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ANALYTICAL COMPUTATION OF ATMOSPHERIC DRAG EFFECTS

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Summary. An analytical interpretation of the satellite orbital element perturbation under influence of a drag has been developed. Some useful formulae for the perturbations of the semi-major axis are given. The agreement with observed values is very good.

S. Šegan, ANALITIČKO IZRAČUNAVANJE UTICAJA OTPORA ATMOSFERE NA KRETANJE SATELITA - Data je analitička teorija poremećaja velike poluose putanje satelita usled dejstva otpora atmosfere. Uz pomoć algebarskog sistema izvedene su osnovne transformacije i dobijeni konačni izrazi. Slaganje teorijskih rezultata sa posmatranjima satelita Interkosmos 10 je veoma dobro.

1. INTRODUCTION

The method of modeling of the Earth's gravitational field can not be applied for the modeling of the density distribution of the thermosphere, i. e. the modeling of this type can not be useful due to ireversibility and impossibility of linearization.

The first attempt to make an analytical theory of the orbital perturbations and by reverse process, to determine the characteristic constants of the model, was made by Sehnal (1977, 1983a). In this paper the analytical theory is further developed. Our theory gives the analytical formulae and a linear expression for the constants. This last condition must be taken into account by the model itself.

A large part of the transformations present in the theory are time consuming and very complicated. Therefore, we used the computer algebra manipulation to make necessary transformations.

The computation of the drag effects in the motion of an artificial satellite is performed by the method of variation of the elements, well-known from the celestial mechanics. The changes of the elements are expressed in this case by the Lagrange equations of motion in the Gaussian form (with the components of the disturbing acceleration (King–Hele, 1959; Sterne, 1960)); the usage of canonical elements (Brower–Hori theory, 1961) would have here only a limited validity since the mathematical description of the density of the atmosphere must be given in a very simple form only.

The Lagrange equations lead to a system of differential equations which is integrable over one revolution, generally in a simple case only. The complications are being caused by the mathematical expression for the atmospheric density. The usage of the aeronomic models (CIRA72,(1972); CIRA86,(Hedin, 1986); MSIS, (Hedin et al., 1977); DTM, (Barlier et al., 1977); C, (Köhnlein, 1980)) is impossible for such an analytical work since the mathematical description of the total density is very complicated, because each atmospheric constituent in these models has to be separately discussed.

The simple atmospheric model was used by King–Hele (1959) who subsequently developed his theory taking into account the atmospherical bulge and a satellite orbit of a high eccentricity (King–Hele, 1962, ..., 1986 , King-Hele and Walker, 1960, 1969, 1970).

To improve the theory, several attempts were made, e.g., Henrard (1986) sugests the decrease of the atmospheric density as a power function

$$\rho = \rho_0 \left(\frac{q_0 - s}{r - s} \right)^{\tau}$$

s, τ to be chosen. Still a simple formula for ρ_0 in his model is necessary.

Fominov (1963, 1974) made the first attempt to develop a theory based on a general model, expressible in the orbital elements and time variables (e.g. true anomaly). He developed the theory, but his model is rather hypothetical, since he computed no real values of the defining constants:

$$\rho = \overline{\rho}\ (\nabla_\gamma + \nabla_\psi + \nabla_\theta),$$

$\overline{\rho}$ — height profile,

ρ_0 — chosen constant, not the density at perigee,
∇_γ — correction of height dependence,
∇_ψ — day-to-night variation,
∇_θ— latitudinal variation.

Vykutilova (1983) uses also a model simpler than one in this paper. (She is using a general expression for this type of models). She gives some numerical results but does not quote the constants, characterizing the model itself.

The aim of this paper was to show that the theory can be developed. All constants were taken from the model TD (Sehnal, 1986).

2. BASIC EQUATIONS

Consider a particle P moving under the attraction of a force μ/r^2 per unit mass; let the polar coordinates of P be (r, ν) so that, setting its radial acceleration equal to the force per unit mass, we have

$$\ddot{r} - r\dot{\nu}^2 = -\frac{\mu}{r^2}, \tag{2.1}$$

or, for an unperturbed orbit in a vector form ($\vec{r}$ is the position vector relative to the centre of attraction)

$$\ddot{\vec{r}} + \mu\frac{\vec{r}}{r^3} = 0. \tag{2.2}$$

Suppose now that this orbit is perturbed by a force $\vec{f}$ (per unit mass), so that

$$\ddot{\vec{r}} + \mu\frac{\vec{r}}{r^3} = \vec{f}. \tag{2.3}$$

The plane of the orbit is now liable to variation; we can express the changes of the orbital elements in terms of $\vec{f}$. We resolve it firstly into components: a radial one, a transverse one and a normal one to the osculating plane (*Fig.1*). Changes in a and e depend solely on f_r and f_t , i.e. on forces in the orbital plane

$$\dot{a} = \frac{2\,r\,f_t\sqrt{\frac{p}{\mu}} + 2\,a\,e\,\dot{e}}{1-e^2}, \tag{2.4}$$

$$\dot{e} = \sqrt{\frac{p}{\mu}}(f_r\,\sin\nu + f_t\,(\cos\nu + \cos E)),$$

where

p — parameter of the ellipse

a — semi–major axis and
e — eccentricity of orbit.

Since the tangent to the orbit makes an angle ψ with the transverse direction along which f_t acts, we have

$$f_t = \frac{\sqrt{\frac{\mu}{p}}}{V}(f_T\,(1 + e\cos\nu) + f_N\, e\cos\nu),$$

$$f_r = \frac{\sqrt{\frac{\mu}{p}}}{V}(f_T\, e\sin\nu - f_N\,(1 + e\cos\nu)),$$

where V is the velocity

$$V^2 = \mu(\frac{2}{r} - \frac{1}{a}).$$

Therefore

$$\dot{a} = \frac{2a^2 v f_T}{\mu},$$

$$\dot{e} = \frac{2af_T(e + \cos\nu) - f_N r\sin\nu}{aV}.$$

For a satellite of a mass M the aerodynamic drag force is

$$\frac{D}{M} = \frac{\rho v^2\delta}{2},$$

where

$$\delta = \frac{C_d A}{M},$$

acting in a direction opposite to the velocity V relative to ambient air, thus

$$f_T = -\frac{\rho v^2\delta}{2}. \tag{2.5}$$

We neglected here small component f_N of a drag normal to the orbital plane. Thus we have

$$\dot{a} = -\frac{a\rho\delta v^3}{\mu}, \tag{2.6}$$

$$\dot{e} = -\rho\delta v(e + \cos\nu), \tag{2.7}$$

or since

$$e + \cos\nu = \frac{a}{r}(1 - e^2)\cos E, \tag{2.8}$$

the last two equations become

$$\frac{\mathrm{da}}{\mathrm{dE}} = -a^2\rho\delta(1 + e\cos E)^{\frac{3}{2}}(1 - e\cos E)^{\frac{1}{2}} \tag{2.9}$$

and

$$\frac{\mathrm{de}}{\mathrm{dE}} = -a\rho\delta\frac{1 + e\cos E}{(1 - e\cos E)^{\frac{1}{2}}}(1 - e^2)\cos E,$$

i.e.

$$\Delta a = -a^2\delta\int_0^{2\pi}\frac{(1 + e\cos E)^{\frac{3}{2}}}{(1 - e\cos E)^{\frac{1}{2}}}\rho\mathrm{dE}, \tag{2.10}$$

$$\Delta e = -a\delta\int_0^{2\pi}\frac{(1 + e\cos E)(1 - e^2)\cos E}{(1 - e\cos E)^{\frac{1}{2}}}\rho\mathrm{dE}.$$

Since the air density ρ is unknown and its expression taken from different density models is very complicated, the process of integration is very laborious.

3. AIR DENSITY

On the heights which are the most important for our study, i.e. between 150–500 km, the surfaces of constant density tend to be spheroidal with approximately the same ellipticity as the Earth, $\varepsilon = 0.00335$. We assume that for any given geocentric latitude the air density varies exponentially with distance from the Earth's centre:

$$\mathrm{d}\rho = -\rho\frac{\mathrm{dr}}{\mathrm{H}},$$

$$\rho = \rho_0 e^{\frac{-(r-r_0)}{H}}, \tag{3.1}$$

where H is the density scale height and r_0 is the perigee distance of a satellite. Our task is to obtain the air density at any point on a satellite orbit.

For the oblate spheroid of the equatorial radius R_e and the ellipticity ε we have the radial distance R

$$R = R_e(1 - \varepsilon\sin^2\varphi + o(\varepsilon^2)), \tag{3.2}$$

where φ is geocentric latitude , and, the perigee distance

$$r_p = R_e^p(1 - \varepsilon \sin^2 \varphi_p),$$
$$R_e^p = r(1 + \varepsilon \sin^2 \varphi_p). \tag{3.3}$$

Now, we choose R so that the spheroid defined by (3.1) passes trough the initial perigee point of the satellite:

$$R = r_{p_0}(1 + \varepsilon \sin^2 \varphi_{p_0})(1 - \varepsilon \sin^2 \varphi), \tag{3.4}$$

and from (3.1)

$$\rho = \rho_{p_0} e^{\frac{-(r-R)}{H}}. \tag{3.5}$$

Since $\sin\delta \equiv \sin\varphi = \sin i \sin u$, where i is the inclination of the orbit, u is argument of the latitude of the perigee,

$$e^{\frac{-(r-R)}{H}} = e^{\frac{-(r-r_{p_0})}{H}} + \frac{\varepsilon r_{p_0} \sin i \cos 2u}{2H} - \frac{\varepsilon r_{p_0} \sin^2 i \cos 2\omega}{2H}. \tag{3.6}$$

At this point King–Hele (1964) derived expressions for the eccentricities between 0.0 and 0.2:

$$\rho = ke^{\frac{-a(1-e\cos E)}{H}}(1 + e\cos 2(\omega + E)) - 2ce \sin 2(\omega + E) \sin E + \frac{c^2}{4}(1 + \cos 4(\omega + E))$$
$$-ce^2(\cos 2\omega + 2\cos 2(\omega + E) - 3\cos 2(\omega + E)\cos 2E) - c^2 e \sin 4(\omega + E) \sin E$$
$$+o(c\varepsilon, ce^3, c^2e^2),$$

where

$$k = \rho_{p_0} e^{(\frac{r_{p_0}}{H} - c\cos 2\omega_0)},$$
$$c = \frac{\varepsilon r_{p_0}}{2H} \sin^2 i, \tag{3.7}$$

under the assumption that we have a value ρ_{p_0} .

