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A QUASI BIENNAL OSCILLATION IN SOLAR ACTIVITY AND GEOPHYSICAL PHENOMENA: PRELIMINARY RESULTS

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Summary: The Quasi Biennal Oscillation (QBO) is well known in several meteorological parameters and has been also identified by several authors in the Earth rotation fluctuations.

In this paper we first indicate for the atmospheric angular momentum and Earth rotation variations, the unstable nature of QBO both in phase and amplitude.

In a second step indications on the existence of QBO are identified in solar activity and geomagnetic index (Aa).

Keywords: Earth rotation; atmospheric angular momentum; sunspots; geomagnetic field.

D. Djurović: REZIME - Kvazi-dvogodišnja oscilacija (QBO) meteoroloških parametara je poznata pojava. Neki autori su je identifikovali i u neravnomernostima Zemljine rotacije.

U ovome radu je ukazano da su faza i amplituda QBO u Zemljinoj rotaciji i ugaonom momentu Zemljine atmosfere nestabilni.

Osim toga, otkrivene su i indikacije da QBO postoji u Sunčevoj aktivnosti i fluktuacijama geomagnetnog indeksa (Aa).

1. INTRODUCTION

The Quasi Biennal Oscillation (QBO) let automaticaly think to the zonal wind inferences in the equatorial stratosphere at barometric altitudes higher than 100 mbar; the typical altitudes being at 50 and 30 mbar.

Since the discovery of QBO by Reed et al. (1961), Veryard et al. (1961), it has been considered many times and its main characteristics are more or less well known (Dunkerton et al., 1985; Naujokat, 1986) but its physical origin remains at the level of hypothesis. Other studies demonstrated that QBO exists also in mean temperatures (Nastrom et al., 1975), in the meridional winds (Groves, 1975) and in the column ozone (Funk et al., 1962; Angell et al., 1973). The presence of QBO in various meteorological parameters of the troposphere is also proved by different studies (Trenberth, 1980); this last review is important to understand our objective to search for relationships between the QBO of the Atmospheric Angular Momentum (AAM) and QBO of the Universal Time (UT_1).

In the Earth rotation (ER) the biennal fluctuation has been discovered, by Iijima et al.(1966), in the differences between atomic time and universal time as determined by 6 selected observatories. For the period 1955.0 – 1969.0, it has been confirmed in the series $UT_2 - A_3$ data computed by the Bureau International de l'Heure (Iijima et al., 1972); they estimated the amplitude (A) and period (P) of the QBO in the universal time to be A = 9.4 ± 0.7 msec, P = 26 months. The origin was attributed to the QBO winds observed in the equatorial stratosphere (Iijima et al., 1966).

In summary we can say that QBO is existing in meteorological parameters (the stratosphere, the troposphere) and in the Earth rotation. Besides, indications exist on the relations between stratospheric and tropospheric QBO (Trenberth, 1980) but they are not yet well sufficiently argumented. The results presented in this paper argue in favour of the relationship between solar activity, atmospheric and Earth rotation QBO's.

2. DATA AND METHODS OF ANALYSIS

The data sets are composed of the following series of observations:

- $UT_1 - UTC$ from 1967.0 to 1985.0, one value (in seconds) every 5 days, published in the Annual Reports of the Bureau International de l'Heure (BIH) for the years 1967 to 1985. (Referenced as BIH Reports).

- W, the daily Wolf numbers (1967.0 - 1985.0) published by the Sunspot Index Data Center (A. Koeckelenbergh, 1986).
- Aa, the geomagnetic index (1976.0 - 1983.0) in microtesla, as defined and described in P.N. Mayaud (1973, 1981); the one value every 3 hours series of data has been provided by M. Menvielle from the Institut de Physique du Globe, Paris.
- AAM, the daily value at 00h UT of the Atmospheric Angular Momentum (1976.0 - 1985.0), in 10^{26} Kg m^2s^{-1}, published by Rosen et al (1981) and Rosen (1985, personal communication).

The smoothing and the one side filtering of the data were performed by the Whittaker-Robinson-Vondrak (WRV) method (Whittaker et al., 1946; Vondrak, 1969).

