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ESTIMATES OF THE MUTUAL PERTURBATIONS IN THE ORBITAL ELEMENTS OF SOME INTERESTING QUASICOMPLANAR MINOR PLANETS

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Summary. The calculus is performed of the first order perturbations in the motion of one minor planet, produced by another minor planet during their proximity, the conditions being: the angles between the orbital planes of the given minor planets are under 0°.500, their proximity (minimum) distances do not exceed 0.000400 AU (60000 km), the perturbing minor planet masses are estimated as being greater than 10^{-13} of the solar mass and whose proximity duration is over half a day. The likely perturbing effects indicate that the interactions of the minor planets, taking place during proximity, might well be exploited for a more refined estimates of their masses. It proved that sensible changes appear in Ω, ω, i and M.

J. Lazović, M. Kuzmanoski, PROCENE MEĐUSOBNIH POREMEĆAJA U PUTANJSKIM ELEMENTIMA NEKIH INTERESANTNIH KVAZIKOMPLANARNIH MALIH PLANETA — Izvršen je račun poremećaja prvoga reda u kretanju jedne male planete, koje bi mogla da izazove druga mala planeta pri njihovom proksimitetu, kada su: uglovi između putanjskih ravni malih planeta manji od pola stepena, njihove proksimitetske (najmanje) daljine do 0.000400 AJ (60000 km), poremećajne male planete sa ocenjenim masama većim od 10^{-13} Sunčeve mase i čije zbližavanje traje duže od pola dana. Dobijeni mogući poremećajni efekti ukazuju da bi se međuakcije malih planeta oko proksimiteta mogle iskoristiti za pouzdanije ocenjivanje njihovih masa. Pokazalo se da osetne promene imaju: Ω, ω, i i M.

The minimum (proximity) distances, under 0.000400 AU (60000 km) of the quasicomplanar minor planets, with inclinations between their orbits under 0°.500, have been found earlier (Lazović, Kuzmanoski, 1978). The perturbations taking place in some of these pairs have been determined in Lazović, Kuzmanoski (1979a, 1980). Now we have examined mutual perturbations with the rest of the minor planets pairs, whereby the masses of the perturbing minor planets are over 10^{-13} of the solar mass, or the kinematic durations of their proximities are greater than half a day (Lazović, 1979; Lazović, Kuzmanoski, 1979b). The orbital elements are taken from EMP for 1977, as in Lazović, Kuzmanoski (1978). First order calculus of perturbation has been performed according to the formulae given in Lazović (1971). To be more definite the question is about the perturbation that the minor planet j might exert upon the motion of the minor planet k during their proximity. The perturbed minor planet $\mathbf{k}$ will be denoted in bold-face type. The perturbations originating from the major planets have not been taken into account.

