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THE SATELLITE ORBITAL ECCENTRICITY PERTURBATIONS UNDER THE INFLUENCE OF A DRAG

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Summary. By a procedure like that in the previous paper (Šegan, 1987) is developed a special FORTRAN program for the computation of perturbation of the satellite orbital eccentricity. Comparison with observed values is given, too.

S. Šegan, POREMEĆAJ EKSCENTRIČNOSTI PUTANJE SATELITA USLED OTPORA ATMOSFERE. Dobijen je model poremećaja ekscentričnosti satelitske putanje usled dejstva otpora atmosfere. Model je efektivno dat FORTRAN programom. Upoređenje sa posmatranjima pokazuje dobro slaganje.

1. INTRODUCTION

A serious attempt to transform the complicated aeronomic models into an analytically tractable shape were made by many autors (Fominov, 1974; Klinkard, 1984; Roth, 1985; Henrard, 1986).

But, some factors like air density at perigee height are limiting factors for all those theories. In order to overcome it we are taking the Sehnal's approximation (1983) of the total density distribution using the spherical harmonic functions:

$$\rho = \sum_{k=1}^{K} \rho_s^k e^{\frac{r_{p0}-r}{kH}} = \sum_{k=1}^{K}\sum_{i=1}^{I} A_i^{(k)} f_i e^{\frac{r_{p0}-r}{kH}}, K = 3, \tag{1.1}$$

where $A_i^{(k)}$ expresses the geometry while f_i are functions of the physical parameters.

From the Lagrangian equations in the Gaussian form (King-Hele, 1964) we have

$$\frac{\mathrm{de}}{\mathrm{dE}} = -a\rho\delta \frac{1 + e\cos E}{(1 - e\cos E)^{\frac{1}{2}}}(1 - e^2)\cos E,$$

$$\Delta e = -a\delta \int_0^{2\pi} \frac{(1 + e\cos E)(1 - e^2)\cos E}{(1 - e\cos E)^{\frac{1}{2}}} \rho \mathrm{dE}. \tag{1.2}$$

The formulation of the model is given in (1.3) using the Sehnal's notation (1986). The density at a specific surface of constant altitude is given by seven additive terms each of which has its own height dependence.

$$\rho = k_0 f_0 f_x \sum_{n=1}^{7} h_n g_n, \tag{1.3}$$

where

$$f_x = 1 + a_1(F_x - F_b),$$

$$f_0 = a_2 + F_m,$$

$$f_m = \frac{F_b - 60}{160},$$

$$k_0 = 1 + a_3(K_p - 3),$$

K_p - daily geomagnetic index, F_x - solar flux, F_b - mean solar flux, and $a_1, a_2, \ldots$ are the model coefficients.

Some of the functions g_n are time dependent (diurnal, annual, ...) while the other ones describe a dependence of the physical parameters. The height dependence is expressed by the h_n terms,

$$h_n = K_{n0} + \sum_{j=1}^{3} K_{nj} A_j e^{c_j \cos 2u} e^{z_j \cos E},$$

where

$$A_j = e^{\frac{120 + R_e(1 - \epsilon\sin^2 i) - a}{40j}},$$

$$z_j = \frac{ae}{40j}, j = 1,3 \tag{1.4}$$

where R_e is the equatorial radius of the Earth, ε is the flattening of the Earth and i is the orbital inclination.

The expansion can be written in a summation notation to allow an easy multiplication of the series. An analytical form of the individual terms is difficult to achieve. Since formation of the terms is straightforward but very combersome, a computer algebra would be best suited for this purpose.

We have used the algebraic system REDUCE 2 (Hearn, 1973, 1978) on the IBM 370 (4381) computer.