3. THE QUASI BIENNAL OSCILLATION OF THE EARTH ROTATION.

3.1 Remarks on the Earth rotation drift.

Studies of the variations of the Earth rotation angular velocity and, consequently, of the length of the day (LOD) and universal time (UT_1, UT_2) fluctuations were generaly based on the series $UT_2 - A3$ and $UT_1 - TAI$ provided by the Bureau International de l'Heure (BIH). In these analysis the drift is often represented by polynomial forms of different degree higher than two; this procedure is theoreticaly not justified and could conduct to controvertial interpretations of the signal structure. For example Fig-1 represents residuals

$$R = UT_1 - TAI - P_n$$

where P_n is a polynomial form of order 3 (curve a) or 2 (curve b); evidently the two curves (a) and (b) are completely different. According to studies made by Stephenson et al. (1982, 1984) and Mignard (1986), a second order polynomial form is justified to model the secular drift of UT because it is the consequence of a constant decelaration generated by the tides and an other phenomena whose the origin is unknown. Of course the linear divergence between the two time scales is due to the duration difference between the astronomical (UT_1) and atomic (TAI) seconds.

On Fig-1(b) the origin of the quasi-sinusoidal fluctuation, with an amplitude of the order of 200 msec and a period ranging between 15 and 20 years, is not known. The same remark has to be done for the 5 years fluctuation of about 50 msec observed on Fig-1(a). The numerical experiments here above suggest the following question: could the two mentionned quasi-sinusoidal fluctuations be generated by unadequate mathematical representation of the drift?

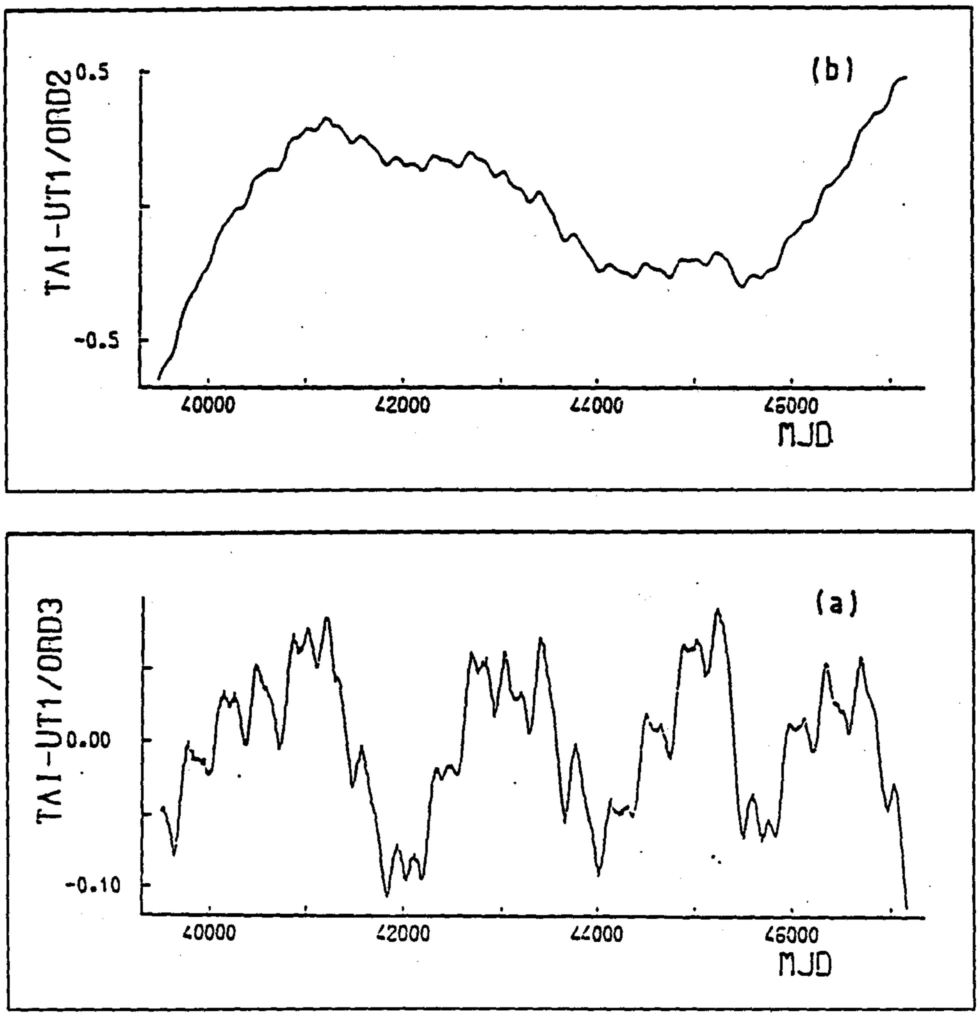

Figure 1: Residuals $R = UT_1 - TAI - P_n$ where P_n is a polynomial form of order 3 (a) and order 2 (b). Vertical scales are in seconds; MJD is set for Modified Julian Date.

