

PRELIMINARY ANALYSIS OF THE SECULAR AND MAIN HARMONIC
TERMS IN THE POLAR MOTION

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Summary. Linear secular terms have been identified in both polar coordinates, independently of whether the latter have been derived from raw or smoothed observational data. After elimination, amplitudes and phases of the main harmonic terms in the polar motion have been determined. Our results back up assumption Chandler's cycle being non-unique. Instead of one single peak, two close ones have unequivocally been identified within that cycle in spectro of polar coordinates. Their periods are: 1.171 and 1.198 years, respectively. In the Kimura's z-term, besides the well known annual cycle, a semi-annual one has been identified, whose amplitude is 3 to 4 times weaker.

N. Pejović, PRELIMINARNA ANALIZA SEKULARNOG I GLAVNIH HARMONIJSKIH ČLANOVA U KRETANJU POLA — U sirovim i izravnatim koordinatama pola identifikovan je sekularni član u obe koordinate. Posle njegove eliminacije određene su amplitude i faze glavnih harmonijskih članova u kretanju pola. Na osnovu naših rezultata potkrepljena je pretpostavka da poznati Čandlerov ciklus nije jedinstven. Umesto jednog pika, u spektrima koordinata pola pouzdano su utvrđena dva bliska. Njihove periode su: 1.171 i 1.198 godina. U Kimurinom z-članu pored poznatog godišnjeg ciklusa identifikovali smo i polugodišnji ciklus čija je amplituda 3—4 puta manja.

INTRODUCTION

Since the foundation of the ILS (International Latitude Service) in 1899 up to now observational material of an exceptional scientific value has been collected. Many reputed researchers have worked on the harmonic analysis of this material. By all of them the existence has been confirmed of two periodic components: circular component of the free nutation — detected by Chandler — supposed to be excited by the non-coincidence of the Earth's rotational axis with the axis of symmetry of its masses (the cause of the non-coincidence of these axes for the time being unknown), and the annual elliptic component of the forced nutation, accounted for by shifts in the masses over the Earth's surface (air and water streams). However, many more periodic components are found by the harmonic analysis, whose source is thought to lie in the specific geophysical composition

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of the Earth. Due to its complexity this problem has not yet received its final solution, such as would allow the position of the Earth's instantaneous pole of rotation to be determined for all past and future times. This is why the trajectories of the Earth's poles, as deduced from the observations, from which the components of the free and forced nutation are removed, assume a very complex shape.

In the present paper we are going to restrain ourselves to the analysis of the main harmonic terms in the polar motion. A more thorough analysis will be the subject of our future work.

Data used: Observational data, acquired at the ILS stations in the period 1899.9 to 1979.0, cleaned up and published in the Publications of the Mizusawa International Latitude Observatory (Yumi and Yokoyama, 1980). Raw and smoothed (by Vondrak's method, 1969) polar coordinates have in parallel been treated.

SECULAR TERM

Let X_0 and Y_0 be the coordinates of the mean pole and x and y the coordinates of the instantaneous pole with respect to CIO (Conventional International Origin). The raw polar coordinates will be marked by the subscript u and the smoothed ones by the subscript v .

With the purpose of removing the secular term from the data used, as required by the harmonic analysis, mean three-annual values x_u, y_u, x_v and y_v have been formed and their first-, second- and third-order differences calculated.

The mean squares S_1 of the first-, S_2 of the second- and S_3 of the third-order differences in the raw and smoothed series are presented in Table I.

TABLE I

Mean square	x_u	y_u	x_v	y_v
S_1	0.0013	0.0010	0.0006	0.0021
S_2	0.0046	0.0037	0.0016	0.0074
S_3	0.0171	0.0141	0.0051	0.0289

It is evident from the above data that $S_1 < S_2 < S_3$ in all series of coordinates. Hence it follows that the linear approximation of the secular term appears as the best one.

The regression coefficients have been calculated according to the least square method. The free terms C_0 and the linear terms C_1 of the raw and smoothed series are given in Table II.

TABLE II

Regres. coeff.	x_u	y_u	x_v	y_v
C_0	0.01788	-0.01547	0.01846	-0.01567
C_1	0.00065	0.00346	0.00064	0.00348

By using the data from Table II we found that the mean pole moves at the rate $dX_0/dt = +0.''0006/\text{year}$ and $dY_0/dt = +0.''0035/\text{year}$, or by the resulting velocity $dr/dt = 0.''0036/\text{year}$, with an accuracy of $\pm 0.''0002/\text{year}$.

