^2_k c^2_{1k}\,\varepsilon^2_{u1k} + b^2_k c^2_{2k}\,\varepsilon^2_{u2k} + b^2_k c^2_{3k}\,\varepsilon^2_{u3k} + $$

$$+ b^2_k c^2_{4k}\,\varepsilon^2_{u4k} + b^2_k c^2_{5k}\,\varepsilon^2_\alpha \qquad (10)$$

where the markations are:

$$c_{1k} = \frac{p_k}{2\,(1+p_k)^2}\,(\mathrm{tg}\, s_k + \mathrm{tg}\, d_k + \mathrm{tg}\, m_k)$$

$$c_{2k} = \frac{p_k}{2\,(1+p_k)^2}\,(\mathrm{tg}\, s_k - \mathrm{tg}\, d_k + \mathrm{tg}\, m_k)$$

$$c_{3k} = \frac{p_k}{2\,(1+p_k)^2}\,(-\,\mathrm{tg}\, s_k + \mathrm{tg}\, d_k + \mathrm{tg}\, m_k)$$

$$c_{4k} = \frac{p_k}{2\,(1+p_k)^2}\,(-\,\mathrm{tg}\, s_k - \mathrm{tg}\, d_k + \mathrm{tg}\, m_k)$$

$$c_{5k} = \frac{-\,2\,p_k}{(1+p_k)^2}\,\mathrm{tg}\, m_k$$

If we suppose that there are

$$\Delta\, u_{1k} = \Delta\, u_{2k} = \Delta\, u_{3k} = \Delta\, u_{4k} = \Delta\, u_k$$

we obtain

$$\Delta\,(\varphi_v - \delta)_k = \frac{1}{2}\,(\Delta\, i_E + \Delta\, i_W) + a_k \Delta\, \delta + b_k\,(\Delta\, \alpha - \Delta\, u_k) \qquad (11)$$

If we suppose that the registration accidental errors (ε_{uik}) are the same, then we have

$$\varepsilon^2_{(\varphi_v-\delta)k} = \frac{1}{4}\left(\varepsilon^2_{iE} + \varepsilon^2_{iW}\right) + a^2_k\,\varepsilon^2_\delta + \frac{b^2_k p^2_k}{(1+p_k)^4}\,(\mathrm{tg}^2\, s_k + \mathrm{tg}^2\, d_k +$$

$$+ \mathrm{tg}^2\, m_k)\,\varepsilon^2_{uk} + b^2_k c^2_{5k}\,\varepsilon^2_\alpha \qquad (12)$$

The coefficient in front of ε^2_{uk} we will mark with $b^2_k\,c^2_{6k}$.

As the coefficients c_k are small values, from (9) and (10) resp. (11) and (12) we conclude that the accuracy of the values α and u_k must not be very high, because its influence on $(\varphi_v - \delta)$ is relatively small. As we say just now, it is valid for δ too.

The inclination errors have relatively largest weight. It was found that these errors — systematical and accidental alike — are fairly variable, and therefore we must try to have a high quality level on the horizontal axis or classical levels to be replaced with more reliable electronical levels.

If we observe on more (n) threads, then in expressions (6) — (12) we replace the corresponding values with averages from every position — naturally, in that case if this simplification is admissable.

5. For the first numerical example, we use the data in (2), p. 504. The observations were done on five threads. In Table 1 there are latitudes from (2) and corresponding values on the basis of our formula (3). We use terms with q and q^2, only.

There are differences between individual values because the mean value of m for every calculation were used in (2), while we used for every thread measured value of m. The mean values of φ are in good agreement.

In the same table there are the coefficients for the error equations.

As the second example, the results of observations in (4), p. 265 were used. There are measurements on three threads (the last is incomplete). The results of our calculations are in the Table 2. The mean latitude is obtained so that the latitude from the third thread has weight of 0,5.

From these tables we can conclude:

a) Star coordinates (α and δ), which we use for the calculation of q_k, do not need to have a high accuracy. It is especially valid for α. In case if the declination systematical error is for instance 1'', it in the first example causes the error of 0,''001 and 0,''01 in the second example. If the right ascension systematical error is 1^s, in this case in the first example the error will be 0,''0002 and 0,''002 in the second example.

b) If the systematical errors at every registrations are the same, then — as we see from (11) — their influence is similar to the influence of $\Delta\alpha$, namely: it is small. Consequently, the inclination error may be the most important.

c) From (10) resp. (12) and our tables it has resulted that accidental error of $(\varphi_v - \delta)$ depends on inclination error and errors of registered passage moments, and it is practically independent of star coordinate accidental errors. — If we suppose that the bubble position is determined with $\pm 0,''20$ (it is for the onesecond level), we get $\pm 0,''15$ for ε_i. It is assumed that $\varepsilon_u = \pm 0,^s1 = \pm 1,''5$. If the coordinate error influence is neglected, then, from (12), for the first example we obtain

$$\varepsilon(\varphi_v - \delta)_1 \approx \pm 0,''12$$

and for the second one

$$\varepsilon(\varphi_v - \delta)_2 \approx \pm 0,''14$$

From these we can see that the inclination error acts an important role, while the weight of registration error is really smaller.