After expansion of all factors of the integrand in the formula (1.2) as a power series in e and E and their transformation we have

$$\Delta e = -a\delta k_0 f_0 f_x \sum_{\substack{n=1\\n\neq 3}}^{5} (g_n K_{n0} \int_0^{2\pi} L_e \mathrm{dE} + \mathrm{g_n} \sum_{\mathrm{j=1}}^{3} \mathrm{K_{nj} A_j} \int_0^{2\pi} \mathrm{C_j L_e e^{z_j \cos E} dE})$$

$$+ \sum_{n=3,6,7} (K_{n0} \int_0^{2\pi} g_n L_e \mathrm{dE} + \sum_{\mathrm{j=1}}^{3} \mathrm{K_{nj} A_j} \int_0^{2\pi} \mathrm{g_n C_j L_e e^{z_j \cos E} dE}). \tag{1.5}$$

Using the FORTRAN program DRAGe we computed Δe for a sample of (different) heights using formulae (1.5) . We used for the balistic coefficient δ an average value

$$\delta = 2.2 \frac{effective\ cross - section}{mass} \approx 0.160 \frac{cm^2}{g}.$$

We choose two cases for formula (1.5) – expanding it into series with powers of small parameters, i.e. e and c: first – to the 4th order of them and second – to the 3rd order. Some results for the heights of 200 km and 300 km can be seen in the table T1 and on the *Fig*.1 and *Fig*.2.

While the accuracy achieved seems good, the results (especially *Fig*.1 and *Fig*.2) indicate that more attention should be given to the problem of the definition of the model density distribution coefficients.

For the changes of physical parameters ($Kp, t, ...$) values Δe are presented by *Fig*.3, and 4 . We can see that our formula (1.5) is very sensitive to the changes in the solar flux and the mean solar flux difference.

Table **T1**: The perturbations of the satellite orbital eccentricity under influence of a drag – variation with geomagnetic index K_p (left side) and with solar flux difference (right side).

K_p	Δe_{200}	ρ	Δe_{200}	ρ	Δe_{200}	ρ	Δe_{200}	ρ
0.0	-1.46	11	-0.11	1.0	-3.07	22	-0.22	2.1
0.5	-1.69	12	-0.12	1.2	-3.15	23	-0.23	2.2
1.0	-1.92	14	-0.14	1.3	-3.23	23	-0.24	2.3
1.5	-2.15	16	-0.16	1.5	-3.31	24	-0.24	2.3
2.5	-2.38	17	-0.17	1.7	-3.39	25	-0.25	2.4
3.0	-2.62	19	-0.19	1.8	-3.47	25	-0.25	2.4
3.5	-2.85	21	-0.21	2.0	-3.55	26	-0.26	2.5
4.0	-3.08	22	-0.23	2.1	-3.64	26	-0.26	2.5
4.5	-3.31	24	-0.24	2.3	-3.72	27	-0.27	2.6
5.0	-3.54	26	-0.26	2.5	-3.80	28	-0.28	2.6
5.5	-3.77	27	-0.27	2.6	-3.88	28	-0.28	2.7
6.0	-4.01	29	-0.29	2.8	-3.96	29	-0.29	2.8
6.5	-4.24	31	-0.31	3.0	-4.04	29	-0.30	2.8
7.0	-4.47	32	-0.33	3.1	-4.12	30	-0.30	2.9
7.5	-4.70	34	-0.34	3.3	-4.20	30	-0.31	2.9
8.0	-4.93	36	-0.36	3.4	-4.28	31	-0.31	3.0
8.5	-5.17	37	-0.38	3.6	-4.36	32	-0.32	3.0
9.0	-5.40	39	-0.39	3.8	-4.44	32	-0.33	3.2
9.5	-5.63	41	-0.41	3.9	-4.52	33	-0.33	3.2

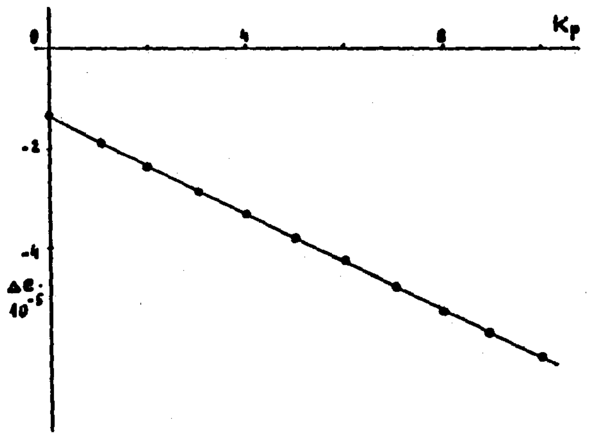

Fig. 1: Some results for the heights of 200 km.Changes Δe with geomagnetic index K_p.