TABLE I

Minor planets (j, $\mathbf{k}$)	I	10^6 ρ_{min}	m_j
(16) Psyche, **(1245) Calvinia**	0°253	181	3.4×10^{-11}
(21) Lutetia, **(367) Amicitia**	0.187	265	2.9×10^{-12}
(24) Themis, **(1462) Zamenhof**	0.303	391	5.7×10^{-12}
(39) Laetitia, **(251) Sophia**	0.225	218	1.7×10^{-11}
(43) Ariadne, **(211) Isolda**	0.408	250	1.4×10^{-12}
(47) Aglaja, **(1541) Estonia**	0.156	178	1.4×10^{-12}
(50) Virginia, **(1335) Demoulina**	0.292	158	2.7×10^{-13}
(76) Freia, **(1692) Subbotina**	0.387	89	1.9×10^{-12}
(79) Eurynome, **(1200) Imperatrix**	0.064	273	1.2×10^{-12}
(84) Klio, **(227) Philosophia**	0.184	254	3.1×10^{-13}
(111) Ate, **(1092) Lilium**	0.496	196	1.6×10^{-12}
(143) Adria, **(469) Argentina**	0.335	99	2.1×10^{-13}
(163) Erigone, **(1874) Kacivelia**	0.018	288	2.4×10^{-13}
(171) Ophelia, **(1581) Abanderada**	0.208	112	6.3×10^{-13}
(212) Medea, **(406) Erna**	0.137	359	9.5×10^{-13}
(215) Oenone, **(1851) Lacroute**	0.007	282	1.4×10^{-13}
(243) Ida, **(1848) ≡ 1933 *QD***	0.345	303	9.0×10^{-14}
(335) Roberta, **(873) ≡ Mechthild**	0.261	149	4.7×10^{-13}
(376) Geometria, **(1374) Isora**	0.129	394	2.1×10^{-13}
(379) Huenna, **(461) Saskia**	0.464	369	4.1×10^{-13}
(379) Huenna, **(1635) Bohrmann**	0.392	328	4.1×10^{-13}
(384) Burdigala, **(722) Frieda**	0.229	280	1.8×10^{-13}
(534) Nassovia, **(452) Hamiltonia**	0.072	168	1.2×10^{-13}
(554) Peraga, **(557) Violetta**	0.453	356	9.5×10^{-13}
(577) Rhea, **(765) Mattiaca**	0.393	245	1.6×10^{-13}
(586) Thekla, **(277) Elvira**	0.461	291	2.7×10^{-13}
(848) Inna, **(1363) Herberta**	0.151	210	3.4×10^{-14}
(962) Aslög, **(1802) Zhang Heng**	0.187	294	1.1×10^{-14}
(1079) Mimosa, **(1100) Arnica**	0.496	98	2.6×10^{-14}
(1135) Colchis, **(1381) Danubia**	0.143	228	3.9×10^{-14}
(1289) Kutaïssi, **(1635) Bohrmann**	0.332	370	5.2×10^{-14}
(1331) Solvejg, **(938) Chlosinde**	0.443	140	5.2×10^{-14}
(1487) Boda, **(1581) Abanderada**	0.361	168	3.4×10^{-14}
(1635) Bohrmann, **(993) Moultona**	0.041	183	9.9×10^{-15}
(1810) Epimetheus, **(1169) Alwine**	0.084	325	2.5×10^{-15}
(1856) Růžena, **(1651) ≡ 1936 *HD***	0.367	125	4.3×10^{-15}

In Table I are listed 36 different minor planets pairs, which on this occasion have been investigated, with the inclinations I between their respective orbital planes, the minimum distances ρ_{min} (in 10^{-6} AU units) and the masses m_j of the perturbing minor planets, taken from our previous papers. The pair (215) Oenone and **(1851) Lacroute** has this time been investigated around its second proximity, where we have a greater distance than the one associated with the first proximity. Namely, according to our criteria $\rho_{min} < 0.000400$ AU, there are two proximities of the orbits of this pair. The perturbing effects related to the first, closer proximity, have already been found (Lazović, Kuzmanoski, 1979a). The pair (534) Nassovia and (**452**) **Hamiltonia** displays two proximities also. The first (shorter) one has been entered in Table I. The second proximity is 0.000364 AU. The perturbations associated with both these proximities have been investigated. In Tables II through V are first the values corresponding to the smaller proximity distance, then those relevant to their greater proximity distance. Havin regard to what has been said in the introductory part of the present paper, the perturbing effects have been calculated bi-laterally, i. e. the roles of the perturbing and the perturbed minor planets have been changed with the pairs: (16, 1245), (43, 211), (84, 227), (143, 469), (1487, 1581). Thereby, the values of the masses from Lazović (1979) have been used: $m_{211} = 1.9\times10^{-12}$, $m_{227} = 3.6\times10^{-13}$, $m_{469} = 4.7\times10^{-13}$, $m_{1245} = 1.2\times10^{-13}$, $m_{1581} = 6.9\times10^{-14}$. As a result we now got in all 42 combinations of the minor planets to be investigated. No noticeable perturbing effects have been obtained with four of these minor planets pairs: (376, **1374**), (1635, **993**), (1810, **1169**), (1856, **1651**). Thus, possible measurable changes in the orbital elements have been found with 32 different pairs.

The integration step w, depending on the pair under investigation, ranged from 0.001 to 3.0 days, whereas the time interval ΔT, around the proximity within which the investigations were accomplished, amounted from 3 to 652.6 days. The highest values of ΔT and w are associated with three minor planets pairs: (16) Psyche and (**1245**) **Calvinia,** (24) Themis and (**1462**) **Zamenhof,** (39) Laetitia and (**251**) **Sophia.** On the other hand there we have the smallest value $w = 0^{d}.001$ for these pairs. Here we were obliged to divide the whole time interval into several sub-intervals in order to perform the corresponding numerical integration. One can see from the enclosed Tables that the perturbing effects in these three pairs were the strongest.