We are taking the Sehnal's method of approximation of the total density distribution using the spherical harmonic functions and linearize them:

$$\rho = \sum_{k=1}^{K} \rho_s^k e^{\frac{r_{p_0}-r}{kH}} = \sum_{k=1}^{K}\sum_{i=1}^{I} A_i^{(k)} f_i e^{\frac{r_{p_0}-r}{kH}}, K = 3, \tag{3.8}$$

where $A_i^{(k)}$ expresses the geometry while f_i are functions of the physical parameters.

The formulation of the model is given in (3.9) in the Sehnal's notation (1986). The density at a specific surface of constant altitude is given by seven additive terms each of which has its own height dependence.

$$\rho = \rho_{p_0} f_0 f_x \sum_{n=1}^{7} h_n g_n, \tag{3.9}$$

where is

$$\begin{aligned} f_x &= 1 + a_1(F_x - F_b), \\ f_0 &= a_2 + F_m, \\ f_m &= \frac{F_b - 60}{160}, \\ k_0 &= 1 + a_3(K_p - 3), \end{aligned}$$

K_p - daily geomagnetic index, F_x - solar flux, F_b - mean solar flux, and $a_1, a_2, ...$ are model coefficients.

Some of the functions g_n are time dependent (diurnal, annual, ...) while the other ones describe a dependence of the physical parameters. The height dependence is expressed by the h_n terms,

$$h_n = K_{n0} + \sum_{j=1}^{3} K_{nj} A_j e^{c_j \cos 2u} e^{z_j \cos E},$$

where

$$\begin{aligned} A_j &= e^{\frac{120 + R_e(1 - \varepsilon \sin^2 i) - a}{40j}}, \\ z_j &= \frac{ae}{40j}, j = 1,3 \end{aligned} \tag{3.10}$$

where R_e is the equatorial radius of the Earth, ε is the flattening of the Earth and i is the orbital inclination.

We take into account the fact, that the latitude φ and the local time t are variable during one revolution and introduce relations (see *Fig.*2):

$$\sin t \cos \varphi = \cos u \sin(\Omega - \alpha_0) + \sin u \cos(\Omega - \alpha_0) \cos i,$$

$$\cos t \cos \varphi = \cos u \cos(\Omega - \alpha_0) - \sin u \sin(\Omega - \alpha_0) \cos i,$$

$$\sin u = \sqrt{1 - e \cos E}(-e \sin \omega + \sin \omega \cos E + \sqrt{1 - e^2} \cos \omega \sin E),$$

$$\sin u = \sin \omega(-e + \sum_{j=0}^{+\infty} e^j (1 - e^2) \cos^{j+1} E) + \sqrt{1 - e^2} \cos \omega \sin E \sum_{j=0}^{+\infty} e^j \cos^j E,$$

$$\cos u = \sqrt{1 - e \cos E}(-e \cos \omega + \cos \omega \cos E - \sqrt{1 - e^2} \sin \omega \sin E),$$

$$\cos u = \cos\omega(-e + \sum_{j=0}^{+\infty} e^j(1-e^2)\cos^{j+1} E) - \sqrt{1-e^2}\sin\omega\sin E\sum_{j=0}^{+\infty} e^j\cos^j E.$$

The small parameter c_j (see Eq. (3.7)) allows us to develop the exponential function into a power series:

$$e^{c_j \cos 2u} = \sum_{n=0}^{+\infty} \frac{(c_j\cos 2u)^n}{n!}. \tag{3.11}$$

The expansion can be written in a summation notation to allow an easy multiplication of the series. An analytical form of the individual terms is difficult to achieve. Since formation of the terms is strightforward but very combersome, a computer algebra would be best suited for this purpose.

We have used the algebraic system REDUCE 2 (Hearn, 1973, 1978) on the IBM 370 computer under TSO operating system.

The samples of the programs are given in *App.B*. Now we can rewrite the basic equations for the density (3.10) as

$$\rho = \sum_{n=1}^{7} g_n K_{n0} + \sum_{n=1}^{7} g_n \sum_{j=1}^{3} K_{nj} A_j C_j e^{z_j \cos E}, \tag{3.12}$$

where C_j represents an expanding into power series

$$C_j = 1 + c_j\cos 2u + \frac{c_j^2}{2}\cos^2 2u + \frac{c_j^3}{3!}\cos^3 2u + o(c_j^4),$$

while the expressions for the terms $\cos^k 2u, k = 1,3$ can be seen in *App.B*, too. After expansion of all factors of the integrand in formula (3.10) as power series in e and E and their transformation we have

$$\Delta a = -a^2\delta k_0 f_0 f_x \sum_{\substack{n=1\\ n\neq 3}}^{5} (g_n K_{n0}\int_0^{2\pi} L_a \mathrm{dE} + \mathrm{g_n}\sum_{\mathrm{j}=1}^{3}\mathrm{K_{nj}A_j}\int_0^{2\pi}\mathrm{C_j L_a e^{z_j\cos E} dE})$$

$$+ \sum_{n=3,6,7}(K_{n0}\int_0^{2\pi} g_n L_a \mathrm{dE} + \sum_{\mathrm{j}=1}^{3}\mathrm{K_{nj}A_j}\int_0^{2\pi}\mathrm{g_n C_j L_a e^{z_j\cos E} dE}). \tag{3.13}$$

At this moment we must notate some of the properties of the Bessel functions. Namely, the solution of the differential equation

$$z^2\frac{\mathrm{d^2y}}{\mathrm{dz^2}} + z\frac{\mathrm{dy}}{\mathrm{dz}} - (z^2+u^2)y = 0.$$

is a Bessel modified function and has a series expansion

$$B_n(z) = \frac{\sum_{m=0}^{+\infty} (\frac{z}{2})^{n+2m}}{m!(n+m)!}$$

which is sometimes useful in practice when z is small. The following asymptotic expansion for $B_n(z)$ is available for large values of z :

$$B_n(z) = e^{\frac{z}{\sqrt{2\pi z}}}(1 + \sum_{l=2}^{+\infty} \prod_{m=1}^{l-1} (-\frac{4n^2 - (2m-1)^2}{8mz})).$$

It can be written as an integral in a form (see *App.B*)

$$B_n(z) = \frac{1}{2\pi} \int_0^{2\pi} e^{z \cos E} \cos nE \mathrm{dE}, \tag{3.14}$$

as it appears in our theory. What is about the terms with $\sin E$? These terms vanish after the integration over one revolution and the formula (3.13) has then a form such as in *App.B*.

4. CIRCULAR ORBITS

In this case $e = 0$ and

$$L_a = 1,$$

$$e^{z_j \cos E} = 1,$$

$$(\Delta a)_{e=0} = -a^2 \delta k_0 f_0 f_x (\sum_{\substack{n=1 \\ n\neq 3}}^{5} (g_n K_{n0} \int_0^{2\pi} \mathrm{dE} + g_n \sum_{j=1}^{3} KnjA_j \int_0^{2\pi} C_j \mathrm{dE}))$$

$$+ \sum_{n=3,6,7} (K_{n0} \int_0^{2\pi} g_n \mathrm{dE} + \sum_{j=1}^{3} K_{nj} A_j \int_0^{2\pi} C_j g_n \mathrm{dE}),$$

where

$$C_j = 1 - \frac{c_j}{6}(3 - 2c_j) + c_j^2(1 + c_j)(3\cos^2\omega - \cos^4\omega) \tag{4.1}$$

and g_3, g_6 have a multiplying factor $\cos E$ and are vanishing after an integration. The last term for $n = 7$ then will be the same as the others and

$$(\Delta a)_{e=0} = -2\pi a^2 \delta k_0 f_0 f_x \sum_{\substack{n=1\\ n\neq 3,6}}^{7} (g_n K_{n0} + \sum_{j=1}^{3} g_n K_{nj} A_j C_j), \tag{4.2}$$

5. COMPARISON WITH THE METHOD OF KING-HELE

The equation (3.7) for Δa after the expansion the integrands as a power series in e and utilizing an integral representation of the Bessel function becomes:

$$\Delta a = -2\pi\delta a^2 k e^{\frac{-a}{H}}$$

$$(B_0 + 2eB_1 + \frac{3}{4}e^2(B_0 + B_2) + \frac{1}{4}e^3(3B_1 + B_3)$$

$$+c(B_2 + 2eB_3 - \frac{1}{8}e^2(3B_0 + 2B_2 - 17B_4))\cos 2\omega$$

$$+\frac{c^2}{4}(B_0 + 2eB_1 + (B_4 - e(B_3 - 3B_5)\cos 4\omega) + o(e^4, ce^3, c^2e^2)). \tag{5.1}$$

From the equations (3.13) and (5.1) we can derive the expressions for perigee distance, orbital period and time. The coefficient k in equation (5.1) has a multiplying factor ρ_{p_0} which is unknown. We will determine it from some of the recently published models of the Earth's atmospheric density distribution (CIRA72,(1972); CIRA86,(Hedin, 1986); MSIS, (Hedin et al., 1977); DTM, (Barlier et al., 1977); C, (Köhnlein, 1980))

In this case for the circular orbits we have

$$(\delta a)_{e=0} = -2\pi\delta a^2 k e^{-\frac{a}{H}}(B_0 + cB_2\cos 2\omega + \frac{c^2}{4}(B_0 + B_4)\cos 4\omega).$$

Using a FORTRAN program (like DRAG in *App.B*) we computed for a sample of (different) heights using formulae (3.13) and (5.1) . We used for the balistic coefficient δ an average value

$$\delta = 2.2\frac{\mathit{effective\ cross-section}}{\mathit{mass}} \approx 0.156\frac{cm^2}{g}.$$

We choose two cases for formula (3.13) – expanding it into series with powers of small parameters, i.e. e and c: first – to the 4th order of them and second – to the 3rd order. Some results for the heights 200 km and 300 km can be seen in the tables T1 and T2 and on the *Fig*.3 and *Fig*.4.

It is clear that for different models of density distribution King–Hele's theory (1964) yield different curves $\Delta a = \Delta a(e), e \subset (0.0, 0.4)$ because the formula (5.1) has a multiplying factor

$$k = k(\rho_{p_0}),$$

$$\rho_{p_0} = \rho_{p_0}(model).$$

While the accuracy achieved seems good, the results (especially *Fig*.3 and *Fig*.4) indicate that more attention should be given to the problem of the definition of ρ_{p_0} and H (scale height), i.e. to the definition of the model density distribution.

For the changes of physical parameters ($Kp, t, ...$) values Δa are presented in the tables T3 , T4 and by *Fig*.5, 6,7 and 8 . We can see that our formula (2.13) is very sensitive to the changes in the solar flux and mean solar flux difference.

