

PERTURBING ACTION OF CERES, PALLAS, JUNO AND VESTA
AT THEIR PROXIMITIES WITH OTHER SELECTED MINOR PLANETS

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Summary. Perturbing effects of the first four asteroids during their proximities with other numbered minor planets are examined for cases of proximity distances being less than 0.001 AU. From among 32 asteroid pairs examined, there were 13 pairs with perturbing effects on orbital elements larger than $10''$. Hereby, we encountered the hitherto largest interaction effects, associated with proximities of the minor planets. The very largest changes found are: $\Delta M = +57''.93$ and $\Delta\omega = -65''.34$ with the asteroid (1023) Thomana at its proximity with (1) Ceres; $\Delta\omega = +55''.80$, $\Delta\Omega = -57''.50$, $\Delta i = -15''.86$ and $\Delta\mu = -0''.00068$ with (217) Eudora at its proximity with (3) Juno; $\Delta\phi = +6''.02$ with asteroid (2091) Sampo at its proximity with (1) Ceres.

J. Lazović i M. Kuzmanoski, POREMEĆAJNO DEJSTVO PLANETOIDA CERES, PALLAS, JUNO I VESTA NA IZABRANE MALE PLANETE U OKOLINI NJIHOVOG PROKSIMITETA — Razmatrana su poremećajna dejstva prva četiri planetoida na druge numerisane male planete u okolini njihovog proksimiteta kada su im najmanje međusobne daljine manje od 0.001 AJ. Od ukupno ispitana 32 para asteroida ima 13 parova sa poremećajnim promenama putanjskih elemenata većim od $10''$. Ovde smo dobili najveće efekte interakcija u okolini proksimiteta malih planeta, koje smo do sada ispitali. Za najveće promene našli smo: $\Delta M = +57''.93$ i $\Delta\omega = -65''.34$ za asteroid (1023) Thomana pri proksimitetu sa (1) Ceres; $\Delta\omega = +55''.80$, $\Delta\Omega = -57''.50$, $\Delta i = -15''.86$ i $\Delta\mu = -0''.00068$ za (217) Eudora pri proksimitetu sa (3) Juno; $\Delta\phi = +6''.02$ za asteroid (2091) Sampo u okolini njegovog proksimiteta sa (1) Ceres.

Examination is performed of the perturbing action during proximity of 32 pairs numbered asteroids (j, k), whose minimum mutual distances $\rho_{\min} < 0.001$ AU. These pairs, the inclinations between of their orbital planes, true anomalies of their proximity points and their proximity distances are found in Lazović, Kuzmanoski (1982). There $j = 1, 2, 3$ and 4 indicate the first four numbered minor planets: (1) Ceres, (2) Pallas, (3) Juno and (4) Vesta, while k is related to other minor planets which conform to the above condition concerning proximity distance. The orbital elements are borrowed from EMP for 1982. The formulae in Lazović (1971) are used in the calculus of perturbations.

The values of masses of the perturbing asteroid j in solar mass units are those in Lazović, Kuzmanoski (1980a): (1) Ceres 5.9×10^{-10} , (2) Pallas 1.1×10^{-10} , (3) Juno 1.0×10^{-10} and (4) Vesta 1.2×10^{-10} . The perturbed minor planets k are indicated in bold-face type. The integration step w ranged from 0.00005 to 5.0 days, and the examined time interval ΔT around proximity assumed values from 62.5200 to 1078.8000 days.

The perturbing effects on the orbital elements of the second minor planet k in the asteroid pair considered, produced by the action of the first minor planet j , are given in the enclosed Tables: in Table I for the time interval before proximity, in Table II for the time interval after proximity and in Table III for the entire time interval of the sensible perturbing action around proximity.