A visual inspection of Fig-1(b) indicates that the 5-year period ondulations exist too on the main wave of 15 to 20-year period; the ratio of the amplitudes let the 5-year component to appear small. However, the 5-year fluctuation remains dominant even if the drift is removed with a polynomial form of degree 4 or 5 or in the residuals of the smoothing of $UT_1 - TAI$ by the Whittaker-Robinson-Vondrak (WRV) method with a filtering parameter $\varepsilon \leq 10^{-14}$.

Such a 5-year period has also been identified by Chao (1988), who pointed out that the cross-correlation between the variations of LOD and occurences of El-Nino Southern Oscillations (ENSO) varies with a period of 5 years, by Trenberth (1976) in the atmospheric circulation and by Courtillot (1976) in the geomagnetic field charges. In conclusion we certainly can not claim neither the 5-year nor the 15 to 20-year variation observed in the Earth rotation are generated by unadequate modeling of $UT_1 - TAI$ drift; of course the fluctuation of 15 to 20-year is less known because the length of analysed series is too short but, we can not *a priori* accept that it is generated by the mathematical modeling of the drift.

3.2 QBO in $UT_1 - TAI$

The discussion above has been also done to have a better understanding of the method applied to exhaust the QBO from the oscillations of 5 years and above, including the drift; in a first step we use the WRV method with a filtering factor $\varepsilon = 10^{-13}$.

The relation of Kun-Yi et al. (1981)

$$\varepsilon P^6 = 64\pi^6 A'(1 - A')^{-1}$$

where P is the period and A' the frequency response of the filter, allows to confirm this choice.

In the residuals of a smoothing with $\varepsilon = 10^{-13}$ the amplitude of the biennal term is reduced by about 20% while the amplitudes of components higher than 4 years are reduced by 95%. The limit of 4 years is choosen because the period of the 5-year fluctuation could decrease till 4 years as lower limit. However, with $\varepsilon = 10^{-13}$ the WRV filtering does not modify the seasonal variation of residuals among which the annual one is twice the amplitude of the semi-annual and QBO components; aiming to amplify QBO and to reduce the seasonal effects we computed the differences of residuals:

$$DR(i) = R(i + \tau) - R(i)$$

with a time lag τ = 365 days; it leads to multiply the amplitudes of the sinusoidal components in R(i) by a factor

$$A = 2\sin(\pi\tau/P);$$

the seasonal terms are practically eliminated while the amplitude of QBO is amplified by a factor 2.

The differences DR(i) are presented on Fig-2; they really suggest the presence of a very perturbed QBO with a period ranging from 600 to 850 days (1,6 to 2,3 years) and an amplitude changing from 3 to 30 msec.

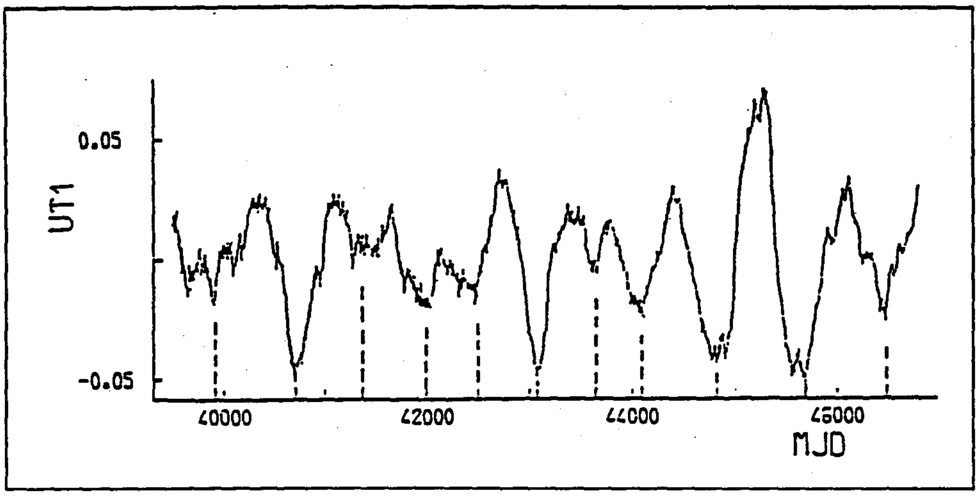

Figure 2: The curve of the residuals $DR(i) = R(i+\tau) - R(i)$, where $\tau = 365$ days suggests the existence of a perturbed QBO from 600 to 850 days; R(i) are the residuals deduced by the WRV method ($\varepsilon = 10^{-13}$) applied on the series $UT_1 - TAI$. The vertical scale is in seconds; MJD is set for Modified Julian Date.