It is only afterwards that we were acquainted with Markovitz (1982) having obtained results, close to ours own. From our results, as well as from those of Markovitz, there follows that a linear secular motion of the mean pole is present, a matter still considered as questionable.

MAIN HARMONIC TERMS

Upon removing the secular term from the data treated, Fourier integral transformations have been used in the identification of the main harmonic terms: the annual and Chandler's term.

The trial period has been varied within the limits of 60 up to 760 days, with an increment of 0.5 days. The initial period is, in fact, twice the value of the interval for which the data are given. Through such a choice of the trial periods the effects, known as leakage and aliasing, have been reduced.

The obtained values of the period P_s , the amplitude A_s and phase F_s of the annual term are given in the columns 2—4 of Table III. A double peak has come forth at the place where Chandler's term has been expected. The periods P_c, P'_c , the corresponding amplitudes A_c, A'_c and phases F_c, F'_c are listed in the columns 5—10 of the same Table. The periods are given in years. The phases are referred to the Julian date D.J. = 2415020.

TABLE III

Coord.	P_s	A_s	F_s	P_c	A_c	F_c	P'_c	A'_c	F'_c
x_u	1.000	0.''094	253.9	1.171	0.''105	59.4	1.197	0.''114	16.4
x_v	1.000	0.076	171.7	1.171	0.107	332.2	1.197	0.118	284.5
y_u	1.000	0.094	253.8	1.171	0.105	68.8	1.199	0.115	15.4
y_v	1.000	0.077	170.3	1.171	0.107	340.8	1.199	0.117	285.1

The mean square errors $\sigma(x_u), \sigma(y_u), \sigma(x_v), \sigma(y_v)$ of the coordinates x_u, y_u, x_v, y_v are equal $\pm 0.''01$.

By using the expression:

$$\sigma^2_A = (2 - \pi/2) \frac{2}{2n + 1} \sigma^2_x$$

the value ± 0.0003 is obtained as the mean square error of the amplitude for the unsmoothed series and the value ± 0.0002 for the smoothed ones.

The mean square errors of the phases, calculated by the formula of

$$\sigma_F \approx 57^\circ.296 \frac{\sigma_x}{A} \sqrt{\frac{2}{2n + 1}}$$

the annual and Chandler's terms lie in the interval from ± 0.2 to ± 0.3 .

The correction to the amplitude ΔA , considering it to represent unbiased estimation of the mathematical expectation, has been calculated according to the formula:

The amplitudes in Table III are corrected for ΔA .

$$\Delta A = \sqrt{\frac{\pi}{2n + 1}} \sigma_x.$$

The used expressions for the mean square error of the amplitude and phase and the correction to the amplitude are according to Djurović (1979).

The spectra, obtained by the Fourier analysis, are illustrated in Fig. 1 for the series x_u, y_u and in Fig. 2 for the series x_v, y_v . The abscissae reproduce the trial period P in years. The amplitude is given in seconds of arc.

The spectra in the interval P from 0.2 to 0.8 years, ten times enlarged, are found in the same Figures. By these results one is induced to suspect the presence of the short period terms as well whose amplitude does not exceed 0."006. This point will be the subject of our future study.

The duplicity of the Chandler's term has been put under scrutiny by comparing the theoretical and the calculated peak widths.

In Table IV are presented the theoretical $\Delta \varepsilon$ and the calculated $\Delta \varepsilon'$ peak widths (in years) of the main harmonic terms.

TABLE IV

Peak widths	$P_c=1.000$	$P_c=1.171$	$P'_c=1.199$
$\Delta \varepsilon$	0.025	0.035	0.036
$\Delta \varepsilon'$	0.025	0.040	0.031