6. COMPARISON TO THE OBSERVATIONS OF INTERKOSMOS 10

The INTERKOSMOS 10 satellite (1973–82A) was launched on October 10, 1973, lived for 1340 days and its main purpose was to collect ionospheric data. We used NASA two–line orbital elements to compute changes in the orbital semi–major axis between February 14, 1975 and February 23, 1977.

We have all needed physical parameters too for the same time interval: solar flux, mean solar flux, geomagnetic index A_p (see *Fig*.9). The obtained changes of the semi–major axis are consequences of computation by two different ways: by using changes in the mean daily motion corrected for a radiation pressure and 1) from observed eccentricities or 2) from corrected eccentricities which are avaliable from the perigee parameter $Q = a(1-e)$, which is a smoothed perigee height.

The change of the perigee height was derived at first by correcting for gravitational perturbations (odd and even zonal harmonics, luni– solar effects) and for the solar radiation pressure effect (the third zonal harmonics effect is already subtracted in the published elements).

The values from computation by this method and by formula (3.13) are presented in the table T5 and in the *Fig*.10.

7. CONCLUSIONS

It can be seen from the analyses that the usage of a computer algebra manipulation give us posibilities to develop an effective analytical drag theory, especially for the satellites with small to moderate eccentricities.

The FORTRAN procedure (see *App.B*) is altered to an approximation of the changes of the semi–major axis in perigee altitude region where significant drag effects are experienced.

In the view of the fact that the formulas (3.13) and (5.1) were implemented on a computer, the formula (3.13) is more efficient because: it does not need a knowledge of a ρ_{p_0} and H; the algorithm is simple and straightforward; the required machine execution time is 5–10 times greater than in the usages of the CIRA72 (in the form CAD, Oliver,1980) , MSIS (Hedin et al., 1977), DTM (Barlier et al., 1977), C (Köhnlein, 1980) for computations of the densities and of the scale heights for the usage the King–Hele's theory (1964).

We checked the accuracy of the reduced model (to e^3, c^3, c^2e, e^2c) and, we did not find any substantial difference.

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APPENDIX A

Tables and Figures

Table T1: The perturbations of the satellite orbital semi-major axis under influence of a drag for different values of the eccentricities. The perigee height is 200 km. KH–King-Hele's theory, OT – the theory developed in this paper.The basic physical parameters are:

Geomagnetic index	$K_p = 2.500$
Solar flux	$F = 80.000$
Mean solar flux	$F_b = 80.000$
The balistic coefficient	$\delta = 0.183$
Local time	$LT = 0.000$
Equatorial radius of the Earth	$R_e = 6378.160$
Ellipticity of the Earth	$\varepsilon = 335 * 10^{-5}$

eccentricity	$\Delta a(KH)$ $\approx e^3$ [m]	$\Delta a(OT)$ $\approx e^3$ [m]
0.00	-427.2	-605.0
0.01		-174.8
0.02	- 83.3	-114.2
0.03		- 92.6
0.04		- 81.2
0.05	- 57.5	- 74.3
0.06		- 69.9
0.08		- 64.9
0.10	- 48.1	- 62.7
0.12		- 61.9
0.14		- 61.9
0.16	- 46.6	- 61.7
0.18		- 61.7
0.20		- 61.7
0.25	- 51.2	- 61.6
0.30		- 61.7
0.35	- 62.6	- 61.9
0.40		- 62.0

Table T2: The perturbations of the satellite orbital semi-major axis under influence of a drag for different values of the eccentricities. The perigee height is 300 km. KH–King-Hele's theory, OT – the theory developed in this paper.The basic physical parameters are:

Geomagnetic index	$K_p = 2.500$
Solar flux	$F = 80.000$
Mean solar flux	$F_b = 80.000$
The balistic coefficient	$\delta = 0.183$
Local time	$LT = 0.000$
Equatorial radius of the Earth	$R_e = 6378.160$
Ellipticity of the Earth	$\varepsilon = 335 * 10^{-5}$

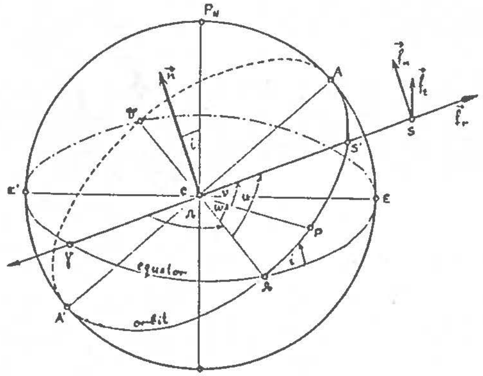
eccentricity	$\Delta a(KH)$ $\approx e^3$ [m]	$\Delta a(OT)$ $\approx e^2$ [m]	$\Delta a(OT)$ $\approx e^3$ [m]
0.00	-40.9	-53.7	-55.4
0.01	-13.8	-18.1	-19.2
0.02	-9.5	-10.8	-11.9
0.03	-7.9	-7.3	-8.4
0.04	-7.1	-5.4	-6.5
0.05	-6.5	-4.3	-5.3
0.06	-6.1	-3.5	-4.6
0.08	-5.7	-2.7	-3.7
0.10	-5.4	-2.2	-3.2
0.12	-5.3	-1.9	-2.9
0.14	-5.3	-1.7	-2.7
0.16	-5.3	-1.5	-2.6
0.18	-5.3	-1.4	-2.5
0.20	-5.4	-1.4	-2.4
0.25	-5.8	-1.3	-2.3
0.30	-6.3	-1.2	-2.2

Table T3: The perturbations of the satellite orbital semi-major axis under influence of a drag – variation with geomagnetic index K_p. KH–King-Hele's theory, OT– the theory developed in this paper.

LST	Δa_{OT}	Δa_{KH} 200km	ρ	Δa_{OT}	Δa_{KH} 250km	ρ	Δa_{OT}	Δa_{KH} 350km	ρ
0.0	-221.5	-101.4	223	-64.3	-38.3	66	-4.7	-5.5	8
0.5	-225.3	-104.4	224	-64.0	-39.4	68	-2.7	-5.6	8
1.0	-231.1	-107.3	234	-65.6	-40.6	70	-2.8	-5.8	8
1.5	-236.8	-110.3	240	-67.2	-41.7	72	-2.8	-6.0	8
2.5	-242.6	-113.3	246	-68.9	-42.8	73	-2.9	-6.2	9
3.0	-254.2	-119.4	258	-72.2	-45.1	77	-3.0	-6.5	9
3.5	-259.9	-122.4	263	-73.8	-46.3	79	-3.1	-6.9	9
4.0	-265.7	-125.5	269	-75.4	-47.4	80	-3.2	-6.9	9
4.5	-271.5	-128.7	275	-77.1	-48.6	82	-3.2	-7.1	10
5.0	-277.3	-131.9	281	-78.7	-49.8	84	-3.3	-7.3	10
5.5	-283.1	-135.2	287	-80.4	-51.1	86	-3.4	-7.5	10
6.0	-288.8	-138.7	293	-82.0	-52.4	87	-3.4	-7.7	10
6.5	-294.6	-142.5	299	-83.6	-53.8	89	-3.5	-7.9	11
7.0	-300.4	-146.7	304	-85.3	-55.4	91	-3.6	-8.2	11
7.5	-306.2	-151.6	310	-86.9	-57.3	93	-3.6	-8.5	11
8.0	-311.9	-157.6	316	-88.6	-59.6	94	-3.7	-8.9	11
8.5	-317.7	-165.1	322	-90.2	-62.6	96	-3.8	-9.5	11
9.0	-323.5	-174.8	328	-91.8	-66.4	98	-3.9	-10.2	12

Table T4: The perturbations of the satellite orbital semi-major axis under influence of a drag for different latitudes and altitudes. KH–King-Hele's theory, OT– the theory developed in this paper.

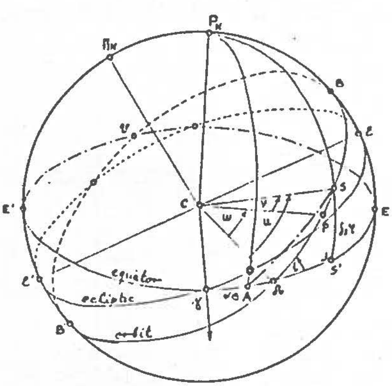
ϕ	Δa_{OT}	Δa_{KH} 200km	ρ	Δa_{OT}	Δa_{KH} 250km	ρ	Δa_{OT}	Δa_{KH} 350km	ρ
-85	-183.9	-109.2	237	-53.7	-41.7	70	-3.4	-6.0	8
-75	-187.6	-109.4	238	-53.2	-41.7	71	-1.3	-6.0	8
-65	-199.2	-109.5	238	-56.5	-41.7	71	-1.7	-6.0	8
-55	-216.0	-109.7	239	-61.3	-41.6	71	-2.2	-6.0	8
-45	-236.8	-110.3	240	-67.2	-41.7	72	-2.8	-6.0	8
-35	-259.7	-111.8	243	-73.7	-41.9	72	-3.5	-6.0	9
-25	-281.6	-114.3	247	-80.0	-42.5	74	-4.1	-6.1	9
-15	-299.0	-118.3	255	-84.9	-43.4	76	-4.6	-6.2	9
-5	-308.7	-123.8	265	-87.7	-44.8	78	-4.9	-6.4	9
5	-308.7	-130.8	278	-87.7	-46.7	81	-4.9	-6.7	10
15	-299.0	-139.1	293	-84.9	-48.9	85	-4.6	-7.0	10
25	-281.6	-148.5	311	-80.0	-51.4	90	-4.1	-7.3	11
35	-259.7	-158.3	329	-73.7	-54.0	94	-3.5	-7.7	11
45	-236.8	-168.0	346	-67.2	-56.6	98	-2.8	-8.1	12
55	-216.0	-176.8	362	-61.3	-59.0	102	-2.2	-8.4	12
65	-199.2	-184.1	374	-56.5	-61.0	105	-1.7	-8.7	12
75	-187.6	-189.3	383	-53.2	-62.3	107	-1.3	-8.9	13
85	-181.7	-191.9	387	-51.5	-63.0	108	-1.1	-9.0	13



F1

*Fig.*1
The perturbation forces acting on an satellite

$\vec{f_r}$ – *radial*
$\vec{f_n}$ – *normal*
$\vec{f_t}$ – *transverse*



F2

*Fig.*2
The stellite orbit on the celestial sphere

$\odot$ – *position of the Sun*
γ – *equinox*
Ω – *ascending node*
$\alpha_\odot$ – *right ascension of the Sun*
A – *projection of the Sun to the equator*
S – *position of the satellite*
P – *projection of the perigee*
ω, ν, u, i – *satellite orbital elements*

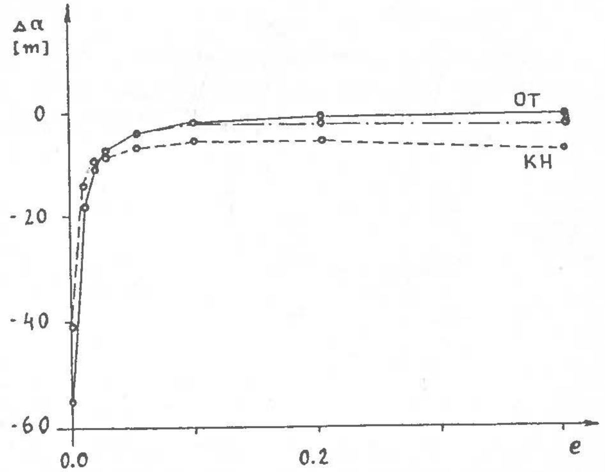


Fig.3
The perturbations of the satellite orbital semi-major axis under influence of a drag for different values of the eccentricities. The perigee height is 200 km. KH–King-Hele's theory, OT – the theory developed in this paper.