We also know that the seasonal components in R(i) are not pure sinusoids but their deviations are too small (Djurovic, 1979) to explain the QBO perturbations as observed in Fig-2.

To have an insight in the cyclic structure of the residuals R(i) and to compare the phases of QBO observed in $UT_1 - TAI$ with other phenomena analysed later on, the Fourier periodogramme has also been computed and is presented at Fig-3. Besides the peaks A and B, due to the seasonal terms, we observe a large energy dissipation between 450 and 1000 days with a maximum of 10 msec at the period of 824 days (2,26 ans). This result confirms the above mentioned instability of the QBO period. Both the amplitude and the period are close to the values determined by Iijima et al. (1972).

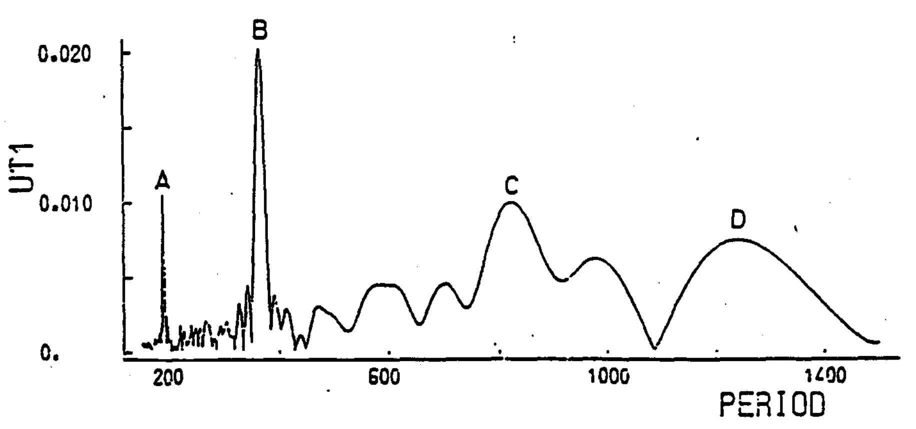

Figure 3: Fourier periodogramme of $UT_1 - TAI$ filtered by WRV ($\varepsilon = 10^{-13}$). Peaks A and B are due to seasonal contributions while a large energy dissipation is observed between 450 and 1000 days. The vertical scale is in seconds; the periods are in days.

Some doubt on the statistical signification of the peak C could be advocated by the presence of the peak D; however D is expected because a part of the 5-year fluctuation remains in the residuals R(i) and if the WRV filtering reduces to 5% the 5-year component we must keep in mind that its amplitude is close to 50 msec.

4. QBO IN THE TROPOSHERIC ZONAL CIRCULATION.

The AAM series considered in this analysis does not take into account the stratospheric winds (Rosen et al., 1981) because the meteorological wind soundings are not performed above an altitude of 50 mb; thus the QBO circulation observed in the tropical stratosphere, with a maximum intensity around 20 mb (Naujokat, 1986) is practically not included in our AAM data set. However, we must note that, taking into account the statistical accuracy and precision achieved in such an analysis, AAM represents practically the whole atmosphere; indeed 95% of the atmospheric mass are below the altitude of 50 mb.

The contribution of the stratosphere in AAM is larger than the mass ratio, but it remains yet small with respect to the tropospheric contribution. Rosen et al. (1985) estimate the contribution of the global stratospheric circulation to the annual fluctuation of LOD to be 20% of the tropospheric one.

Although such an analysis does not exist for QBO, taking into account the remarks above, it seems that the explanations of Iijima et al. (1966, 1972) about the origin of QBO in $UT_2 - A3$ is not quite acceptable. Having in mind the Trenberth's paper (1980) we considered as reasonable to search for QBO in AAM.