It appears from Table III that the largest changes take place in the mean anomaly $\Delta M = +57''.93$ and the perihelion argument $\Delta \omega = -65''.34$ with the minor planet (1023) **Thomana** at its proximity with (1) Ceres. The largest changes in the longitude of the ascending node $\Delta \Omega = -57''.50$, the inclination $\Delta i = -15''.86$ and the mean motion $\Delta \mu = -0''.00068$ are

TABLE I

Minor planets (j, k)	ΔM	$\Delta \omega$	$\Delta \Omega$	Δi	$\Delta \varphi$	$10^6 \Delta \mu$
1, 102	+ 1''.60	-11''.16	+ 9''.99	+ 1''.34	+ 0''.13	- 195''
1, 103	+ 6.17	+ 1.40	- 0.13	+ 0.01	+ 3.72	- 156
1, 393	- 2.94	- 0.32	+ 3.42	+ 2.43	- 0.13	- 560
1, 512	+ 9.44	+ 8.90	- 11.93	+ 0.29	+ 3.74	+ 1251
1, 1023	+ 32.23	- 49.99	+ 7.44	+ 1.91	+ 4.13	- 1589
1, 1440	+ 1.36	- 9.26	+ 4.58	+ 0.20	+ 1.73	- 491
1, 1775	- 0.17	- 2.30	+ 3.04	+ 1.18	- 0.34	- 129
1, 2091	+ 27.61	- 31.77	+ 11.70	+ 1.04	+ 4.30	- 1368
1, 2116	+ 40.06	- 38.77	+ 6.53	+ 0.89	+ 3.31	- 42
2, 387	+ 0.34	+ 0.19	- 0.56	- 0.06	+ 0.02	- 51
2, 1193	- 0.50	+ 1.77	- 0.46	+ 0.12	+ 0.46	- 170
2, 1910	- 8.37	+ 6.96	+ 0.57	- 0.13	- 0.25	- 103
3, 217	+ 31.84	+ 14.89	- 31.75	- 8.76	+ 5.94	- 2833
3, 299	- 0.42	- 21.88	+ 22.41	- 0.13	+ 0.04	- 5
3, 370	- 0.79	- 1.08	+ 1.74	+ 0.31	+ 0.04	0
3, 1185	- 1.00	+ 3.42	- 2.14	+ 0.12	+ 0.09	+ 39
3, 1221	- 0.71	- 0.27	+ 0.39	- 0.38	+ 0.28	- 13
3, 1224	- 0.20	- 5.97	+ 5.18	+ 0.40	+ 0.56	+ 121
3, 1530	- 0.31	- 3.45	+ 3.54	+ 0.23	+ 0.14	+ 52
3, 1942	- 0.59	+ 0.42	+ 0.01	+ 0.19	- 0.03	- 32
3, 2094	- 2.07	+ 1.75	- 0.06	0.00	+ 0.08	+ 4
3, 2265	- 1.77	+ 1.08	+ 0.09	- 0.04	+ 0.08	- 26
4, 99	- 0.39	+ 1.78	- 0.56	+ 0.22	+ 0.45	- 180
4, 344	- 0.78	+ 0.02	- 0.02	+ 0.01	+ 0.40	- 114
4, 428	- 0.91	+ 5.30	- 4.73	+ 0.41	+ 0.12	- 74
4, 749	+ 9.05	- 1.12	- 3.93	+ 0.87	+ 1.99	+ 644
4, 1056	- 3.18	+ 1.94	- 0.06	+ 0.48	+ 0.53	+ 85
4, 1137	- 0.44	+ 6.46	- 5.67	- 0.31	+ 0.21	- 89
4, 1278	+ 0.30	+ 1.13	- 0.97	+ 0.43	+ 0.26	- 91
4, 1378	+ 0.24	+ 18.60	- 19.46	0.00	- 0.33	- 99
4, 2015	+ 1.71	- 0.72	- 0.47	+ 0.23	+ 0.23	- 42
4, 2171	- 7.20	- 0.72	+ 4.99	- 1.22	+ 1.39	+ 423

stated with the minor planet (217) **Eudora** at its proximity with (3) Juno, its $\Delta\omega = +55''.80$ being also large. It can be seen from this Table that the largest change in the angle of eccentricity $\Delta\varphi = +6''.02$ appears in the orbit of the minor planet (2091) **Sampo** at its proximity with (1) Ceres. There are, according to Table III, 13 asteroid pairs in which perturbing changes in orbital elements of the second asteroid k exceed $10''$.