In Tables II to IV are listed the minor planets pairs in which relatively greater changes of the orbital elements are obtained with the second minor planet $\boldsymbol{k}$, due to the action of the first j, namely: in Table II for the time interval up to the instant of proximity, in Table III for the interval after the moment of proximity and in Table IV for the whole time interval of the perceptible perturbing action during proximity. The change of the mean anomaly ΔM, due to the perturbing action, has here been a result of only the first term of the increment of the mean anomaly due to its variation. The so called second term of the change of the mean anomaly, owing to the double integral of the mean motion derivative, does not affect the perturbing increment of the mean anomaly. It is only with the pairs (16, **1245**) and (39, **251**) that the second term assumed the values $-0''.01$ and $+0''.01$ respectively, in both cases after the proximity passages. The change of the argument of perihelion $\Delta\omega$ was obtained using the changes of the longitude of perihelion and the longitude of the ascending node according the formula $\Delta\omega = \Delta\tilde{\omega} - \Delta\Omega$. By means of the found values of the changes of the longitude of perihelion and of the mean anomaly, listed in Tables II to IV, one is able to easily derive the changes of the mean longitude ΔL according to the formula $\Delta L = \Delta\tilde{\omega} + \Delta M$. In all the considered examples we obtained $|\Delta L| < |\Delta\omega|$. The largest change of the mean longitude is found with the pair (16, **1245**), $\Delta L = -0''.45$ for the entire interval of the sensible effect. Otherwise, we have with this pair $\Delta L = -3''.29$ up to the instant

TABLE II

Minor planets (*j*, *k*)	ΔM	$\Delta\omega$	$\Delta\Omega$	$\Delta\tilde{\omega}$	Δi	$\Delta\varphi$	$10^5\,\Delta\mu$
16, 1245	+10".94	—1".40	—12".83	—14".23	+1".27	—1".28	+ 33"
21, 367	+ 0.01	+0.32	— 0.29	+ 0.03	+0.01	+0.02	+ 2
24, 1462	— 1.06	—1.43	+ 2.29	+ 0.86	—0.09	+0.03	— 6
39, 251	— 6.01	+8.61	— 1.92	+ 6.69	+0.37	+0.45	—343
43, 211	— 0.03	+0.05	— 0.02	+ 0.03	+0.02	+0.03	— 20
47, 1541	+ 0.26	—0.45	+ 0.24	— 0.21	+0.02	+0.02	0
50, 1335	— 0.01	+0.04	— 0.04	0.00	—0.01	—0.02	+ 16
76, 1692	+ 0.19	+0.54	— 0.64	— 0.10	+0.04	+0.05	+ 10
79, 1200	+ 0.02	—0.16	+ 0.12	— 0.04	0.00	+0.02	— 7
111, 1092	— 0.18	+0.01	+ 0.12	+ 0.13	+0.03	+0.03	— 14
143, 469	+ 0.02	—0.07	+ 0.04	— 0.03	+0.01	+0.02	— 11
171, 1581	— 0.33	—0.60	+ 1.01	+ 0.41	0.00	0.00	— 4
211, 43	+ 0.06	—0.09	+ 0.03	— 0.06	—0.03	+0.04	+ 26
212, 406	+ 0.02	+0.09	— 0.12	— 0.03	0.00	+0.01	+ 1
335, 873	+ 0.17	—0.40	+ 0.20	— 0.20	+0.02	0.00	— 14
379, 1635	+ 0.10	+0.09	— 0.18	— 0.09	0.00	—0.01	+ 2
469, 143	— 0.46	+0.60	— 0.10	+ 0.50	—0.02	—0.02	+ 23
534, 452	— 0.25	+0.14	+ 0.09	+ 0.23	+0.01	0.00	0
534, 452	— 0.08	+0.12	— 0.03	+ 0.09	0.00	0.00	0
554, 557	— 0.07	+0.04	+ 0.05	+ 0.09	+0.01	0.00	— 2
586, 277	+ 0.03	+0.02	— 0.04	— 0.02	0.00	0.00	+ 1
1289, 1635	— 0.05	—0.01	+ 0.06	+ 0.05	0.00	0.00	0
1487, 1581	+ 0.02	—0.19	+ 0.17	— 0.02	0.00	0.00	— 1
1581, 1487	— 0.03	+0.40	— 0.38	+ 0.02	0.00	+0.01	+ 2