As for the $UT_1 - TAI$ data, AAM series has been filtered by the WRV method with $\varepsilon = 10^{-13}$; the corresponding Fourier periodogramme of residuals is given in Fig-4. Besides the peaks A1 and B1, due to the seasonal fluctuations, we observe a weak maximum around 700 days (C1) that could be attributed to the QBO. To improve its detection in residuals we cleaned the high frequencies using the WRV filtering with $\varepsilon = 10^{-11}$; doing that, in the smoothed residuals $R^*(i)$ the semi-annual component is removed, the annual one is reduced by 70% while that one at a frequency of 2 years is reduced by only 5%. After the WRV filtering, the differences of smoothed residuals

$$DR^*(i) = R^*(i+\tau) - R^*(i)$$

were computed with a time lag of 365 days; the annual term is then eliminated while QBO is amplified. The representation of $DR^*(i)$ on Fig-5 let appear more clearly the QBO but it would be useful to repeat the computation on a longer series of AAM observations. For the time being we accept that Fig-4 and 5 could be considered only as weak indications of the presence of QBO in AAM.

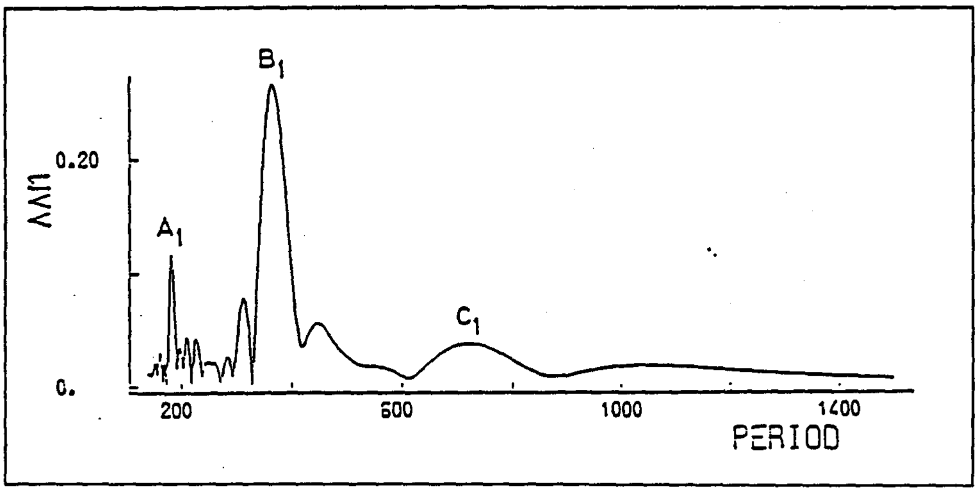

Figure 4: Fourier periodogramme of AAM filtered by WRV ($\varepsilon = 10^{-13}$). Peaks A1 and B1 are due to seasonal contributions; peaks C1 at 700 days could be attributed to QBO. The vertical scale is in $10^{26} Kg\ m^2 s^{-1}$; the periods are in days.

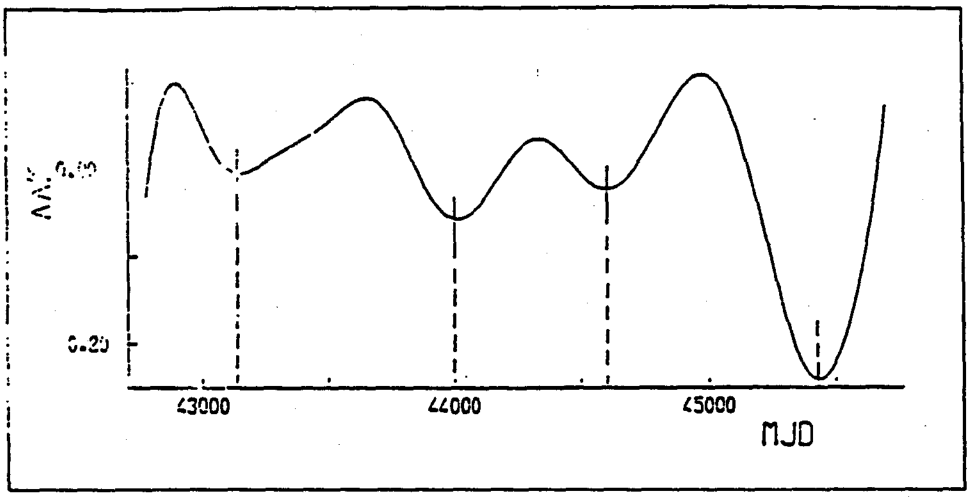

Figure 5: The curve of the residuals $DR^*(i) = R^*(i+\tau) - R^*(i)$, where $\tau = 365$ days suggests the existence of QBO with a mean period of 760 days. $R^*(i)$ are the residuals of a double filtering by the WRV method ($\varepsilon = 10^{-13}, \varepsilon = 10^{-10}$) applied on AAM. The vertical scale is in $10^{26} Kg\ m^2 s^{-1}$; MJD is set for Modified Julian Date.