Accordingly, the numerical examples verifies our constatation (in paragraph 4.), that the fundamental problem of this method is the determination of accurate and real inclination.

6. The latitude, from the measurements in the meridian, we get from the formula

$$\varphi = \delta + z + \rho \tag{13}$$

where are: z — the measured zenith distance, and ρ — the refraction influence.

TABLE 1

Threads	VII	VIII	IX	X	XI	Mean values
Latitudes from (2)	50°12′34″,03	50°12′34″,18	50°12′34″,32	50′12′34″,02	50°12′34″,02	50°12′34″,12
Latitude from our formula (3)	33,88	34,17	34,29	34,03	34,07	34,09
a_k	—0,0013381	—0,0013386	—0,0013388	—0,0013383	—0,0013383	—0,0013384
b_k	— ,9835354	— ,9835349	— ,9835347	— ,9835348	— ,9835351	— ,9835351
b_kc_{1k}	— ,0165162	— ,0176706	— ,0188141	— ,0197830	— ,0207435	— ,0187056
b_kc_{2k}	— ,0136977	— ,0121714	— ,0090766	— ,0082562	— ,0053853	— ,0099637
b_kc_{3k}	+ ,0136938	+ ,0121652	+ ,0090712	+ ,0082544	+ ,0053752	+ ,0099582
b_kc_{4k}	+ ,0165123	+ ,0176643	+ ,0188088	+ ,0197814	+ ,0207334	+ ,0187010
b_kc_{5k}	+ ,0000079	+ ,0000125	+ ,0000107	+ ,0000035	+ ,0000204	+ ,0000110
$b^2_kc_{6k}$	+0,0009327	+0,0009204	+0,0009200	+0,0009190	+0,0009180	+0,0008980

TABLE 2

Threads	I	II	III	Mean values
Latitude from (3)	59°59′31″,26	59°59′29″,98	59°59′30″,25	59°59′30″,55
Latitude from our formula (3)	31,26	30,01	30,27	30,56
a_k	—0,0112212	—0,0112143	—0,0112158	—0,0112171
b_k	— ,8662839	— ,8662899	— ,8662886	— ,8662874
b_kc_{1k}	— ,0262556	— ,0250311	— ,0235021	— ,0249296
b_kc_{2k}	— ,0203756	— ,0218578	— ,0235021	— ,0219119
b_kc_{3k}	+ ,0203142	+ ,0217964	+ ,0234407	+ ,0218504
b_kc_{4k}	+ ,0261942	+ ,0249697	+ ,0234407	+ ,0248682
b_kc_{5k}	+ ,0001228	+ ,0001228	+ ,0001228	+ ,0001228
$b^2_kc_{6k}$	+0,0022033	+0,0022029	+0,0022036	+0,0021975

The zenith distance is obtained from measurements at two instrument position:

$$z = \frac{1}{2}(r_E - r_W)R + \frac{1}{2}(l_E - l_W)\,L + k \tag{14}$$

where are:

r_E, r_W — micrometric readings (which are corrected because of errors of a micrometric screw), at E resp. W instrument position,

R — the mean value of a revolution,

l_E, l_W — bubble positions at E resp. W instrument positions,

L — the level constant, and

k — the reduction to the meridian.

From (13) and (14) we can obtain

$$\varphi - \delta = \Delta \tag{15}$$

where Δ is the sum of the known, measured values. At the following analysis the mark of this latitude will be φ_m.

7. On the basis of the same principle as in the paragraph 4., we will analyse the influence of errors on the value $(\varphi_m - \delta)$.

The systematical errors have the following influence model:

$$\Delta(\varphi_m - \delta) = \frac{1}{2}(\Delta r_E - \Delta r_W)\,R + \frac{1}{2}(r_E - r_W)\,\Delta R + \frac{1}{2}(\Delta l_E - \Delta l_W)\,L +$$

$$+ \frac{1}{2}(l_E - l_W)\,\Delta L + \Delta \rho \tag{16}$$

and the accidental errors:

$$\varepsilon^2_{(\varphi_v - \delta)} = \frac{1}{4}\left(\varepsilon^2_{rE} + \varepsilon^2_{rW}\right) R^2 + \frac{1}{4}(r_E - r_W)^2\ \varepsilon^2_R + \frac{1}{4}\left(\varepsilon^2_{lE} + \varepsilon^2_{lW}\right)L^2 +$$

$$+ \frac{1}{4}(l_E - l_W)^2\ \varepsilon^2_L + \varepsilon^2_\rho \tag{17}$$

It is apparent from (16), that the value $(\varphi_m - \delta)$ may be charged with influences of numerous systematical errors. In consequence of relatively great values of R and L, it should be expected that the largest influence would be from errors Δr_E, Δr_W and Δl_E, Δl_W.