The values of $\Delta\mu$ in Tables I and II are rather large, but their signs before and after the instant of the proximity are opposite. As a consequence, the resulting total change in the mean motion, produced during the proximity, assumes considerably lower values found in Table III.

Table IV comprises, for each one of the examined minor planets pair (j, k), the larger of two limiting values of their mutual distances q in 10^{-7} AU units, at which the perturbing changes in at least one of the orbital elements begin or stop being felt. The notations $-$ or $+$ indicate that the concerned value pertain to the positions before or after the

TABLE II

Minor planets (j, k)	ΔM	$\Delta\omega$	$\Delta\Omega$	Δi	$\Delta\varphi$	$10^5\Delta\mu$
1, 102	- 3''.67	- 0''.68	+ 1''.28	+ 0''.18	- 1''.50	+ 202''
1, 103	- 3.83	+ 43.05	- 39.41	+ 0.26	- 0.01	+ 183
1, 393	+ 5.48	- 0.54	- 0.02	- 0.01	- 2.42	+ 556
1, 512	- 1.10	+ 22.99	- 24.78	+ 0.60	- 2.39	-1227
1, 1023	+ 25.70	- 15.35	- 15.95	- 4.06	- 2.41	+1572
1, 1440	+ 2.42	- 29.31	+ 27.31	+ 1.21	- 0.78	+ 475
1, 1775	+ 5.81	- 2.61	- 0.59	- 0.22	- 1.08	+ 124
1, 2091	- 60.46	+ 81.63	- 15.62	- 1.27	+ 1.72	+1382
1, 2116	+ 12.29	+ 9.05	- 18.86	- 2.52	+ 1.23	+ 56
2, 387	- 0.27	- 0.09	- 0.01	0.00	- 0.19	+ 51
2, 1193	- 2.37	+ 1.72	+ 1.07	- 0.28	- 0.06	+ 170
2, 1910	- 4.18	+ 5.19	- 1.41	+ 0.33	+ 0.21	+ 102
3, 217	- 34.71	+ 40.91	- 25.75	- 7.10	- 7.81	+2765
3, 299	- 6.61	+ 9.55	- 1.65	+ 0.01	+ 0.45	+ 3
3, 370	- 3.92	+ 3.49	- 0.34	- 0.06	+ 0.21	0
3, 1185	- 0.66	- 0.06	+ 1.23	- 0.07	+ 0.23	- 40
3, 1221	+ 0.76	- 0.48	+ 0.37	- 0.36	- 0.30	+ 12
3, 1224	- 2.40	+ 4.23	- 3.46	- 0.26	+ 0.76	- 120
3, 1530	- 1.92	+ 2.40	- 1.17	- 0.07	+ 0.18	- 51
3, 1942	- 1.11	+ 0.75	0.00	- 0.08	+ 0.09	+ 32
3, 2094	+ 0.03	+ 1.75	- 1.78	- 0.13	- 0.01	- 5
3, 2265	+ 0.32	+ 0.81	+ 0.61	- 0.25	+ 0.03	+ 24
4, 99	- 1.64	+ 1.39	+ 0.84	- 0.34	- 0.04	+ 181
4, 344	+ 0.53	- 1.03	+ 0.49	- 0.37	+ 0.01	+ 112
4, 428	+ 2.02	+ 0.40	- 1.15	+ 0.09	- 0.60	+ 69
4, 749	- 1.58	+ 9.15	- 9.12	+ 2.03	- 1.22	- 604
4, 1056	+ 1.99	- 1.03	- 0.08	+ 0.70	- 0.40	- 99
4, 1137	+ 2.83	- 1.90	- 2.00	- 0.10	- 0.43	+ 103
4, 1278	- 0.19	- 0.24	- 0.50	+ 0.22	- 0.47	+ 98
4, 1378	- 3.93	- 3.14	+ 4.68	0.00	- 1.15	+ 112
4, 2015	+ 0.55	- 0.52	+ 0.48	- 0.24	+ 0.26	+ 42
4, 2171	+ 5.65	- 8.96	+ 5.82	- 1.44	- 1.28	- 460