5. QBO IN SOLAR ACTIVITY AND THE GEOMAGNETIC FIELD

The residuals $\hat{R}(i)$ and $\overline{R}(i)$ after filtering of the Wolf numbers and the geomagnetic index by the WRV method ($\varepsilon = 10^{-13}$) respectively were submitted to the DFT analysis whose periodogrammes are given in Fig-6 and 7. In the first one, related to the Wolf numbers, two maxima (C2, D2) between 600 and 1000 days could be attributed to a QBO in the solar activity.

As it is not the first time that such a biennal fluctuation is detected in the solar activity (see Lamb, 1972), we incline to believe that C2 and D2 (Fig. 6) are not generated by random fluctuations. The peaks A2 and B2, corresponding to periods of 321 and 394 days respectively, probably are associated with the perturbed annual fluctuation.

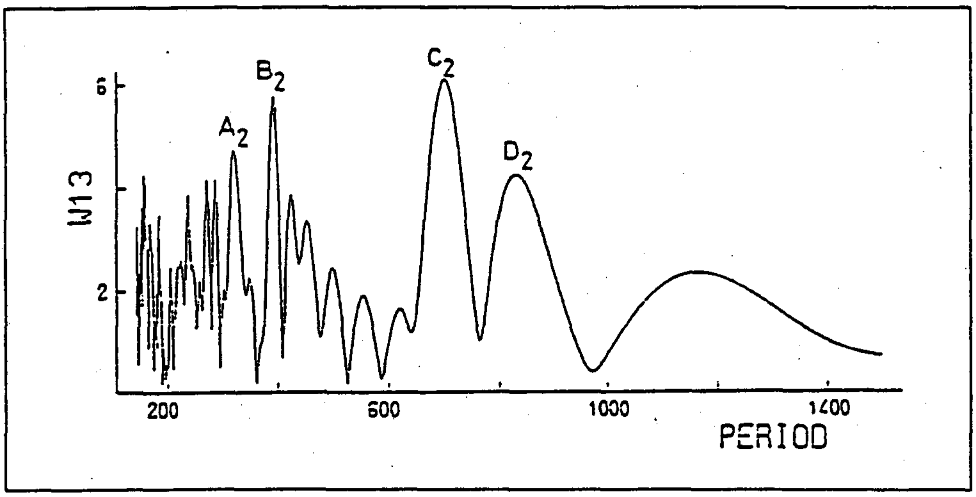

Figure 6: Fourier periodogramme of the Wolf numbers filtered by WRV ($\varepsilon = 10^{-13}$). Peaks (C2, D2) between 600 and 1000 days could be attributed to QBO in the solar activity. The vertical scale gives the Wolf numbers; the periods are in days.

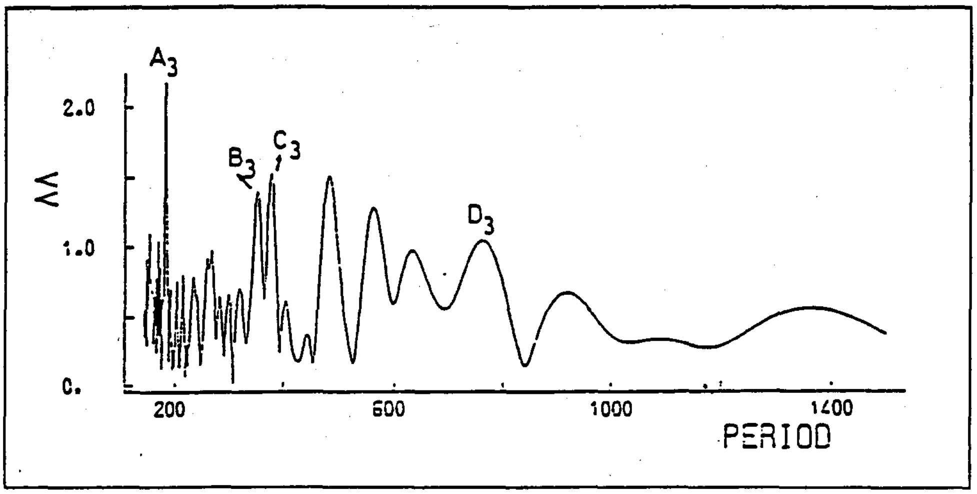

Figure 7: Fourier periodogramme of Aa filtered by WRV ($\varepsilon = 10^{-13}$). Peaks A3 is generated by the semi-annual component; between 450 and 1000 days a large dispersion is observed with 5 maxima in the QBO domain. The vertical scale is in microtesla; the periods are in days.