TABLE III

Minor planets (*j*, *k*)	ΔM	$\Delta\omega$	$\Delta\Omega$	$\Delta\tilde{\omega}$	Δi	$\Delta\varphi$	$10^5\,\Delta\mu$
16, 1245	—9".30	+26".72	—14".58	+12".14	+1".45	+1".14	— 55"
21, 367	—0.01	+ 0.27	— 0.30	— 0.03	+0.02	—0.02	— 2
24, 1462	+0.97	— 3.30	+ 2.51	— 0.79	—0.09	—0.03	+ 5
39, 251	+6.55	— 5.63	— 1.76	— 7.39	+0.35	—0.43	+348
43, 211	+0.05	— 0.05	— 0.01	— 0.06	+0.01	—0.03	+ 20
47, 1541	—0.25	— 0.04	+ 0.25	+ 0.21	+0.01	—0.02	0
50, 1335	—0.01	+ 0.06	— 0.03	+ 0.03	—0.01	+0.02	— 16
76, 1692	—0.21	+ 0.71	— 0.60	+ 0.11	+0.05	—0.05	— 10
79, 1200	—0.02	— 0.08	+ 0.12	+ 0.04	+0.01	—0.02	+ 7
111, 1092	+0.16	— 0.23	+ 0.12	— 0.11	+0.03	—0.03	+ 14
143, 469	—0.03	+ 0.02	+ 0.03	+ 0.05	+0.01	—0.02	+ 11
171, 1581	+0.37	— 1.37	+ 0.92	— 0.45	0.00	0.00	+ 4
211, 43	—0.02	0.00	+ 0.03	+ 0.03	—0.02	—0.04	— 26
212, 406	—0.02	+ 0.14	— 0.11	+ 0.03	+0.01	—0.01	— 1
335, 873	—0.18	+ 0.05	+ 0.17	+ 0.22	+0.01	0.00	+ 14
379, 1635	—0.10	+ 0.28	— 0.19	+ 0.09	—0.01	+0.01	— 2
469, 143	+0.52	— 0.49	— 0.08	— 0.57	—0.03	+0.02	— 23
534, 452	+0.24	— 0.32	+ 0.10	— 0.22	0.00	0.00	0
534, 452	+0.08	— 0.05	— 0.04	— 0.09	—0.01	0.00	0
554, 557	+0.06	— 0.13	+ 0.05	— 0.08	+0.01	0.00	+ 2
586, 277	—0.02	+ 0.05	— 0.03	+ 0.02	+0.01	0.00	— 1
1289, 1635	0.00	— 0.09	+ 0.10	+ 0.01	0.00	0.00	0
1487, 1581	—0.06	— 0.07	+ 0.12	+ 0.05	0.00	0.00	+ 1
1581, 1487	+0.10	+ 0.16	— 0.24	— 0.08	0.00	—0.01	— 2

of proximity, and the value $\Delta L = +2''.84$ following the proximity. From Tables II and III we see that in the intervals preceding and following the proximity passages of the minor planets ☊ and i are subject to the changes of the same sens, i. e. they are either growing or diminishing. On the other hand the mean anomaly M, the longitude of perihelion $\tilde{\omega}$, the angle of eccentricity φ and the mean daily motion μ change, almost all, in opposite directions with respect to the instant of proximity. The argument of perihelion ω displays changes both in the same and opposite directions before and after proximity. This reflected itself in the total changes of the orbital elements given in Table IV for entire proximity areas, so that there appear both larger and smaller changes, or their value is zero. The instant of proximity has been taken as initial one in each of the pairs, $t_p = 0$. The changes $\Delta\mu$ in the mean daily motion are given in $(10^{-5})''$. It can be gathered from Table IV that sensible changes appear in ☊, ω, i and M.