instant of the proximity proper. The largest angular widths, comprising the parts of orbits within which a measurable effect of the gravitational action of the minor planet j on the minor planet k is produced during their proximity, are given in columns Δv_j and Δv_k . In the last column of Table IV are values of the dynamical duration of proximity Δt , which is in fact the time interval within which the perturbing action of the asteroid j on the asteroid k around their proximity is being felt. We find that the longest measurable action is $\Delta t = 1028.4000$ days, produced by the minor planet (1) Ceres on the minor planet (2091) Sampo, the corresponding angular widths in their orbits being $\Delta v_1 = 206^\circ.29736$ and $\Delta v_{2091} = 199^\circ.40963$, respectively. These, at the same time, are the largest yet found. Associated with this asteroid pair is also the largest mutual distance $\rho = 0.5232817$ AU at which sensible changes in the orbital elements due to their close approaching start being felt.

The inclinations between the orbital planes of the celestial bodies investigated, while being different, are mostly higher; the minimum mutual

TABLE III

Minor planets (j, k)	ΔM	$\Delta \omega$	$\Delta \Omega$	Δi	$\Delta \varphi$	$10^5 \Delta \mu$
1, 102	- 2".07	- 11".84	+ 11".27	+ 1".52	- 1".37	+ 7"
1, 103	+ 2.34	+ 44.45	- 39.54	+ 0.27	+ 3.71	+ 27
1, 393	+ 2.54	- 0.86	+ 3.40	+ 2.42	- 2.55	- 4
1, 512	+ 8.34	+ 31.89	- 36.71	+ 0.89	+ 1.35	+ 24
1, 1023	+ 57.93	- 65.34	- 8.51	- 2.15	+ 1.72	- 17
1, 1440	+ 3.78	- 38.57	+ 31.89	+ 1.41	+ 0.95	- 16
1, 1775	+ 5.64	- 4.91	+ 2.45	+ 0.96	- 1.42	- 5
1, 2091	- 32.85	+ 49.86	- 3.92	- 0.23	+ 6.02	+ 14
1, 2116	+ 52.35	- 29.72	- 12.33	- 1.63	+ 4.54	+ 14
2, 387	+ 0.07	+ 0.10	- 0.57	- 0.06	- 0.17	0
2, 1193	- 2.87	+ 3.49	+ 0.61	- 0.16	+ 0.40	0
2, 1910	- 12.55	+ 12.15	- 0.84	+ 0.20	- 0.04	- 1
3, 217	- 2.87	+ 55.80	- 57.50	- 15.86	- 1.87	- 68
3, 299	- 7.03	- 12.33	+ 20.76	- 0.12	+ 0.49	- 2
3, 370	- 4.71	+ 2.41	+ 1.40	+ 0.25	+ 0.25	0
3, 1185	- 1.66	+ 3.36	- 0.91	+ 0.05	+ 0.32	- 1
3, 1221	+ 0.05	- 0.75	+ 0.76	- 0.74	- 0.02	- 1
3, 1224	- 2.60	- 1.74	+ 1.72	+ 0.14	+ 1.32	+ 1
3, 1530	- 2.23	- 1.05	+ 2.37	+ 0.16	+ 0.32	+ 1
3, 1942	- 1.70	+ 1.17	+ 0.01	+ 0.11	+ 0.06	0
3, 2094	- 2.04	+ 3.50	- 1.84	- 0.13	+ 0.07	- 1
3, 2265	- 1.45	+ 0.27	+ 0.70	- 0.29	+ 0.11	- 2
4, 99	- 2.03	+ 3.17	+ 0.28	- 0.12	+ 0.41	+ 1
4, 344	- 0.25	- 1.01	+ 0.47	- 0.36	+ 0.41	- 2
4, 428	+ 1.11	+ 5.70	- 5.88	+ 0.50	- 0.48	- 5
4, 749	+ 7.47	+ 8.03	- 13.05	+ 2.90	+ 0.77	+ 40
4, 1056	- 1.19	+ 0.91	- 0.14	+ 1.18	+ 0.13	- 14
4, 1137	+ 2.39	+ 4.56	- 7.67	- 0.41	- 0.22	+ 14
4, 1278	+ 0.11	+ 0.89	- 1.47	+ 0.65	- 0.21	+ 7
4, 1378	- 3.69	+ 15.46	- 14.78	0.00	- 1.48	+ 13
4, 2015	+ 2.26	- 1.24	+ 0.01	- 0.01	+ 0.49	0
4, 2171	- 1.55	- 9.68	+ 10.81	- 2.66	+ 0.11	- 37