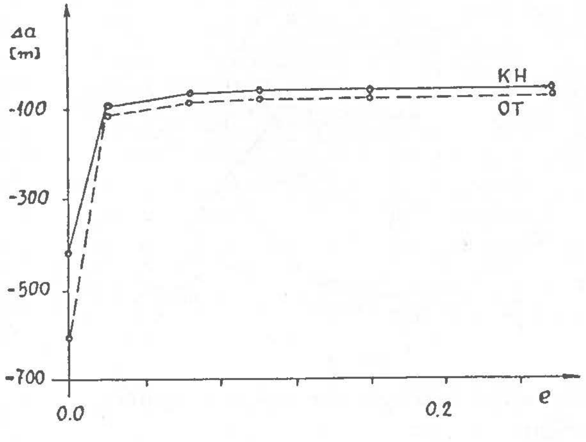


Fig.4
The perturbations of the satellite orbital semi-major axis under influence of a drag for different values of the eccentricities. The perigee height is 300 km. KH–King-Hele's theory, OT – the theory developed in this paper.

Table T5: The perturbations of the satellite orbital semi-major axis under influence of a drag for the satellite INTERKOSMOS 10. The mean perigee height is 273 km. I–observed values, II – the values from theory developed in this paper.

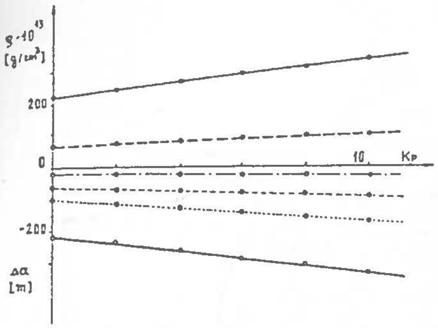
-3888.472	-2113.397	7086.41	0.064893	-3.3
-2208.915	-1988.023	7085.78	0.064770	-3.4
-3398.710	-1900.723	7085.27	0.064632	-4.3
-2524.254	-1865.912	7084.66	0.064625	-2.4
-2595.006	-1730.836	7084.28	0.064558	-3.3
-2692.333	-1868.670	7083.53	0.064456	-3.6
-2771.225	-1737.874	7082.86	0.064258	-2.4
-3038.167	-1581.679	7081.97	0.064116	-3.0
-2610.527	-1610.577	7081.52	0.064081	0.9
-2713.371	-1648.157	7079.25	0.063900	0.4
-3073.924	-1365.980	7078.35	0.063722	0.4
-2795.874	-1515.311	7077.61	0.063599	1.7
-2229.380	-1778.742	7077.20	0.063589	1.3
-3751.791	-1694.448	7076.77	0.063535	2.3
-4148.994	-1607.180	7075.87	0.063176	3.9
-2475.177	-1546.616	7075.25	0.062918	3.4
-2626.568	-1488.236	7074.80	0.062908	3.2
-2288.794	-1488.626	7073.39	0.062877	1.9
-2425.824	-1494.054	7073.04	0.062792	1.1
-2046.989	-1340.752	7072.61	0.062757	0.3
-1958.231	-1459.104	7072.30	0.062722	-3.4
-1471.756	-1503.262	7071.81	0.062767	-3.6
-2245.769	-1439.974	7071.59	0.062700	-3.6
-1892.972	-1582.113	7071.14	0.062589	-3.9
-2063.159	-1565.044	7070.59	0.062511	-2.5
-2056.626	-1488.371	7069.99	0.062424	-2.7
-1640.585	-1445.783	7069.54	0.062359	-0.7
-1625.791	-1513.469	7068.95	0.062308	0.9
-1783.409	-1634.199	7068.35	0.062310	1.7
-2312.697	-1693.406	7067.97	0.062205	0.5
-2310.479	-1464.023	7066.44	0.062023	-2.3
-1356.456	-1423.367	7064.62	0.061825	-4.0
-3474.791	-1582.253	7063.99	0.061741	-3.3
-2043.086	-1575.220	7063.47	0.061697	-3.0
-2798.929	-1578.681	7063.06	0.061653	-2.3
-2398.856	-1500.131	7062.50	0.061606	1.0
-2671.084	-1444.477	7061.50	0.061499	3.2
-2358.024	-1438.867	7061.10	0.061426	2.1
-2609.980	-1596.066	7060.04	0.061293	9.6
-3779.403	-1636.772	7059.07	0.061203	7.0
-2630.064	-1521.116	7058.18	0.061067	2.3
-3305.838	-1259.217	7057.41	0.061055	-1.2
-2842.132	-1533.100	7050.44	0.060984	-1.3
-2243.254	-1433.789	7055.58	0.060788	-2.2
-2517.874	-1433.343	7054.94	0.060711	-2.4
-2354.280	-1454.160	7054.51	0.060659	-1.9
-2684.537	-1476.545	7054.16	0.060512	-2.2

-2286.419	-1482.920	7053.71	0.060499	-1.4
-1526.247	-1529.066	7052.80	0.060348	-2.8
-2491.905	-1502.155	7052.07	0.060175	-1.5
-1840.722	-1532.924	7051.51	0.060033	-1.2
-1666.020	-1666.495	7051.13	0.060041	-1.0
-1134.904	-1834.843	7050.02	0.060017	0.5
-2194.818	-1720.581	7049.78	0.059938	-1.6
-1713.513	-1668.893	7043.30	0.059212	-2.0
-1413.457	-1477.567	7042.58	0.059122	-4.3
-1280.659	-1419.650	7042.33	0.059107	-5.0
-603.814	-1433.886	7042.05	0.059063	-4.9
-64.805	-1597.807	7041.61	0.058982	-4.5
-1976.395	-1469.873	7041.03	0.058891	-4.7
-985.115	-1603.962	7040.59	0.058846	-2.2
-1021.585	-1589.416	7040.33	0.058834	-3.2
-768.073	-1575.010	7039.88	0.058783	-1.9
-828.369	-1603.044	7039.25	0.058665	-1.8
-677.407	-1629.227	7039.01	0.058632	1.0
-715.926	-1790.735	7038.74	0.058614	4.4
-776.714	-1864.773	7038.34	0.058567	4.6
-1024.372	-1947.813	7038.14	0.058529	-1.2
-875.423	-2268.968	7037.93	0.058474	-7.5
-1369.864	-2311.920	7037.71	0.058462	-8.6
-1661.212	-2250.370	7037.39	0.058395	-9.1
-1153.738	-2490.456	7037.00	0.058324	5.4
-1153.048	-2527.335	7036.57	0.058284	0.2
-1183.630	-2509.103	7036.20	0.058198	-1.9
-1367.964	-2522.602	7035.85	0.058187	-0.7
-1149.927	-2594.960	7035.35	0.058154	-2.3
-1228.893	-2878.942	7034.86	0.058146	-4.9
-1575.992	-2786.307	7034.41	0.058041	-6.6
-1294.015	-2787.683	7033.89	0.057990	-8.7
-1172.142	-2768.815	7033.44	0.057862	-4.0
-1193.533	-3146.792	7032.68	0.057738	5.1
-1407.014	-3219.306	7032.32	0.057731	8.6
-1361.957	-3177.355	7032.02	0.057669	10.1
-1585.962	-3194.059	7031.68	0.057267	17.4
-1773.847	-3581.284	7031.37	0.057304	32.2
-1916.894	-3603.043	7030.92	0.057325	35.3
-1785.077	-3371.634	7030.28	0.056685	28.1
-2303.335	-3505.635	7029.81	0.056900	28.9
-2167.212	-3189.797	7029.26	0.056977	20.0
-2224.943	-3001.660	7028.80	0.057105	10.9
-2030.082	-3044.467	7027.91	0.057049	3.9
-2587.349	-2582.847	7026.85	0.057037	-3.9
-2991.470	-2558.621	7026.22	0.056960	-7.6
-1983.613	-2309.596	7025.62	0.056901	-9.5
-1788.491	-2567.627	7025.08	0.056827	-6.0
-2011.222	-2426.952	7024.61	0.056759	-7.5
-2222.400	-2282.115	7023.90	0.056646	-9.0
-2040.401	-2068.845	7022.68	0.056500	-8.8
-2032.443	-2125.333	7022.26	0.056435	-5.1
-1647.862	-1923.765	7020.34	0.056206	-2.2
-2312.294	-2153.711	7019.91	0.056137	-0.9
-2755.289	-1948.122	7019.42	0.056074	1.3
-2813.460	-1957.892	7019.00	0.055990	1.7
-2408.634	-1858.907	7018.27	0.055925	2.4