Experience shows that the accidental errors of star position determination at the micrometric field (ε_r) and at the position of level bubble (ε_l) are dominant factors at the accuracy of determination of φ_m. It is possible to estimate that at zenith-telescopes the excepted accidental error of $(\varphi_m - \delta)$ is less than $\pm$ 0,''20 (for one observation), while at good isolated universal instruments (with special care of temperature protection of levels) is below of $\pm$0,''30.

It is obvious that the main problem of this method is also the determination of correct inclination.

8. With observations of the same star in the prime vertical and the meridian, we can obtain two values:

$$\varphi_v - \delta = \sigma$$

$$\varphi_m - \delta = \Delta$$

At the ideal case, the difference of these values

$$\Delta \varphi = (\varphi_m - \delta) - (\varphi_v - \delta) = \Delta - \sigma \tag{18}$$

is needed zero, but, however, it practically never happens. Both methods give, in a sense of accidental errors, nearly equal accuracies — concerning that there are constatations at (**5**) and (**6**), and our investigations show also the same — but the influences of different systematical errors may be important. The previous analysis has presented that at meridian observations (by Talcott method) the possibility of influence of instrumental and measuring errors is larger.

The expression (18) can be used for the analyse of instrumental constants (striding level constant, Talcott level constants and mean value of micrometric revolution) — if we suppose that there are not other systematical influences. In this case we can establish equations of this sort

$$\Delta \varphi = a_1 \cdot \Delta L_0 + a_2 \cdot \Delta L_{t1} + a_3 \cdot \Delta L_{t2} + a_4 \cdot \Delta R \tag{19}$$

where a_1, a_2, a_3 and a_4 are known values. The index o denotes the striding level, $t1$ the first Talcott level and $t2$ the second Talcott level.

9. From this explanation it is obvious that for the analyse of anomalous refraction we must have experienced observers, well investigated instrument, which is effectly isolated from direct surroundings, and normal observation condition (don't forget: the accuracy of star observations fully depends on the image quality at a field of view). Understandable that these rigorous conditions cannot be answered at the most, but we hope that with good thermical isolation of the instrument and after changing the classical levels with electronical levels, we can approach this condition.

Let us suppose that the values ($\varphi - \delta$) are relieved of systematical influences of instrumental constants, and only noncalculated refractional influences remained. Then, from the previous explanation,

$$\Delta \varphi = \varphi_m - \varphi_v = \Delta - \sigma = - \Delta \rho \tag{20}$$

is resulted, where $\Delta \rho$ is the value from (16), as the systematical error of ($\varphi_m - \delta$).

The expression is valid if there are not anomalous refraction. In that case, φ_v don't depend on refractional influences, while φ_m includes the error $\Delta \rho$ because of the incorrect calculation of normal refraction.

But if there are the anomalous refraction — which is a more real situation — then instead of (20) it is more correct to write

$$\Delta \varphi = - \Delta \rho - \Delta \rho_v + \Delta \rho_m \qquad (21)$$

where are: $\Delta\rho_v$ and $\Delta\rho_m$ the corresponding systematical corrections because of anomalous refraction influences.

It is necessary to estimate the size of influences of $\Delta\rho$, $\Delta\rho_v$ and $\Delta\rho_m$.

Observations in the meridian are not far from the zenith, where ρ (in equation (13)) is very small, so we can really suppose that the influence of normal refraction is correctly calculated and therefore $\Delta\rho = 0$.

We will estimate $\Delta\rho_v$ and $\Delta\rho_m$, using the classical expressions of anomalous refraction influences. This approximation is admissible, because the question is about a narrow zone at the zenith distance only. At this analysis we consider influences of the boundary layer only, which plays the dominant role (**7**).

The influence of anomalous refraction on zenith distances z we can obtain from this relation:

$$\Delta z = \beta_0 \cos (a - a_{00}) \, ln \, n_0 \sec^2 z \qquad (22)$$

where are:

β_0 — the inclination of equal density boundary layer (in second of arc),

a_{00} — the azimuth of the refraction zenith connected with the boundary layer,

a — the azimuth of the observed star, and

n_0 — the refraction index at the boundary layer.