The origin of a possible annual variation of the solar activity is not well understood; is it generated by the annual selection of the observations or by a real annual variation of the solar activity? Nevertheless it has to be associated with the results deduced from the observations of the solar diameter made in Grasse (France); in such a series, Delache et al. (1988) identified a variation of the solar diameter at 320 days and they expect that it could have a physical origin. The same periodic fluctuation has been also identified in the Wolf numbers by Delache et al. (1985).

In the above mentionned papers, Delache et al. (1985, 1988) discussed a clear detection of a 1000 days (2.7 years) fluctuation but they miss to relate it with the QBO in AAM and $UT_1 - TAI$; they also pointed out a less conclusive component around 1.9 years. The two terms detected by Delache could be related with QBO's frequency unstability.

To reduce the high frequency variations of the Wolf numbers, the residuals $\hat{R}(i)$ were filtered by the WRV method with $\varepsilon = 10^{-10}$ while, the annual component was largely removed by computing the quantities

$$D\hat{R}^*(i) = \hat{R}^*(i + \tau) - \hat{R}^*(i)$$

of the smoothed residuals $\hat{R}^*(i)$ with $\tau = 365$ days. As it can be seen on Fig-8, the $D\hat{R}^*(i)$ exhibits a QBO fluctuation largely perturbed both in amplitude and in frequency; this representation explains why in the periodogramme of Fig-6 the two peaks (C2, D2), appear at 600 and 1000 days. It seems they represent a unique perturbed component rather than two real ones. To test this point of view and having in mind that the influence of solar activity on the geomagnetic index is well known, the same analysis as above has been applied to Aa. Aa is indeed an independant observation series from the Wolf numbers and it could allow to test our conclusions about the cyclic structure of the solar activity parameters.

In the periodogramme of Aa (Fig. 7) the peak A3, well pronounced, is generated by the known semi-annual component. The peaks B3 and C3 correspond to the peaks C2 and D2 appearing in the periodogramme of the Wolf numbers (Fig. 6) while between 450 and 1000 days there is a large dispersion of the spectral energy with 5 maxima in the QBO domain.

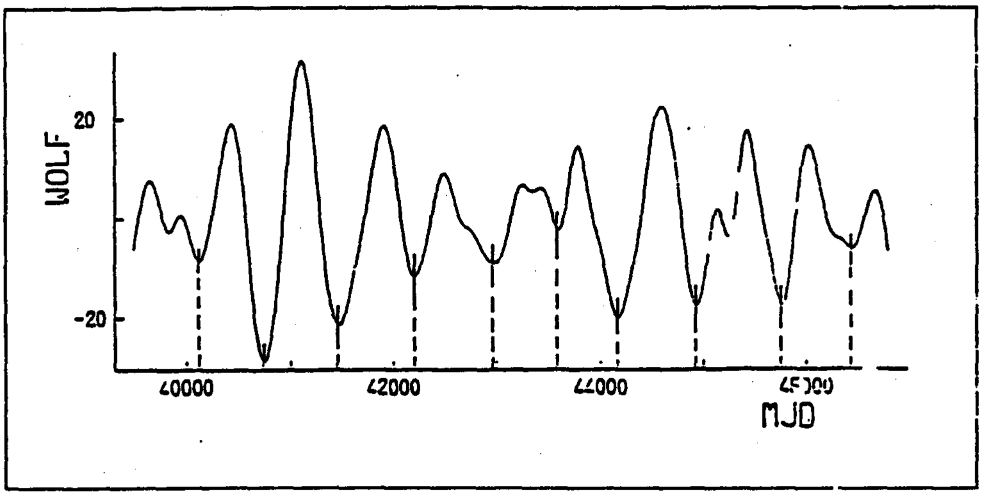

Figure 8: The curve of the residuals $D\hat{R}^*(i) = \hat{R}^*(i+\tau) - \hat{R}^*(i)$ with $\tau = 365$ days let appear a perturbed QBO both in phase and amplitude; the mean period is 710 days. $R^*(i)$ are the residuals of a double filtering by the WRV method ($\varepsilon = 10^{-13}, \varepsilon = 10^{-10}$) applied on the Wolf numbers. The vertical scale gives the Wolf numbers; MJD is set for Modified Julian Date.