TABLE IV

Minor planets (j, k)	ΔM	$\Delta\omega$	Δ☊	$\Delta\tilde{\omega}$	Δi	$\Delta\varphi$	$10^5\,\Delta\mu$
16, 1245	+1″.64	+25″.32	−27″.41	−2″.09	+2″.72	−0″.14	−22″
21, 367	0.00	+ 0.59	− 0.59	0.00	+0.03	0.00	0
24, 1462	−0.09	− 4.73	+ 4.80	+0.07	−0.18	0.00	− 1
39, 251	+0.54	+ 2.98	− 3.68	−0.70	+0.72	+0.02	+ 5
43, 211	+0.02	0.00	− 0.03	−0.03	+0.03	0.00	0
47, 1541	+0.01	− 0.49	+ 0.49	0.00	+0.03	0.00	0
50, 1335	−0.02	+ 0.10	− 0.07	+0.03	−0.02	0.00	0
76, 1692	−0.02	+ 1.25	− 1.24	+0.01	+0.09	0.00	0
79, 1200	0.00	− 0.24	+ 0.24	0.00	+0.01	0.00	0
111, 1092	−0.02	− 0.22	+ 0.24	+0.02	+0.06	0.00	0
143, 469	−0.01	− 0.05	+ 0.07	+0.02	+0.02	0.00	0
171, 1581	+0.04	− 1.97	+ 1.93	−0.04	0.00	0.00	0
211, 43	+0.04	− 0.09	+ 0.06	−0.03	−0.05	0.00	0
212, 406	0.00	+ 0.23	− 0.23	0.00	+0.01	0.00	0
335, 873	−0.01	− 0.35	+ 0.37	+0.02	+0.03	0.00	0
379, 1635	0.00	+ 0.37	− 0.37	0.00	−0.01	0.00	0
469, 143	+0.06	+ 0.11	− 0.18	−0.07	−0.05	0.00	0
534, 452	−0.01	− 0.18	+ 0.19	+0.01	+0.01	0.00	0
534, 452	0.00	+ 0.07	− 0.07	0.00	−0.01	0.00	0
554, 557	−0.01	− 0.09	+ 0.10	+0.01	+0.02	0.00	0
586, 277	+0.01	+ 0.07	− 0.07	0.00	+0.01	0.00	0
1289, 1635	−0.05	− 0.10	+ 0.16	+0.06	0.00	0.00	0
1487, 1581	−0.04	− 0.26	+ 0.29	+0.03	0.00	0.00	0
1581, 1487	+0.07	+ 0.56	− 0.62	−0.06	0.00	0.00	0

There are moments t_- and t_+, expressed in days, reckoned from $t_p = 0$, at which the perturbing changes in at least one of the orbital elements start and cease to be sensible. To these moments correspond the limiting values ρ_- and ρ_+ of the mutual distance of the two minor planets considered, the one distance being linked with the initial and the second with the final moment of sensible interaction. The larger of the two values with each one of the pairs is entered in the second

TABLE V

Minor planets (j, k)	$10^6 \rho$	Δv_j	Δv_k	Δt	Element
16, **1245**	194341₋	95°99708	96°02282	439ᵈ6000	ω, ϖ
21, **367**	1775₊	0.33040	0.31806	1.0900	ω, Ω
24, **1462**	207486₊	36.84040	35.55090	244.6000	ω, ϖ
39, **251**	179354₊	64.93400	68.75067	325.6000	ω, ϖ
43, **211**	2021₊	0.42332	0.49768	1.9000	ϖ
47, **1541**	3531₊	0.67200	0.66390	3.4000	ω
50, **1335**	321₋	0.27771	0.26335	0.6500	ω, Ω
76, **1692**	4196₊	0.43823	0.40216	2.2900	*M*
79, **1200**	1535₋	0.26707	0.30275	1.3300	ω
111, **1092**	3789₊	0.76418	0.80928	3.7750	ω, ϖ
143, **469**	1087₊	0.45199	0.47628	1.8500	ω, ϖ
171, **1581**	1662₊	0.91017	0.91798	4.2500	*M*
211, **43**	1705₋	0.43876	0.37324	1.6750	ω, ϖ
212, **406**	1837₊	0.48192	0.46129	2.2000	ω
335, **873**	1518₊	1.08736	1.12634	4.1000	ω, ϖ
379, **1635**	2296₊	0.78062	0.75381	3.7750	ω
469, **143**	1897₊	1.06202	1.00775	4.1250	*M*
534, **452**	1292₊	0.85443	0.85266	4.4500	*M*
534, **452**	1225₋	0.84741	0.84564	3.9250	ω
554, **557**	2642₊	0.99328	1.01375	3.1750	ω, ϖ
586, **277**	2695₊	0.54095	0.52589	3.0000	ω
1289, **1635**	505₊	1.05688	1.05564	4.5750	ω
1487, **1581**	445₋	0.56202	0.56589	3.9750	ω, Ω
1581, **1487**	476₋	0.63707	0.63270	4.4750	ω