-2382.060	-2174.946	7017.51	0.055848	8.9
-2838.663	-2124.165	7017.19	0.055794	0.3
-3577.916	-2134.060	7016.77	0.055756	-1.1
-3134.672	-2232.312	7015.30	0.055580	-1.8
-2946.473	-2113.690	7014.73	0.055556	-6.6
-2472.828	-2009.158	7014.19	0.055608	-5.9
-2015.695	-2184.591	7013.75	0.055529	-3.2
-2318.177	-2114.561	7012.51	0.055297	-2.6
-2163.290	-2048.353	7011.97	0.055150	-1.8
-2139.771	-2466.510	7011.27	0.055080	-1.7
-2585.827	-2731.100	7009.52	0.054860	-2.5
-3020.788	-2742.936	7008.28	0.054784	-2.2
-2299.122	-2897.794	7007.45	0.054705	-1.6
-2064.807	-2748.396	7006.83	0.054618	0.2
-2057.238	-3168.142	7006.25	0.054546	0.5
-2482.032	-3379.514	7004.69	0.054268	-3.1
-3775.392	-3434.346	7003.98	0.054202	-3.5
-3613.986	-3402.800	7002.40	0.053996	-1.9
-2911.994	-3225.425	7001.38	0.053832	-2.9
-2354.239	-3104.206	7000.55	0.053752	3.1
-2690.929	-2895.674	6999.43	0.053592	3.3
-2312.138	-2860.096	6998.60	0.053383	1.7
-2350.106	-2652.788	6997.48	0.053271	-3.2
-2358.181	-2549.521	6996.86	0.053251	-5.1
-2287.126	-2611.588	6996.35	0.053170	-5.3
-2690.072	-2564.540	6995.70	0.053136	-6.7
-2744.136	-2596.441	6994.65	0.052989	-7.0
-2922.219	-2822.777	6994.29	0.052963	-1.3
-4147.073	-2734.654	6993.96	0.052910	2.3
-2565.095	-2647.618	6991.68	0.052564	2.4
-3371.977	-2444.068	6991.29	0.052519	2.3
-2745.134	-2243.539	6990.71	0.052485	3.1
-2531.766	-2426.822	6990.26	0.052439	11.9
-2769.400	-2435.134	6987.89	0.052144	15.4
-3081.875	-2512.081	6987.48	0.052076	11.4
-2913.415	-2608.423	6983.08	0.051519	6.4
-3195.628	-2237.628	6981.56	0.051319	2.1
-3537.469	-2257.557	6981.32	0.051242	-0.4
-2543.988	-2056.651	6979.98	0.051089	-4.3
-2419.198	-2146.275	6979.62	0.051080	-4.6
-2123.655	-2318.506	6978.77	0.050968	-4.1
-2954.707	-2218.275	6978.28	0.050874	-0.6
-2525.920	-2248.275	6977.80	0.050892	-0.8
-2515.736	-2197.881	6977.43	0.050816	2.6
-2537.411	-2047.638	6977.05	0.050818	-1.2
-2097.579	-2298.806	6976.62	0.050761	1.1
-2262.101	-2121.199	6976.16	0.050716	-3.3
-1928.714	-2061.634	6975.67	0.050595	-4.5
-2001.010	-1959.205	6975.19	0.050535	-5.6
-698.957	-1844.614	6974.76	0.050471	-4.8
-1782.046	-1972.063	6974.25	0.050405	-5.1
-1450.102	-1732.737	6973.81	0.050302	-1.9
-1345.581	-2002.797	6973.41	0.050268	-1.9
-1600.377	-2053.377	6972.76	0.050162	-0.8
-2012.188	-1808.218	6972.24	0.050080	0.3
-1622.119	-1718.780	6971.64	0.050041	-0.6
-1255.898	-1835.548	6969.70	0.049823	-0.5

-1878.780	-1610.985	6969.05	0.049689	-2.0
-1566.108	-1922.676	6968.49	0.049634	-0.8
-2557.692	-1794.581	6967.61	0.049443	9.3
-2379.937	-1841.401	6967.05	0.049363	11.5
-2232.204	-1803.194	6966.42	0.049297	5.9
-2423.713	-1761.699	6965.27	0.049149	6.1
-3163.162	-1751.400	6964.33	0.049011	4.6
-2467.947	-1895.217	6961.96	0.048669	4.1
-2483.403	-1987.780	6961.58	0.048677	1.5
-2807.339	-2029.572	6961.23	0.048623	0.2
-3119.607	-882.301	6992.21	0.048580	-0.5
-3253.551	-2049.995	6960.26	0.048445	-2.2
-3138.526	-2007.559	6959.45	0.048357	-2.4
-2830.910	-1932.334	6959.12	0.048307	-2.4
-2757.008	-2008.050	6958.18	0.048182	-3.5
-2642.036	-1998.702	6957.83	0.048142	-3.5
-2674.183	-2068.061	6957.03	0.048061	-3.2
-2773.181	-2324.678	6956.62	0.047957	-1.3
-3472.390	-2308.525	6952.61	0.047552	-1.8
-3948.463	-2279.845	6952.10	0.047506	-1.8
-3091.904	-2222.348	6951.22	0.047412	-1.9
-3327.478	-2065.158	6950.86	0.047313	-2.6
-3308.466	-2057.059	6949.99	0.047238	-2.0
-3069.993	-2002.558	6944.44	0.046222	-0.8
-3690.271	-2041.209	6943.60	0.046159	-1.7
-3740.132	-1998.880	6940.89	0.045831	-1.6
-3678.857	-1914.983	6940.26	0.045786	-1.7
-2963.813	-1963.981	6939.61	0.045725	-2.4
-2990.645	-1937.679	6938.28	0.045639	-1.2
-2551.047	-2089.030	6935.69	0.045353	-1.3
-2621.612	-1908.472	6935.11	0.045246	-2.4
-2502.509	-1898.833	6934.53	0.045184	-2.1
-1495.646	-2066.501	6934.00	0.045108	-3.5
-1562.912	-2272.862	6933.47	0.045100	-3.6
-2385.086	-2537.820	6930.81	0.044862	-4.9
-2431.904	-2566.758	6930.28	0.044730	-4.3
-2255.688	-2566.285	6929.87	0.044653	-5.0
-1587.094	-2518.206	6929.46	0.044590	-5.0

SUM OF PERTURBATIONS: OBSERVED= -151694.7 FROM THEORY = -136005.4



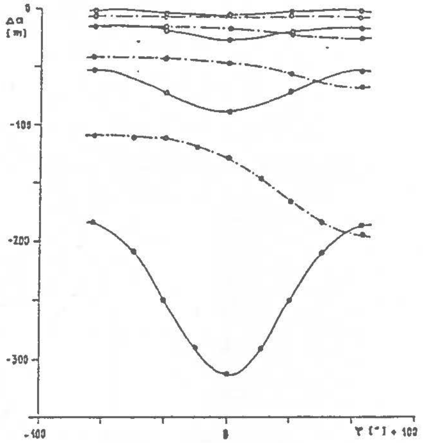
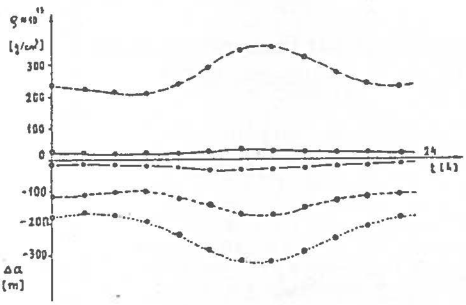


Fig.5

The perturbations of the satellite orbital semi-major axis under influence of a drag – variation with geomagnetic index K_p. KH–King-Hele's theory, OT– the theory developed in this paper; ρ – density

Fig.6

The perturbations of the satellite orbital semi-major axis under influence of a drag – latitudinal variation.



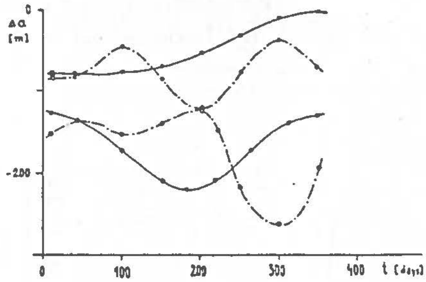


Fig.7

Daily variation of the perturbations of the satellite orbital semi-major axis under influence of a drag for different perigee distances.

Fig.8

Annual variation of the perturbations of the satellite orbital semi-major axis under influence of a drag for different perigee distances.

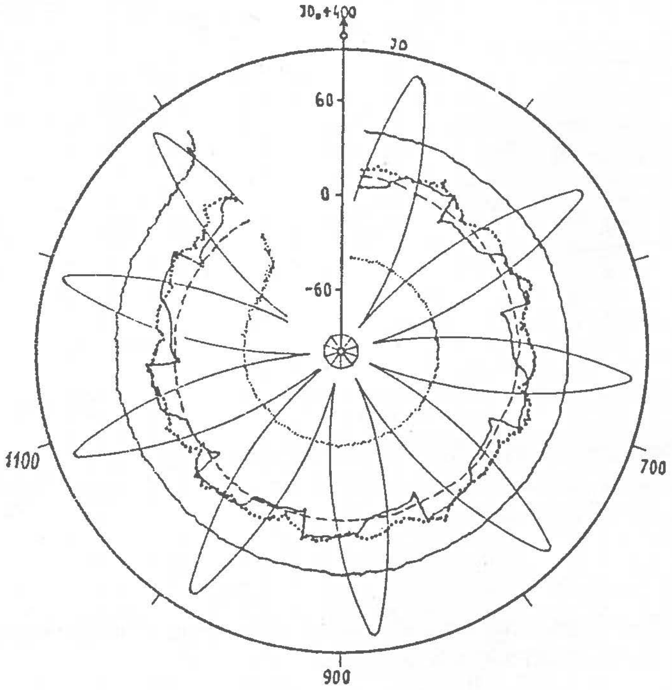


Fig.9
The physical parameters for the observed interval (solar flux, geomagnetic index, latitude, local solar time ...): February, 1975 to February, 1977.

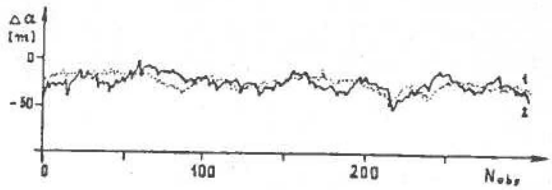


Fig.10
The perturbations of the satellite orbital semi-major axis under influence of a drag for the satellite INTERKOSMOS 10. The mean perigee height is 273 km. 1 – the values from theory developed in this paper, 2–observed values.