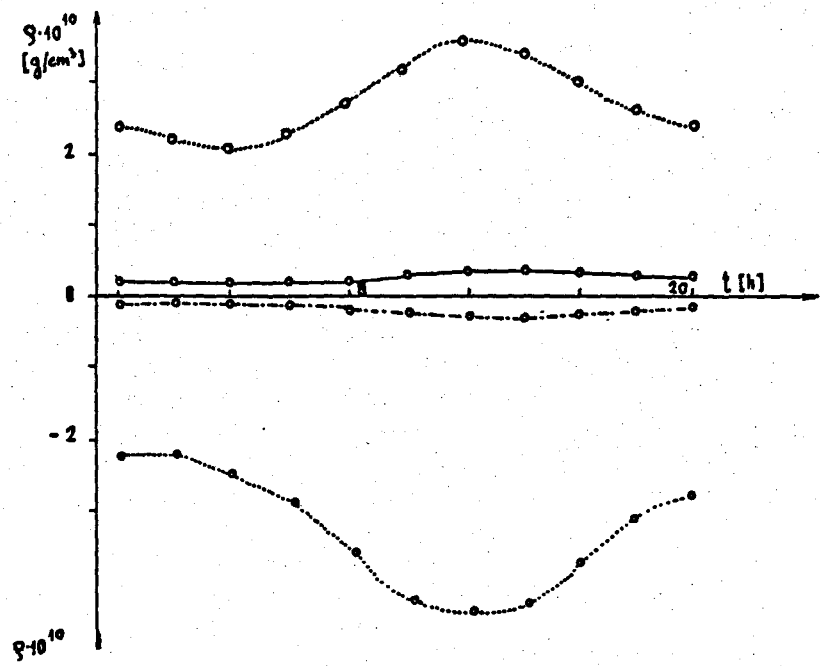

Fig. 2: Diurnal changes of Δe for 200 km and 300 km.

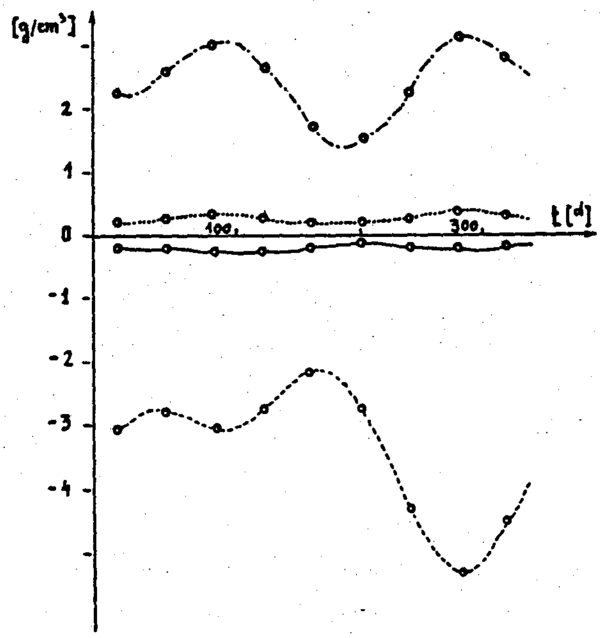

Fig. 3: Annual changes of Δe for the same altitudes (200 km and 300 km).

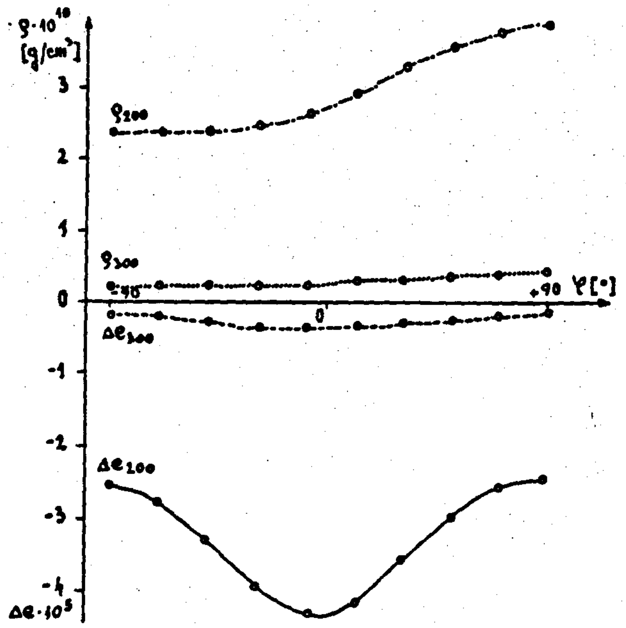

Fig. 4: Latitudinal variation of Δe.