column of Table V, given in 10^{-6} AU units, with the corresponding index $-$ or $+$, depending upon whether that value is related to positions before or after the proximity passage. In the third and the fourth column of the same Table are given the maximum angular widths Δv_j and Δv_k of the parts of orbits, over which the measurable perturbing action of the minor planet j upon the minor planet k around of their proximity can be stated. The fifth column comprises the durations Δt of this sensible gravitational action of the minor planets, $\Delta t = t_+ - t_-$, termed the dynamical proximity duration, which should be distinguished from the kinematic proximity duration as defined in Lazović, Kuzmanoski (1979b). By the kinematic proximity duration is termed the time interval around of proximity during which the mutual distances of the two minor planets are $\rho < 0.000400$ AU. The limiting distance 0.000400 AU has been adopted for the reason that at the lesser distances sensible mutual perturbations might be expected. It turned out that the dynamical proximity duration was, as a rule, longer than the kinematic proximity duration, except with the pairs (50, **1335**), (384, **722**), (577, **765**) and (1245, **16**). This is quite understandable for the limiting value of ρ in all of these pairs was less than 0.000400 AU. It is to be noted that throughout we had $\Delta t < \Delta T$, i. e. the investigated intervals ΔT were longer than the intervals Δt of the perceivable mutual perturbations of the minor planets considered. In the ultimate column of Table V are listed the orbital elements of the minor planet k affected for longer interval of time than other elements by the change caused by the action of the minor planet j. The values Δt in the preultimate column are related to those orbital elements. The greatest values of the limiting mutual distance ρ at which sensible changes are still obtained, the angular widths Δv_j and Δv_k and the dynamical proximity duration Δt, as can be seen in Table V, are associated with the minor planets pairs (16)Psyche and (**1245**) **Calvinia**, (24) Themis and (**1462**) **Zamenhof**, (39) Laetitia and (**251**) **Sophia**; thus with those in which the greatest mutual perturbations during proximity are found. At the same time we see that the perturbing masses, used for them, and listed in Table I, were the greatest. Having regard to the values of ρ in Table V we see that the mutual perturbations in the minor planets motions can manifest themeselves at considerably greater mutual distances than 0.000400 AU adopted earlier. In the above indicated three pairs the limiting ρ even several hundreds times is greater than 0.000400 AU, while the dynamical proximity duration is several hundreds, and even thousands times longer than their kinematic proximity duration, found earlier in Lazović, Kuzmanoski (1979b).

We proceed now by giving perturbations for some of the pairs not entered in our Tables on account of their small amounts and lower number of the orbital elements affected.

For the minor planets pair (84, **227**) the following amounts of perturbations have been obtained: $\Delta M = +0\overset{''}{.}01$ till the instant of the proximity passage t_p and $+0\overset{''}{.}01$ after that instant. Accordingly, for the whole time interval around the proximity there is no any measurable perturbing change in the mean anomaly. $\Delta\tilde{\omega}$ is $-0\overset{''}{.}01$ until proximity and $+0\overset{''}{.}01$ after it, thus we again have zero change over the complete interval around the proximity. $\Delta\mu$: $-0\overset{''}{.}00002$ until t_p and $+0\overset{''}{.}00002$ after t_p, therefore the final result is 0. As we can see there are no changes in the orbital elements extending over the whole proximity area, but they are confined to limited portions of it, whereby their amounts are hardly noticeable.

(163, **1874**), $\Delta\Omega$: $-0\overset{''}{.}02$ until t_p, $-0\overset{''}{.}01$ after t_p, $-0\overset{''}{.}03$ in all.