This also indicates the existence of QBO in the solar activity and the geomagnetic field as well as the perturbed nature of the QBO period.

6. STATISTICAL IN-(DEPENDANCE) OF THE QBO FLUCTUATIONS

The comparisons of the QBO phases computed by DFT does not conduct to deterministic conclusions: in AAM the phase is not well determined because the maximum C1 is small; in W and Aa periodogrammes different maxima exist that could be attributed to QBO; nevertheless we are giving in Table 1 the phases associated with the closest maximum to 2.2 years and we note that they do not diverge so much.

Since the DFT phases are relative to the sinusoidal approximation of the perturbed QBO's, they are not sufficiently conclusive. To obtain complementary informations whether QBO fluctuations observed in the 4 series above (UT1, AAM, W, Aa) are mutualy dependant or not the cross-correlation functions $C(\tau)$, τ being the phase difference in days, have been estimated. For different combinations of series of residuals R(i) the maxima of $C(\tau)$ are given in Table 2.

According to the Student's t-criterion, all the maxima of $C(\tau)$ are statisticaly significant with a probality greater than P, as given in the last column.

The cross-correlation results suggest the conclusion that QBO observed in the four series of data are statisticaly dependant. Moreover we note that the phase lag τ is also biennal (1.8 - 2.4 years). In this work the last result is not analysed.

Table - 1.

Periods and phases of the maxima deduced by DFT for UT_1, AAM, W, Aa series.

Series	Maximum	Periods in days	Phases
UT_1	C	824	291°
AAM	C1	720	221°
W	D2	832	259°
Aa	D3	762	288°

Table - 2.

Maximum value and phase lag τ of the cross-correlation functions.

Series	Max	τ(days)	N-data	P(%)
UT_1/W	0.23	670	961	99
UT_1/AAM	0.9	830	272	99
UT_1/Aa	0.43	800	935	99
W/Aa	0.53	875	920	99

To verify if our results could be not generated by the mathematical approach of filtering and smoothing, we simulated a series of $UT_1 - TAI$ residuals R(i) obtained after the elimination of the parabolic drift; computed residuals are composed of 4 sinusoids whose the amplitudes and phases are given in Table 3. A gaussian noise with a variance $\sigma^2 = 4$ has been added. The same procedure of filtering with WRV method, $\varepsilon = 10^{-13}$ and $\varepsilon = 10^{-10}$, does not rise to any presence of a biennal term and assure us that the QBO detected in the 4 series are not generated by the filtering and smoothing of data.

Table - 3.

Periods, amplitudes and phases of the 4 components of a theoretical signal. (The phases are randomely choosen).

Period (years)	Amplitude (msec)	Phase (1976.0) (degrees)
0.5	9	212°
1.0	20	60°
5.0	50	127°
15.0	100	223°

7. DISCUSSION

From the present analysis it appears that QBO is difficult to point out clearly; QBO is displayed with a large amplitude and frequency dispersion. Its period varies between 450 and 1000 days.

Some of our results confirm past conclusions about the existence of QBO recognized in UT_1 (Iijima et al., 1966, 1972; Carta et al., 1982), and in solar activity (Lamb, 1972). In the Wolf numbers our periodogramme (Fig. 6) displays two peaks (C2, D2) which can not be attributed to random fluctuations or observational errors.

The geomagnetic index Aa is very sensitive to the corpuscular radiation of the Sun which is directly correlated with the Wolf numbers; thus the existence of maxima distributed between 450 and 850 days is not surprising.

In the equatorial zone the QBO stratospheric circulation is a well-known phenomena since many years (Reed et al., 1961; Veryard,1961; Naujokat, 1986). Its physical connexion with tropospheric biennal phenomena is not proved but they are indications that it exists as it is reviewed by Trenberth (1980). Thus the high correlation UT_1/AAM and the maximum C1 observed in the periodogramme AAM (Fig. 4), although its amplitude is small, conducts us to accept it as the indications of the presence of QBO in AAM; the amplitude in AAM seems too small to be considered as the origin of QBO in the Earth rotation (by exchange of angular momentum between the atmosphere and the solid Earth); it could let to search, for the QBO observed in UT_1 and AAM, a common external origin as for example variations in the solar activity; this assumption is suggested by other analysis establing relations between solar activity and geophysical phenomena (Djurovic et al., 1988, 1989).

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