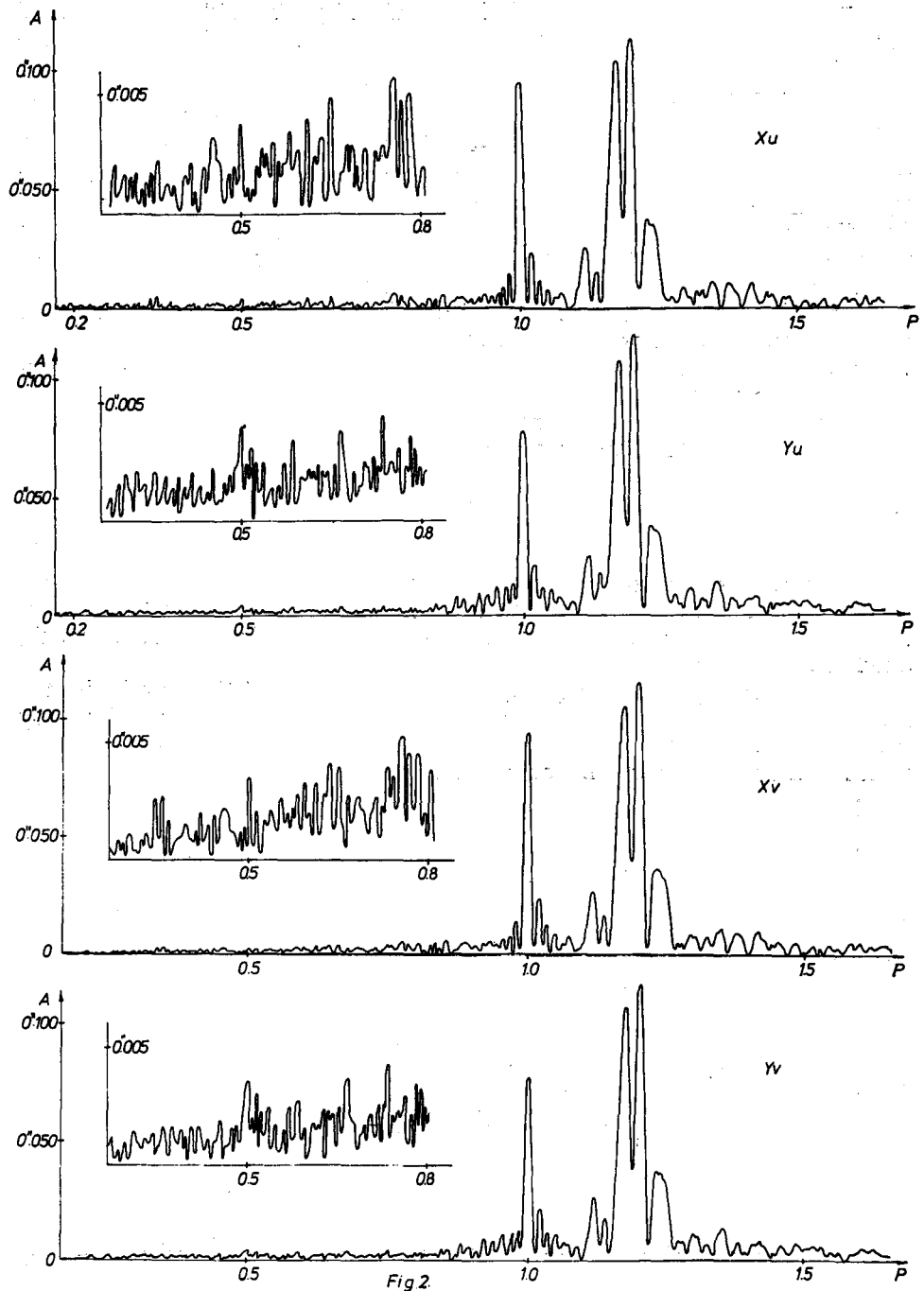


Fig 2.

Evidently, the theoretical and the calculated peak widths of the annual and both Chandler terms are approximately equal. By this fact, the assumption of the Chandler term being twofold, is favoured. If it were considered as a single one, with observed width $\Delta\epsilon' = 0.071$, it would be two times wider than the corresponding theoretical value $\Delta\epsilon$ (Table IV).

For the amplitudes of the primary peaks to be accurately determined, the differences of periods ΔP of any adjacent terms have to be greater or equal to $\Delta\epsilon$, a condition met only with the annual term. However, the two Chandler peaks are 0.028 years apart, i.e. less than $\Delta\epsilon$. Accordingly, the accurate values of their amplitudes (provided there were two peaks) cannot be determined from the given data by the present method.

From the same data Shouxian, Yingmin and Shuhe (1982) deduced a double Chandler term with the periods 1.171 and 1.198 years respectively. Their amplitudes are in close agreement with our results, arrived at by quite a different method.

With regard to the above results we presume that, what we are faced with, is not one single, but two close free nutations of the Earth's axis.

THE KIMURA'S Z-TERM

It is well known that the Kimura's z-term can be approximated by a sinusoid of an annual period. However, in analysing the ILS data we were able to identify, in addition to the annual, a semi-annual component as well, which heretofore was unknown.