(215, **1851**), $\Delta\Omega$: $-0\overset{''}{.}06$ until t_p, $-0\overset{''}{.}06$ after t_p, $-0\overset{''}{.}12$ in all; $\rho = 0.000688_-$, $\Delta t = 0\overset{d}{.}7500$.

(227, **84**), ΔM: $-0\overset{''}{.}01$ up to t_p, $+0''01$ after t_p, $0\overset{''}{.}00$ in all; i changes by only $\Delta i = -0\overset{''}{.}01$ in the moment $+0\overset{d}{.}1000$.

(243, **1848**), $\Delta\omega$: $-0\overset{''}{.}04$ up to t_p, $-0\overset{''}{.}02$ after t_p, $-0\overset{''}{.}06$ in all; $\Delta\Omega$: $+0\overset{''}{.}03$ up to t_p, $+0\overset{''}{.}03$ after t_p, $+0\overset{''}{.}06$ in all; $\rho = 0.000581_+$, $\Delta t = 1\overset{d}{.}0500$.

(379, **461**), $\Delta\omega$: $-0\overset{''}{.}08$ up to t_p, $-0\overset{''}{.}10$ after t_p, $-0\overset{''}{.}18$ in all; $\Delta\Omega$: $+0\overset{''}{.}09$ up to t_p, $+0\overset{''}{.}09$ after t_p, $+0\overset{''}{.}18$ in all; $\rho_- = \rho_+ = 0.000985$, $\Delta t = 0\overset{d}{.}7000$.

(384, **722**), $\Delta\Omega$: $+0\overset{''}{.}02$ up to t_p, $+0\overset{''}{.}01$ after t_p, $+0''03$ in all; $\rho = 0.000373_-$, $\Delta t = 0\overset{d}{.}2000$.

(577, **765**), $\Delta\Omega = +0\overset{''}{.}02$ for only 0.1000 days after proximity, $\rho = 0.000378_+$.

(848, **1363**), $\Delta\omega$: $+0\overset{''}{.}04$ up to t_p, $+0\overset{''}{.}06$ after t_p, $+0\overset{''}{.}10$ in all; $\Delta\Omega$: $-0\overset{''}{.}05$ up to t_p, $-0\overset{''}{.}05$ after t_p, $-0\overset{''}{.}10$ in all; $\rho = 0.000442_-$, $\Delta t = 0\overset{d}{.}8000$.

(962, **1802**), Ω changes by only $\Delta\Omega = -0\overset{''}{.}01$ in the moment $+0\overset{d}{.}0500$.

(1079, **1100**), $\Delta\omega$: $-0\overset{''}{.}10$ up to t_p, $-0\overset{''}{.}09$ after t_p, $-0\overset{''}{.}19$ in all; $\Delta\Omega = +0\overset{''}{.}09$ up to t_p, $+0\overset{''}{.}10$ after t_p, $+0\overset{''}{.}19$ in all; $\rho = 0.000538_+$, $\Delta t = 0\overset{d}{.}7500$.

(1135, **1381**), $\Delta\Omega$: $+0\overset{''}{.}01$ up to t_p, $+0\overset{''}{.}01$ after t_p, $+0\overset{''}{.}02$ in all; $\rho = 0.000403_+$, $\Delta t = 0\overset{d}{.}5500$.

(1245, **16**), $\Delta\omega$: $-0''03$ up to t_p, $-0''05$ after t_p, $-0''08$ in all; $\Delta\Omega$: $+0''04$ up to t_p, $+0''04$ after t_p, $+0''08$ in all; i changes by only $\Delta i = -0''01$ in the moment $+0^d0500$; $\rho = 0.000336_+$, $\Delta t = 0^d7500$.

(1331, **938**), $\Delta\omega$: $+0''03$ up to t_p, $+0''01$ after t_p, $+0''04$ in all; $\Delta\Omega$: $-0''02$ up to t_p, $-0''02$ after t_p, $-0''04$ in all; $\rho = 0.000563_+$, $\Delta t = 0^d6000$.

The estimated orbital elements changes of the now investigated quasicomplanar minor planets point once again to the significant possibility of using their interactions, taking place duing proximities, for a more reliable estimate of their masses.

The calculations were carried out on the IBM 360/44 of the Computing Centre of the Institute for Mathematics in Beograd

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