distances are larger; the perturbing minor planets have appreciably larger masses, the durations of the occurrence are considerably longer and the perturbations stronger than those found in our earlier investigations (Lazović, Kuzmanoski, 1979, 1980a, 1980b, 1981). We deem the results of the present work as making more complete our knowledge of the close approaches of the minor planets and of their dynamical consequences.

The calculations were carried out on the IBM 360/44 of the Computing Centre of the Institute for Mathematics in Beograd.

TABLE IV

Minor planets (<i>j, k</i>)	$10^7 \rho$	Δv_j	Δv_k	Δt
1, 102	1591271+	16°.07134	15°.31930	74°.5000
1, 103	2413654+	45.23778	44.31199	201.9000
1, 393	2821093+	17.32525	16.14341	82.5000
1, 512	2690706-	39.12731	33.39273	163.9000
1, 1023	1529524-	17.88248	19.05657	86.5000
1, 1440	3177143+	46.23232	47.66138	199.9000
1, 1775	2032614+	18.40262	17.48958	89.5000
1, 2091	5232817-	206.29736	199.40963	1028.4000
1, 2116	1467991+	22.64537	21.98913	98.5000
2, 387	2697355+	11.23362	11.88361	46.5000
2, 1193	3893653+	12.87713	13.46369	48.3000
2, 1910	1240669+	8.48609	9.11957	54.5000
3, 217	1337603+	6.67905	7.00374	41.5200
3, 299	1519526-	25.08576	24.75865	87.5000
3, 370	1281920+	13.69466	13.25261	55.5000
3, 1185	1109175-	20.38640	19.26146	58.5000
3, 1221	740312+	4.50401	3.51007	16.1000
3, 1224	1380529+	20.42225	19.34664	76.5000
3, 1530	1210843+	16.87404	15.74817	72.5000
3, 1942	353620-	1.99765	1.89318	9.6000
3, 2094	2071960+	19.57019	17.91566	72.5000
3, 2265	1483566+	12.61532	12.44571	78.5000
4, 99	938674+	19.83819	20.74509	66.5000
4, 344	1441025+	14.86246	14.67321	61.5000
4, 428	1273398-	14.45058	14.45528	55.9000
4, 749	945039-	43.20115	41.62326	187.9000
4, 1056	1608442+	39.99096	38.23022	169.9000
4, 1137	1857070+	54.17366	55.40458	181.9000
4, 1278	2115317+	28.61702	29.99325	87.9000
4, 1378	1460568+	52.57508	52.67071	207.9000
4, 2015	664663-	10.94886	10.86687	42.5000
4, 2171	1080173+	47.22006	45.75464	205.9000

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