The z-term spectrum (Fig. 3a), along with those of the amplitude, the period and the phase of the harmonic terms (Table V), were calculated by the method of Fourier transformations. This method was applied to the z-term residuals after the linear trend has been removed,

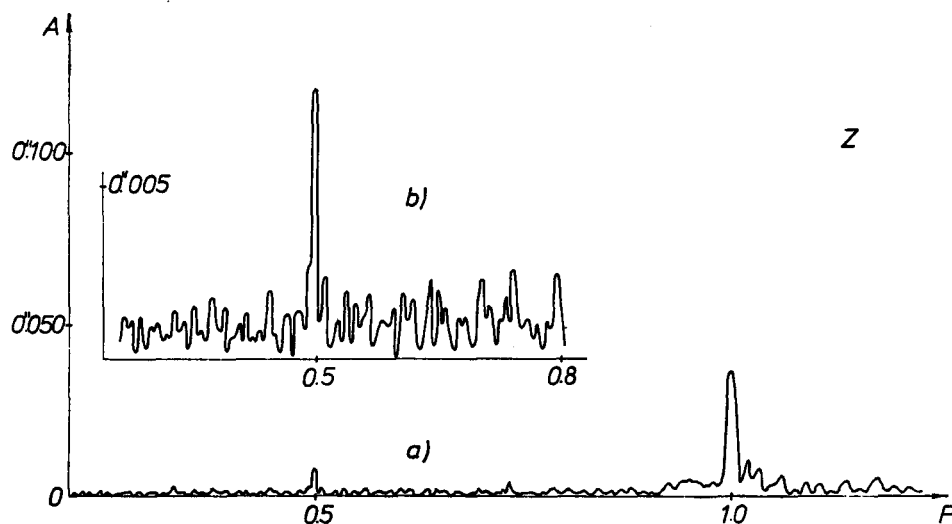


Fig.3.

The notations in Table V mean, respectively: P, A and F — the period, amplitude and phase of the annual term; P', A' and F' — the period, amplitude and phase of the semi-annual term. The phases are related to the Julian date D.J. = 2415020. The periods are expressed in years.

TABLE V

P	A	F	P'	A'	F'
1.000	0."037	5°8	0.499	0."008	32.3

The abscissa axis P in Fig. 3 represent the trial period in years, and that of the ordinate the amplitude in second of arc. The annual and the semi-annual peaks are clearly distinguishable (Fig. 3a). The semi-annual peak becomes particularly pronounced in the enlarged part of the same Figure (3b).

In order to provide an estimate of what is the statistical meaning of the amplitude A', two alternate hypotheses, H_0 and H_1 , have been tested. The first H_0 hypothesis, implied that the results of the spectral analysis, for the frequency $\omega = 2\pi/0.499$, represented a white noise. The implication of the second hypothesis H_1 was: the data embraced a harmonic, with the frequency ω .

It could be established, by employing the Schuster and the Walker criteria, with the significance levels 0.05 and 0.01, respectively that the H_0 hypothesis was to be turned down.

We learn from Kulikov's monography (1962), dedicated to the problems of longitude and latitude variations, that different authors have obtained different values for the amplitude and phase of the z-term. This is explicable by the fact that various authors have used different observational. The origin of the z-term remains unknown even to our days.

Fikera (1971) claims to have found the z-term being dependent on the catalogues used in the latitude observations. Other authors, however, come forward with other interpretations.

The calculations were carried out on the IBM 360/44 of the Computing Centre of the Institute for Mathematics in Beograd.

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REFERENCES

Djurović, D. 1979, Mathematical treatment of the astronomical observations, University of Belgrade (in Serbian).

- Fichera, E. 1971, Studio del catalogo SIL di Melchior-Dejaiffe dalle osservazioni 1935—1947, Istituto Universitario Navale, Napoli.
- Kulikov, K. A. 1962, *Variations of latitudes and longitudes, monograph*, Moscow (in Russian).
- Markovitz, Wm. 1982, Polar motions for 1825—1981 and 1727 — 31 analyzed by new methods, Report to IAU Commissions 19 and 31, Patras.
- Shouxian, W., Yingmin, H., Shuhe, W. 1982, A modulation phenomenon on Chandler's polar motion. *Scientia Sinica (Series A)* XXV, 4, 413—421.
- Vondrak, J. 1969, *Bull. Astron. Inst. Czechoslovakia*, 20, 349.
- Yumi, S., Yokoyama, K. 1980, *Results of the ILS in a Homogeneous system 1899.9—1979.0*, Mizusawa.