APPENDIX B

FORTRAN Program DRAG

```
C     ............................D.R.A.G.............................
C     PROGRAM FOR THE CALCULATION ATMOSPHERIC DRAG EFFECTS ANALYTICALY..
C     INPUT:............................................................
C      1)      SATELLITE    DATA                             UNITS:
C              - AX.......SEMI-MAJOR AXIS...................KM...........
C              - ANAG.....INCLINATION OF THE ORBIT..........DEG..........
C              - AN.......RIGHT ASCENSION OF THE NODE.......DEG..........
C              - W........ARGUMENT OF THE PERIGEE...........DEG..........
C              - EC.......ECCENTRICITY.......................   ..........
C              - AM.......MEAN ANOMALY......................DEG..........
C              - DMM......DAILY MEAN MOTION.................R/D..........
C              - FI.......GEOCENTRIC LATITUDE...............DEG..........
C              - Q........PERIGEE PARAMETER.................KM...........
C              - ADFPUM...ATMOSPHERIC DRAG FORCE PER UNIT MASS...........
C      2)      TIME AND PHYSICAL PARAMETERS
C              - YEAR.....YEAR IN THE POINT OF QUESTION.....YEAR AC......
C              - DAY......DAY IN THE YEAR...................DAYS.........
C              - F........SOLAR FLUX........................E-22W/M2(S/C)
C              - FB.......MEAN SOLAR FLUX...................            .
C              - AP.......GEOMAGNETIC INDEX KP...........................
C              - ST.......LOCAL TIME........................HOURS........
C              - AS.......RIGHT ASCENSION OF THE SUN........HOURS........
C      3)      TOTAL DENSITY MODEL PARAMETERS (CONSTANTES AND RELATIONS)
C              - HN ......................
C              - GN ......................
C              - P3,P4,P5,P6,P7 ..........
C              - AA, BB, B1,B2,B3,B4,B5 ....
C              - AK(7,4) .................
C              - AK0 .....................
C              - F0, FM, FX ..............
C      - BP(I),I=1,9 ARE THE BESSEL POLINOMES OF THE ORDER I; 0=BP(0)..
C     OUTPUT............................................................
C      1)      - SUMAHG...CHANGE OF THE SEMI-MAJOR AXIS.....CM...........
C     ..................................................................
      SUBROUTINE DRAG(SUMAHG)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION BP(10),CW(10),GN(5)
      COMMON /CHANGE/ EC,EC2,EC3,CJ,CJ2,CJ3,CINC,SINC,CW,SW,SDP3
      COMMON /BIND/ AA,BB,B1,B2,B3,B4,B5
      COMMON /CONST/ DTOR,RTOD,PI,TWOPI
      COMMON /AKON/ AK(7,4)
      DATA P3/263.D0/,P4/-263.D0/,P5/-29.41D0/,P6/8.0913D0/,P7/10.0813
     . D0/
  900 FORMAT(................)
  901 FORMAT(................)
      CALL CONS
      READ(5,*) IORDER, ADFPUM
COMMENT IORDER=<2 FOR THE CALCULATION ONLY TO EC**2,CJ**2
      READ(8,900) AX,ANAG,AN,W,FI,EC
      READ(8,901) DAY,F,FB,AP,ST,AS
      P3= P3*TWOPI/365.D0
      P4= P4*TWOPI/365.D0
      P5= P5*TWOPI/365.D0
      P6= P6*TWOPI/24.D0
      P7= P7*TWOPI/24.D0
      FI=FI*DTOR
      DAY=DAY*TWOPI/365.
      W=W*DTOR
```

```
      ANAG=ANAG*DTOR
      AN=AN*DTOR
      AS=AS*15.D0*DTOR
      AK0=1.D0+.04762D0*(AP-3.D0)
      FM=(FB-60.D0)/160.D0
      F0=0.2875D0+FM
      FX=1.D0+.007D0*(F-FB)
        CINC=DCOS(ANAG)
        SINC=DSIN(ANAG)
        ANAS2=2.D0*(AN-AS)
        P72=2.D0*P7
        SDP3=DSIN(DAY-P3)
        EPSINC=.00335*SINC*SINC/2.D0
      SW=DSIN(W)
      CW(1)=DCOS(W)
       DO 9 I=1,8
    9  CW(I+1)=CW(I)*CW(1)
      AA=(FM/3.D0+1.D0)*DSIN(AN-AS-P6)
      BB=(FM/3.D0+1.D0)*CINC*DCOS(AN-AS+P6)
      CB=15.D0*FM+1.D0
      B1=CB*DCOS(P72)*DSIN(ANAS2)
      B2=CB*DCOS(P72)*DSIN(ANAS2)*CINC**2
      B3=2.D0*CB*DCOS(P72)*DCOS(ANAS2)*CINC
      B4=-CB*DSIN(P72)
      B5=-CB*DSIN(P72)*CINC**2
      GN(1)=1.
      GN(2)=FM+471.D0/10000.D0
      GN(5)=DSIN(2.*(DAY-P5))*(7.*FM+1.D0)
      GN(4)=(7.*FM+1.D0)*DSIN(DAY-P4)
      EC2=EC*EC
      EC3=EC*EC2
      PARMC=EPSINC*AX*(1.D0-EC)
      SUMAHG=-TWOPI*AX*AX*ADFPUM*AK0*F0*FX*1.D7
      SUMAA=0.D0
      SUMAB=0.D0
      G3L=SW*SDP3*SINC*EC/2.D0
      G6L=(AA*CW(1)+BB*SW)*EC/2.D0
      G7L=(B1-B2+B4+B5)/2.+3./16.D0*EC2*(3.*B1+5.*B2+3.*B4-5.*B5-
     . 8.*CW(2)*(B1+B2+B4-B5)-8.*B3*SW*CW(1))
      DO 10 I=1,5
      IF(I.EQ.3) GO TO 10
      SUMAA=SUMAA+GN(I)*(1.D0+3.*EC2/4.D0)*AK(I,1)
   10 CONTINUE
      SUMAA=SUMAA+AK(3,1)*G3L+AK(6,1)*G6L+AK(7,1)*G7L
      DO 20 J=1,3
      HEIGHT=40.D0*J
      CJ=PARMC/HEIGHT
      CJ2=CJ*CJ
      CJ3=CJ*CJ2
      ARA=(120.D0+6378.16*(1.D0-EPSINC)-AX)/HEIGHT
      AJ=DEXP(ARA)
      ARZ=AX*EC/HEIGHT
      CALL BPL(ARZ,BP,0)
      CALL GCHL(HNG3L,HNG6L,HNG7L,HNG15L,BP,0,G3L,G6L,G7L,IORDER)
      DO 30 I=1,5
      IF(I.EQ.3) GO TO 30
      SUMAB=SUMAB+GN(I)*AK(I,J+1)*AJ*HNG15L
   30 CONTINUE
```

```
      SUMAB=SUMAB+AJ*(AK(3,J+1)*HNG3L+AK(6,J+1)*HNG6L+AK(7,J+1)*HNG7L)
 20   CONTINUE
      SUMAHG=SUMAHG*(SUMAA+SUMAB)
      WRITE(6,*) SUMAHG
 88   CONTINUE
      RETURN
      END
        SUBROUTINE CONS
        IMPLICIT REAL*8(A-H,O-Z)
        COMMON /CONST/ DTOR,RTOD,PI,TWOPI
        PI=3.141592654D0
        TWOPI=2.D0*PI
        DTOR=PI/180.D0
        RTOD=180.D0/PI
        RETURN
        END
      SUBROUTINE GCHL(HNG3L,HNG6L,HNG7L,HNG15L,BP,O,G3L,G6L,G7L,IORDER)
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION BP(10),CW(10)
      COMMON /CHANGE/ EC,EC2,EC3,CJ,CJ2,CJ3,CINC,SINC,CW,SW,SDP3
      COMMON /BIND/ AA,BB,B1,B2,B3,B4,B5
      ANS1=SDP3*EC2*SINC*SW*(3/8.*BP(1)+9./8.*BP(3))+SDP3*SINC*SW*BP
     . (1)
      HNG3L=CJ*EC*SDP3*SINC*SW*CW(2)*(-BP(2)+5.*BP(4))+CJ*EC*SDP3*