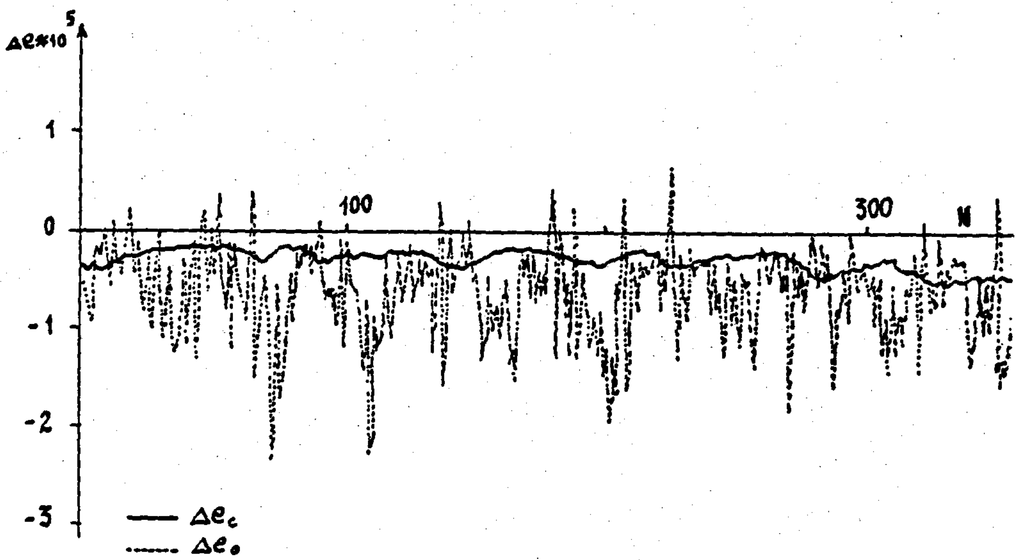

Fig. 5: Perturbations of the orbital eccentricity of the ANS satellite under influence of a drag; doted line–observed values, solid line–calculated values.

The change of the perigee height was derived firstly by correcting for gravitational perturbations (odd and even zonal harmonics, luni– solar effects) and secondly for the solar radiation pressure effect (the third zonal harmonics effect is already subtracted in the published elements).

2. COMPARISON TO THE OBSERVATIONS OF ANS SATELLITE

The ANS satellite was the first Netherlands Astronomical Satellite (1974 70A) launched on August 30, 1974 which decayed on June 14, 1977. The data of its sun-sinhronous orbit with the orbital plane perpendicular to the direction to the Sun were already the subject of a detailed analysis (Wakker and al., 1981; Sehnal, 1982).

We had at our disposal the orbital data from the NASA centre. The area–to–mass ratio is necessary for computing the atmospheric drag effects and we take it from the data for the satellite dimensions and mass as given by Wakker (1978) and Sehnal (1982).

We have all needed physical parameters too for the same time interval: solar flux, mean solar flux, geomagnetic index A_p. The obtained changes of the eccentricities are consequences of the computation by using changes in the mean daily motion corrected for a radiation pressure and from the observed values of semi–major axis and eccentricities.

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The values from computation by this method and by formula (1.5) are presented by the *Fig*.5.

3. CONCLUSIONS

It can be seen from the analyses that analytical interpretation give us posibilities to compute a drag, especially for the satellites with small to moderate eccentricities.

The FORTRAN procedure is altered to an approximation of the changes of the eccentricities in perigee altitude region where significant drag effects are experienced. The algorithm is simple and straightforward; we checked the accuracy of the reduced model (to e^3, c^3, c^2e, e^2c) and, we did not find any substantial difference.

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