For the meridian, where is $a = 0$, we obtain

$$\Delta z_m = \beta_0 \cos a_{00} \ln n_0 \sec^2 z \qquad (23)$$

The size of anomalous refraction influences on the star passage moments u_i through the prime vertical is

$$\Delta u_i = \Delta z_v \sec \varphi = \pm \beta_0 \sin a_{00} \ln n_0 \sec^2 z \sec \varphi \qquad (24)$$

where the sign plus belongs to the western part, and the sign minus to the eastern part of the first vertical.

From (13) we conclude that φ_m is fully burdened with Δz, and for this reason

$$\Delta \rho_m = \Delta z_m$$

Δu_i, from (24), partially influences φ_v only. It is obvious from (9). Since the coefficients $b_k c_{ik}$ are small values, we may expected that $\Delta \rho_v$ also will be small. Let us control this constation on the numerical examples at the paragraph 5.

We will suppose that the position of the atmospheric layers are invariable between passages through the eastern and the western part of the prime vertical. In this case the following relations exist:

$$\Delta u_1 = - \beta_0 \sin a_{00} \ln n_0 \sec^2 z \sec \varphi$$
$$\Delta u_2 = - \beta_0 \sin a_{00} \ln n_0 \sec^2 z \sec \varphi$$
$$\Delta u_3 = + \beta_0 \sin a_{00} \ln n_0 \sec^2 z \sec \varphi$$
$$\Delta u_4 = + \beta_0 \sin a_{00} \ln n_0 \sec^2 z \sec \varphi$$

which, on the basis of (9), gives for the first example

$$\Delta (\varphi_v - \delta)_{k1} = + 0{,}06\, \beta_0 \sin a_{00} \ln n_0 \sec^2 z_1 \sec \varphi_1 = \\ = 0{,}09\, \beta_0 \sin a_{00} \ln n_0$$

and for the second example

$$\Delta (\varphi_v - \delta)_{k2} = + 0{,}09\, \beta_0 \sin a_{00} \ln n_0 \sec^2 z_2 \sec \varphi_2 = \\ = 0{,}18\, \beta_0 \sin a_{00} \ln n_0$$

If we presume that the atmosferic layer position is changeable between two passages — which is a more real conception — then the influence of $\Delta (\varphi_v - \delta)$ is probably still smaller.

Considering that the possible value of $\beta_0 \sin a_{00} \ln n_0$, at normal conditions and at the zenith zone, is not so great (0,''1—0,''2) and if we observe at places with moderate latitudes, then we can suppose that $\Delta \rho_v = 0$. This supposition is not far from the reality.

On the basis of all these, instead of (21) we can use

$$\Delta\varphi = \Delta\rho_m$$

consequently, we obtain the anomalous refraction influence at the meridian. At this we need not take any supposition about the mathematical model of the anomalous refraction influences, which is very important (connection with this see(**1**)).

10. The basic idea of this method was done somewhat earlier (**8**), the observational programme was worked out, but at the beginning of the adoption of this method it was immediately clear that the levels were the limitting factor at the realisation of this method. For this reason we put off the observational control of the possible analyse of the anomalous refraction influence with this method for a while when we would be able to use a high quality instrument for these investigations.

We remark that for Belgrade we selected stars from Boss's General Catalogue, which zenith distances at the meridian are less then 10' ($+ 44°38' < \delta < + + 44°48'$). It was possible to select 20 stars, from 4,0 to 7,4 magnitudes. If the star zenith distance at the meridian is 10', at the first vertical the zenith distance for the same star is 4°,4 and the hour angle 25^m.

11. At the end, let us come back to these conditions what we marked in the paragraph 1. The proposed method satisfies fully the conditions under points 3) and 4) — what we undestand as very important — then the condition 1) partailly too. The condition 1) is satisfied partially, because the accuracy of the de-

termined value from the observations of one star is insufficient, and therefore we ought to observe several stars, what naturally prolonges the duration of that process.

In our opinion the condition 2) is very important, because, without the fulfilment of this demand, it would be impossible to obtain a real value of anomalous refraction, which is relatively a small value.

The proposed method, on condition that we fulfil the mentioned demands, gives a possibility to analyse the refraction influence at the zenith zone, which is chiefly used for the latitude observations. In our opinion, the obtained results about the anomalous refraction in the narrow zenith zone cannot be extrapolated for the other zones.

On the present occasion we don't forget, as well, that the size of anomalous refraction influence is relatively small, and therefore if we want to single out this value from the astrometrical observational results, we must satisfy a good number of conditions. Undoubtedly it is very hard to determine these influences. But, it is obvious, to be done from our investigations, too.

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