     . SINC*SW*(-1./2.*BP(2)-5./4.*BP(4)-1./4.*O)+2.*CJ*SDP3*
     . SINC*SW*CW(2)*BP(3)+CJ*SDP3*SINC*SW*(-1./2.*BP(1)-1./2.*BP(3))
     . +EC*SDP3*SINC*SW*(3/2.*BP(2)+1./2.*O)
     . +SDP3*CJ2*SINC*SW*CW(2)*(5*BP(1)+2.*BP
     . (3)-BP(5))+SDP3*CJ2*SINC*SW*CW(4)*(-BP(1)-1./2.*BP(3)+3./2.
     . *BP(5))+SDP3*CJ2*SINC*SW*(-BP(1)-1./2.*BP(3))+ANS1
      ANS1=AA*CW(1)*BP(1)+BB*CJ*EC*SW*CW(2)*(-BP(2)+5.*BP(4))+BB*
     . CJ*EC*SW*(-1./2.*BP(2)-5./4.*BP(4)-1./4.*O)
     . +2.*BB*CJ*SW*CW(2)*BP(3)+BB*CJ*SW*(-1./2.*BP(1)-1./2.*BP(3))
     . +BB*EC*SW*(3/2.*BP(2)+1./2.*O)+BB*
     . CJ2*SW*CW(2)*(5*BP(1)+2.*BP(3)-BP(5))+BB*CJ2*SW*CW(4)*(-BP
     . (1)-1./2.*BP(3)+3./2.*BP(5))+BB*CJ2*SW*(-BP(1)-1./2.*BP(3))
     . +BB*EC2*SW*(3/8.*BP(1)+9./8.*BP(3))+BB*SW*BP(1)
      HNG6L=AA*CJ*EC*CW(1)*(3/2.*BP(2)-15./4.*BP(4)+1./4.*O)+AA*CJ*EC
     . *CW(3)*(-BP(2)+5.*BP(4))+AA*CJ*CW(1)*(1./2.*BP(1)-3./2.*BP(3))+
     . 2.*AA*CJ*CW(3)*BP(3)+AA*EC*CW(1)*(3./2.*BP(2)+1./2.*O)+
     . +AA*CJ2*CW(3)*(5*BP(1)+3.*BP(3)-2.*BP(5))+AA*
     . CJ2*CW(5)*(-BP(1)-1./2.*BP(3)+3./2.*BP(5))+AA*EC2*CW(1)*(3
     . /8.*BP(1)+9./8.*BP(3))+ANS1
      ANS6=-2.*B5*EC*CW(2)*BP(3)+B5*EC*(BP(1)+BP(3))
     . +B5*CJ2*CW(2)*(17./4.*BP(2)
     . +BP(4)-3./4.*BP(6)+3.*O)+B5*CJ2*CW(4)*(-17./4.*BP(2)-3./2.*BP
     . (4)+9./4.*BP(6)-5./2.*O)+B5*CJ2*CW(6)*(3/2.*BP(2)-3./2.*BP(6
     . ))+B5*CJ2*(-3./4.*BP(2)-1./4.*BP(4)-1./2.*O)+B5*EC2*CW(2)*(7/
     . 4.*BP(2)-7./4.*BP(4)+3./2.*O)+B5*EC2*(-5./4.*BP(2)+11./16.*BP(4
     . )-15./16.*O)-B5*CW(2)*BP(2)+B5*(1/2.*BP(2)+1./2.*O)
      ANS5=B4*CJ2*CW(2)*(-1./4.*
     . BP(2)-2.*BP(4)+3./4.*BP(6))+B4*CJ2*CW(4)*(17./4.*BP(2)+5./
     . 2.*BP(4)-9./4.*BP(6)+3./2.*O)+B4*CJ2*CW(6)*(-3./2.*BP(2)+3./2.*
     . BP(6))+B4*CJ2*(-1./4.*BP(2)+1./4.*BP(4))+B4*EC2*CW(2)*(-7./
     . 4.*BP(2)+7./4.*BP(4)-3./2.*O)+B4*EC2*(1/2.*BP(2)-17./16.*BP(4)
     . +9./16.*O)+
     . B4*CW(2)*BP(2)+B4*(-1./2.*BP(2)+1./2.*O)+6.*B5*CJ*EC*
     . CW(2)*BP(5)+B5*CJ*EC*CW(4)*(2*BP(3)-6.*BP(5))+B5*CJ*EC*(-
```

```
. 1./2.*BP(1)-3./4.*BP(3)-3./4.*BP(5))+B5*CJ*CW(2)*(BP(2)+2.*BP(4
. ))-2.*B5*CJ*CW(4)*BP(4)+B5*CJ*(-1./2.*BP(2)-1./4.*BP(4)-1./4.*O)
. +ANS6
 ANS4=B4*CJ*EC*CW(4)*(-2.*BP(3)+6.*BP(5))+B4*CJ*EC*(1/2.*BP(1
. )-5./4.*BP(3)+3./4.*BP(5))+B4*CJ*CW(2)*(BP(2)-2.*BP(4))+2.*
. B4*CJ*CW(4)*BP(4)+B4*CJ*(-1./2.*BP(2)+1./4.*BP(4)+1./4.*O)
. +2.*B4*EC*CW(2)*BP(3)+B4*EC*(BP(1)-BP(3))+ANS5
 ANS3=-B3*CJ*SW*CW(1)*BP(4)+2.*B3*CJ*SW*CW(3)*BP(4)+2.*B3*EC*SW*
. CW(1)*BP(3)+B3*CJ2*SW*CW(1)*(-3./4.*BP(2)-1./2.*BP(4)
. +1./4.*BP(6)-1./2.*O)+B3*CJ2*SW*CW(3)*(7/2.*BP(2)+2.*BP(4)-
. 3./2.*BP(6)+2.*O)+B3*CJ2*SW*CW(5)*(-3./2.*BP(2)+3./2.*BP(6))+B3
. *EC2*SW*CW(1)*(-7./4.*BP(2)+7./4.*BP(4)-3./2.*O)+B3*SW*CW(1)*
. BP(2)+B4*CJ*EC*CW(2)*(4*BP(3)-6.*BP(5))+ANS4
 ANS2=2.*B2*EC*CW(2)*BP(3)+B2*EC*(-BP(1)-BP(3))
. +B2*CJ2*CW(2)*(-17./4.*BP(2)
. -BP(4)+3./4.*BP(6)-3.*O)+B2*CJ2*CW(4)*(17./4.*BP(2)+3./2.*BP
. (4)-9./4.*BP(6)+5./2.*O)+B2*CJ2*CW(6)*(-3./2.*BP(2)+3./2.*BP(
. 6))+B2*CJ2*(3/4.*BP(2)+1./4.*BP(4)+1./2.*O)+B2*EC2*CW(2)*(-7./
. 4.*BP(2)+7./4.*BP(4)-3./2.*O)+B2*EC2*(5/4.*BP(2)-11./16.*BP(4)
. +15./16.*O)
. +B2*CW(2)*BP(2)+B2*(-1./2.*BP(2)-1./2.*O)+B3*CJ*EC*SW*
. CW(1)*(BP(3)-3.*BP(5))+B3*CJ*EC*SW*CW(3)*(-2.*BP(3)+6.*BP(
. 5))+ANS3
 ANS1=B1*CJ2*CW(2)*(-1./4.*
. BP(2)-2.*BP(4)+3./4.*BP(6))+B1*CJ2*CW(4)*(17./4.*BP(2)+5./
. 2.*BP(4)-9./4.*BP(6)+3./2.*O)+B1*CJ2*CW(6)*(-3./2.*BP(2)+3./2.*
. BP(6))+B1*CJ2*(-1./4.*BP(2)+1./4.*BP(4))+B1*EC2*CW(2)*(-7./
. 4.*BP(2)+7./4.*BP(4)-3./2.*O)+B1*EC2*(1/2.*BP(2)-17./16.*BP(4)
. +9./16.*O)
. +B1*CW(2)*BP(2)+B1*(-1./2.*BP(2)+1./2.*O)-6.*B2*CJ*EC*
. CW(2)*BP(5)+B2*CJ*EC*CW(4)*(-2.*BP(3)+6.*BP(5))+B2*CJ*EC*(
. 1./2.*BP(1)+3./4.*BP(3)+3./4.*BP(5))+B2*CJ*CW(2)*(-BP
. (2)-2.*BP(4))+2.*B2*CJ*CW(4)*BP(4)+B2*CJ*(1/2.*BP(2)+1./4.
. *BP(4)+1./4.*O)+ANS2
 HNG7L=B1*CJ*EC*CW(2)*(4*BP(3)-6.*BP(5))+B1*CJ*EC*CW(4)*(-2.*
. BP(3)+6.*BP(5))+B1*CJ*EC*(1/2.*BP(1)-5./4.*BP(3)+3./4.*BP(5))
. +B1*CJ*CW(2)*(BP(2)-2.*BP(4))+2.*B1*CJ*CW(4)*BP(4)+B1*CJ
. *(-1./2.*BP(2)+1./4.*BP(4)+1./4.*O)
. +2.*B1*EC*CW(2)*BP(3)+B1*EC*(BP(1)-BP(3))+ANS1
 HNG15L=4.*CJ*EC*CW(2)*BP(3)-2.*CJ*EC*BP(3)+2.*CJ*CW(2)*BP(2)-
. CJ*BP(2)+2.*EC*BP(1)+CJ2*CW(2)*(4.*BP(2)-BP(4)+3.*O)+CJ2*CW(4)
. *(BP(4)-1.*O)+CJ2*(-BP(2)-1./2.*O)+EC2*(3/4.*BP(2)+3./4.*O)+O
 IF(IORDER.LE.2) RETURN
 G3L=G3L+SW*SDP3*SINC*(-9.D0*EC3/8.D0)/2.D0
 G6L=G6L+(AA*CW(1)+BB*SW)*(-9.D0*EC3/8.D0)/2.D0
 HNG3L=HNG3L+CJ*SDP3*EC2*SINC*SW*
. CW(2)*(-9./2.*BP(1)-39./8.*BP(3)+51./8.*BP(5))+CJ*SDP3*EC2*
. SINC*SW*(15./8.*BP(1)+9./8.*BP(3)-3./2.*BP(5))
. +EC*SDP3*CJ2*SINC*SW*CW(2)*(43./2.*BP(2)+25./2.*BP(4)-5./2.
. *BP(6)+25./2.*O)+EC*SDP3*CJ2*SINC*SW*CW(4)*(3/4.*BP(2)-15./2.*
. BP(4)+13./4.*BP(6)+7./2.*O)+EC*SDP3*CJ2*SINC*SW*(-21./4.*BP(2)-
. 9./4.*BP(4)-7./2.*O)+SDP3*CJ3
. *SINC*SW*CW(2)*(5*BP(1)+BP(3)-BP(5))+SDP3*CJ3*SINC*SW*CW(4
. )*(-7./2.*BP(1)+BP(3)+8./3.*BP(5)-1./6.*BP(7))+SDP3*CJ3*SINC
. *SW*CW(6)*(1/2.*BP(1)-1./2.*BP(3)-1./6.*BP(5)+1./6.*BP(7))
. +SDP3*CJ3*SINC*SW*(-7./12.*BP(1)-1./4.*BP(3))+SDP3*EC3*SINC*SW
. *(9./16.*BP(4)-9./16.*O)
 ANS1=BB*CJ*EC2*SW*CW(2)*
```

```
.  (-9./2.*BP(1)-39./8.*BP(3)+51./8.*BP(5))+BB*CJ*EC2*SW*(15./8.
.  *BP(1)+9./8.*BP(3)-3./2.*BP(5))+BB*EC*CJ2*SW*CW(2)*(43./2.*
.  BP(2)+25./2.*BP(4)-5./2.*BP(6)+25./2.*O)+BB*EC*CJ2*SW*CW(4)*(
.  3./4.*BP(2)-15./2.*BP(4)+13./4.*BP(6)+7./2.*O)+BB*EC*CJ2*SW*(-
.  21./4.*BP(2)-9./4.*BP(4)-7./2.*O)
.  BB*CJ3*SW*CW(2)*(5*BP(1)+BP(3)-BP(5))+BB*CJ3*SW*CW(4)*(-7.
.  /2.*BP(1)+BP(3)+8./3.*BP(5)-1./6.*BP(7))+BB*CJ3*SW*CW(6)*(
.  1./2.*BP(1)-1./2.*BP(3)-1./6.*BP(5)+1./6.*BP(7))+BB*CJ3*SW*(-
.  7./12.*BP(1)-1./4.*BP(3))+BB*EC3*SW*(9./16.*BP(4)-9./16.*O)
 HNG6L=HNG6L+AA*CJ*EC2*CW(1)*(21./8.*BP(1)+15./
.  4.*BP(3)-39./8.*BP(5))+AA*CJ*EC2*CW(3)*(-9./2.*BP(1)-39./8.*
.  BP(3)+51./8.*BP(5))+AA*EC*CJ2*CW(1)*(-4.*BP(2)-25./4.*BP(4)
.  +3./4.*BP(6)-3./2.*O)+AA*EC*CJ2*CW(3)*(19.*BP(2)+41./2.*BP(4)-
.  4.*BP(6)+17./2.*O)+AA*EC*CJ2*CW(5)*(3/4.*BP(2)-15./2.*BP(4)+
.  13./4.*BP(6)+7./2.*O)+AA*CJ3*CW(1)*
.  (-13./12.*BP(1)-1./4.*BP(3)+1./2.*BP(5))+AA*CJ3*CW(3)*(
.  15./2.*BP(1)+1./2.*BP(3)-19./6.*BP(5)+1./6.*BP(7))+AA*CJ3*CW(
.  5)*(-4.*BP(1)+3./2.*BP(3)+17./6.*BP(5)-1./3.*BP(7))+AA*CJ3*
.  CW(7)*(1/2.*BP(1)-1./2.*BP(3)-1./6.*BP(5)+1./6.*BP(7))+AA*
.  EC3*CW(1)*(9./16.*BP(4)-9./16.*O)+AA*CJ2*CW(1)*(-BP(1)-BP(3)+
.  1./2.*BP(5))+ANS1
 ANS6=B5*EC*CJ2*CW(4)*(-6.*BP(1)-22.*BP(3)-22.*BP(5)+6.*BP(7)
.  )+B5*EC*CJ2*CW(6)*(-10.*BP(1)+5.*BP(3)+9.*BP(5)-4.*BP(7))+
.  B5*EC*CJ2*(-6.*BP(1)-15./4.*BP(3)-5./4.*BP(5))
.  +B5*CJ3*CW(2)*(43./12.*BP(2)-3./4.*BP(6)+3.*O)
.  +B5*CJ3*CW(8)*(2/3.*BP(4)-1./6.*BP(8)-1./2.*O)+
.  B5*CJ3*CW(4)*(-21./4.*BP(2)+13./6.*BP(4)+13./4.*BP(6)-1./6.*
.  BP(8)-5.*O)+B5*CJ3*CW(6)*(5/2.*BP(2)-10./3.*BP(4)-5./2.*BP(6
.  )+1./3.*BP(8)+3.*O)+B5*CJ3*(-5./12.*BP(2)-1./8.*BP(4)-7./24.*O)
.  +B5*EC3*CW(2)*(7*BP(1)+25./8.*BP(3)-9./8.*BP(5))+B5*EC3*(-
.  25./4.*BP(1)-3.*BP(3)+1./4.*BP(5))
 ANS5=B4*EC3*CW(2)*(-7.*BP(1)-25./8.*BP(3)+9./8.*BP(5))+B4*EC3
.  *(3/4.*BP(1)+1./8.*BP(3)-7./8.*BP(5))+B5*CJ*EC2*CW(2)*(-137./
.  16.*BP(2)-17./2.*BP(4)+141./16.*BP(6)-21./4.*O)+B5*CJ*EC2*CW(4)
.  *(11./2.*BP(2)+19./2.*BP(4)-9.*BP(6)+3.*O)+B5*CJ*EC2*(43./16.*
.  BP(2)+17./16.*BP(4)-15./16.*BP(6)+27./16.*O)+B5*EC*CJ2*CW(2)*
.  (25.*BP(1)+43./2.*BP(3)+21./2.*BP(5)-2.*BP(7))+ANS6
 ANS4=B4*CJ*EC2*CW(2)*(39./16.*BP(2)+21./2.*
.  BP(4)-147./16.*BP(6)+3./4.*O)+B4*CJ*EC2*CW(4)*(-11./2.*BP(2)
.  -19./2.*BP(4)+9.*BP(6)-3.*O)+B4*CJ*EC2*(3/8.*BP(2)-33./16.
.  *BP(4)+9./8.*BP(6)+9./16.*O)+B4*
.  EC*CJ2*CW(2)*(5*BP(1)-9./2.*BP(3)-27./2.*BP(5)+2.*BP(7))+
.  B4*EC*CJ2*CW(4)*(4*BP(1)+21.*BP(3)+25.*BP(5)-6.*BP(7))+B4*
.  EC*CJ2*CW(6)*(10.*BP(1)-5.*BP(3)-9.*BP(5)+4.*BP(7))+B4*EC*
.  CJ2*(-BP(1)-1./4.*BP(3)+5./4.*BP(5))
.  +B4*CJ3*CW(2)*(-7./12.*BP(2)-BP(4)+3./4.
.  *BP(6))+B4*CJ3*CW(8)*(-2./3.*BP(4)+1./6.*BP(8)+1./2.*O)+B4*
.  CJ3*CW(4)*(21./4.*BP(2)-7./6.*BP(4)-13./4.*BP(6)+1./6.*BP(8
.  )+4.*O)+B4*CJ3*CW(6)*(-5./2.*BP(2)+10./3.*BP(4)+5./2.*BP(6)-1.
.  /3.*BP(8)-3.*O)+B4*CJ3*(-1./12.*BP(2)+1./8.*BP(4)-1./24.*O)
.  +ANS5
 ANS3=B3*CJ*EC2*SW*CW(1)*(11./4.*BP(2)+19./4.*BP(4)-9./2.*BP(6
.  )+3./2.*O)+B3*CJ*EC2*SW*CW(3)*(-11./2.*BP(2)-19./2.*BP(4)+9.*BP
.  (6)-3.*O)+B3*EC*
.  CJ2*SW*CW(1)*(-5./2.*BP(1)-9./2.*BP(3)-9./2.*BP(5)+1./2.*BP(
.  7))+B3*EC*CJ2*SW*CW(3)*(10.*BP(1)+19.*BP(3)+19.*BP(5)-4.*BP(
.  7))+B3*EC*CJ2*SW*CW(5)*(10.*BP(1)-5.*BP(3)-9.*BP(5)+4.*BP(7
.  ))+B3*CJ3*SW*CW(1)*(-7./12.*BP(2)+1.
```

```
. /4.*BP(6)-1./2.*O)+B3*CJ3*SW*CW(3)*(4*BP(2)-1./3.*BP(4)-2.*BP
. (6)+1./12.*BP(8)+13./4.*O)+B3*CJ3*SW*CW(5)*(-5./2.*BP(2)+3.*BP
. (4)+5./2.*BP(6)-1./4.*BP(8)-11./4.*O)+B3*CJ3*SW*CW(7)*(-2./3.*
. BP(4)+1./6.*BP(8)+1./2.*O)+B3*EC3*SW*CW(1)*(-7.*BP(1)-25./8.*
. BP(3)+9./8.*BP(5))+ANS4
 ANS2=B2*EC*CJ2*CW(6)*(10.*BP(1)-5.*BP(3)-9.*BP(5)+4.*BP(7))+
. B2*EC*CJ2*(6*BP(1)+15./4.*BP(3)+5./4.*BP(5))
. +B2*CJ3*CW(2)*(-43./12.*BP(2)+3.
. /4.*BP(6)-3.*O)+B2*CJ3*CW(8)*(-2./3.*BP(4)+1./6.*BP(8)+1./
. 2.*O)+B2*CJ3*CW(4)*(21./4.*BP(2)-13./6.*BP(4)-13./4.*BP(6)+1./
. 6.*BP(8)+5.*O)+B2*CJ3*CW(6)*(-5./2.*BP(2)+10./3.*BP(4)+5./2.*BP
. (6)-1./3.*BP(8)-3.*O)+B2*CJ3*(5/12.*BP(2)+1./8.*BP(4)+7./24.*O)
. +B2*EC3*CW(2)*(-7.*BP(1)-25./8.*BP(3)+9./8.*BP(5))+B2*EC3*(
. 25./4.*BP(1)+3.*BP(3)-1./4.*BP(5))+ANS3
 ANS1=B1*EC3*CW(2)*(-7.*BP(1)-25./8.*BP(3)+9./8.*BP(5))+B1*EC3
. *(3/4.*BP(1)+1./8.*BP(3)-7./8.*BP(5))+B2*CJ*EC2*CW(2)*(137./
. 16.*BP(2)+17./2.*BP(4)-141./16.*BP(6)+21./4.*O)+B2*CJ*EC2*CW(4)
. *(-11./2.*BP(2)-19./2.*BP(4)+9.*BP(6)-3.*O)+B2*CJ*EC2*(-43./16.
. *BP(2)-17./16.*BP(4)+15./16.*BP(6)-27./16.*O)
. +B2*EC*CJ2*CW(2)*(-25.*BP(1)-43./2.*BP(3)-21./
. 2.*BP(5)+2.*BP(7))+B2*EC*CJ2*CW(4)*(6*BP(1)+22.*BP(3)+22.*
. BP(5)-6.*BP(7))+ANS2
 HNG7L=HNG7L+
. B1*CJ*EC2*CW(2)*(39./16.*BP(2)+21./2.*BP(4)-147./16.*BP(
. 6)+3./4.*O)+B1*CJ*EC2*CW(4)*(-11./2.*BP(2)-19./2.*BP(4)+9.*BP(
. 6)-3.*O)+B1*CJ*EC2*(3/8.*BP(2)-33./16.*BP(4)+9./8.*BP(6)+9./
. 16.*O)+B1*EC*CJ2*CW(2)*(5*BP(1)-
. 9./2.*BP(3)-27./2.*BP(5)+2.*BP(7))+B1*EC*CJ2*CW(4)*(4*BP(1)
. +21.*BP(3)+25.*BP(5)-6.*BP(7))+B1*EC*CJ2*CW(6)*(10.*BP(1)-
. 5.*BP(3)-9.*BP(5)+4.*BP(7))+B1*EC*CJ2*(-BP(1)-1./4.*BP(3)+5.
. /4.*BP(5))+B1*CJ3*
. CW(2)*(-7./12.*BP(2)-BP(4)+3./4.*BP(6))+B1*CJ3*CW(8)*(-
. 2./3.*BP(4)+1./6.*BP(8)+1../2.*O)+B1*CJ3*CW(4)*(21./4.*BP(2)-7./
. 6.*BP(4)-13./4.*BP(6)+1./6.*BP(8)+4.*O)+B1*CJ3*CW(6)*(-5./2.*BP
. (2)+10./3.*BP(4)+5./2.*BP(6)-1./3.*BP(8)-3.*O)+B1*CJ3*(-1./12.
. *BP(2)+1./8.*BP(4)-1./24.*O)+ANS1
 HNG15L=HNG15L+CJ*EC2*CW(2)*(-
. 7./2.*BP(2)+7./2.*BP(4)-3.*O)+CJ*EC2*(7/4.*BP(2)-7./4.*BP(4)+
. 3./2.*O)+EC*CJ2*CW(2)*(30.*BP(1)+
. 17.*BP(3)-3.*BP(5))+EC*CJ2*CW(4)*(-2.*BP(1)-BP(3)+3.*BP(5))
. +EC*CJ2*(-7.*BP(1)-4.*BP(3))+CJ3*CW(2)*(3*BP(2)-
. BP(4)+3.*O)+CJ3*CW(4)*(BP(4)-1.*O)+CJ3*(-1./2.*BP(2)
. -1./3.*O)+EC3*(3./4.*BP(1)+1./4.*BP(3))
 RETURN
 END
 BLOCK DATA AKONS
 IMPLICIT REAL*8 (A-H,O-Z)
 COMMON/AKON/ AK(7,4)
 DATA AK /-2.1041D-12,-7.8626D-13, 1.4228D-14, 1.9794D-13,
.           2.2387D-13,-2.1152D-12,-1.7174D-13,
.           2.5051D-09, 1.1295D-10, 5.2364D-11,-7.6532D-11,
.          -1.1571D-10, 9.0702D-10, 7.6048D-11,
.          -1.5026D-10,-6.8497D-11, 1.3776D-11, 1.5305D-11,
.           1.6575D-11,-2.2081D-10,-1.8880D-11,
.           7.9489D-11, 6.3063D-11,-3.2733D-12,-7.6109D-12,
.          -9.8514D-12, 9.7056D-11, 8.3009D-12/
 END
```