UDC 523.9-327 OSP

DETAILED TREATMENT OF SYNODIC SOLAR ROTATION

Kubičela A.

Astronomical Observatory, Beograd

Karabin M.

Institute of Astronomy, Faculty of Sciences, Beograd

Received June 25, 1981

Summary. Begining with a previous vectorial result about the synodic solar rotation dependance on the inclination of solar equator the classical definition of synodic solar rotation has been reconsidered in some detail. The angular difference between one synodic and sidereal solar rotation turn, $\Delta\lambda$, as a function of the longitude of the Earth and the inclination of the solar equator, β, indicates a substantially different nature of trigonometric and vectorial definition of synodic solar rotation. Only a long-term mean value of $\Delta\lambda$ does not depend on β. For the extreem case, $\beta = 90°$, $\Delta\lambda$ shows a purely geometrical discontinuity. On contrary, the sideral, apparent annual and synodic rotations of the solar globe are smooth and can be described by the corresponding vectorial angular velocities.

Kubičela A. i Karabin M. DETALJNO RAZMATRANJE SINODIČKE SUNČEVE ROTACIJE — Polazeći od jednog ranije datog rezultata, po kome sinodička Sunčeva rotacija zavisi od nagiba Sunčevog ekvatora, detaljno je razmotrena klasična definicija sinodičke Sunčeve rotacije. Uglovna razlika, $\Delta\lambda$, između jednog sinodičkog i sideričkog Sunčevog obrta, posmatrana kao funkcija longitude Zemlje i nagiba Sunčevog ekvatora, β, ukazuje na bitnu razliku između trigonometrijske i vektorske definicije Sunčeve sinodičke rotacije. Samo dugoročne srednje vrednosti $\Delta\lambda$ ne zavise od β. U ekstremnom slučaju, $\beta = 90°$, $\Delta\lambda$ ispoljava jedan čisto geometrijski diskontinuitet. Nasuprot ovome, Sunčeva siderička, prividna godišnja i sinodička rotacija su neprekidne i mogu se opisati odgovarajućim vektorskim uglovnim brzinama.

INTRODUCTION

In our first attempt to interpret solar rotational velocities as vectors (Kubičela and Karabin, 1982) one of the results suggested is the dependance of synodic solas rotation on the inclination of the solar equator. As that seemed inconsistent with the usual scalar treetment of synodic solar rotation some further investigations were needed.

Therefore in this paper we repeat a part of the text and Figure 1 from the mentioned paper (copyright by Reidel Publishing Company) and in an extended analysis we look for a possible explaination of that controversal result.

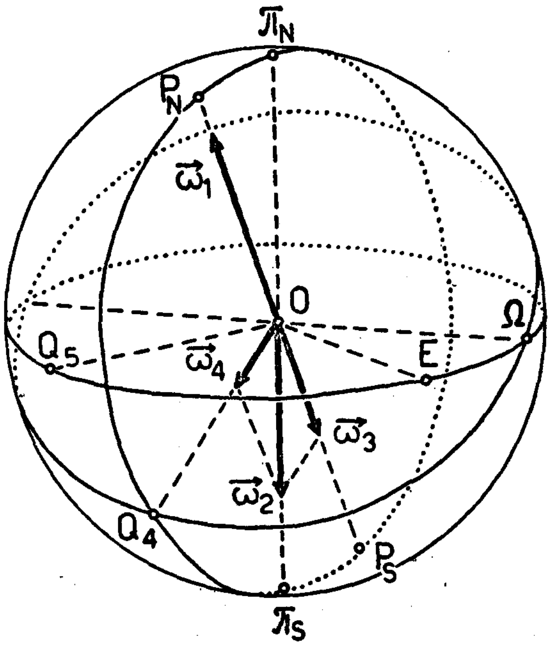

Fig. 1. Projection, ω_3, of the apparent angular velocity introduced by the Earth's revolution, ω_2, onto the solar rotation axis, $P_N - P_S$. 0 is the centre of the Sun, ω_1 is the angular velocity of the sidereal solar rotation and $0\pi_N$ and $0\pi_S$ are the directions toward the north and south ecliptic poles respectively.

COLINEAR COMPONENT OF SYNODIC SOLAR ROTATION

The line-of-sight component of solar rotation is usually expressed as

$$V_1 = R\,\omega_1 \cos\varphi_m \sin\lambda_m \cos B_0, \tag{1}$$

where ω_1 is the angular velocity of sidereal solar rotation. R is radius of the Sun, φ_m and λ_m are heliographic latitude and longitude of the observed photospheric point, M, and B_0 is heliographic latitude of the centre of the solar disk.

The Earth's revolution introduces an apparent angular velocity of the opposite direction to the solar rotation. Its line-of-sight component expressed in the ecliptic coordinate system is analogous to (1)

$$V_a = R\,\omega_2 \cos b_m \sin(l_m - L),$$

where ω_2 represents the sidereal orbital angular velocity of the Earth, b_m and l_m are heliocentric ecliptic latitude of any given point at the Sun, and L is heliocentric longitude of the Earth. Angular velocity ω_2 is a periodic function of time according to the second Kepler's law: $\omega_2 = 2\pi\, ab\, P^{-1} r^{-2}$, where r is Sun-Earth radiusvector, a and b are the semiaxes of the Earth's orbit and P is the period of its revolution.

The annual change of ω_2 is about 3.3%. Such a variability is to be assumed in other angular velocities derived from it.

In order to clarify the influence of $\boldsymbol{\omega}_2$ on $\boldsymbol{\omega}_1$ it is necessary to project ω_2 onto the rotation axis of the Sun. Let vector $\boldsymbol{\omega}_1$, Figure 1, be the angular velocity of sidereal solar rotation. The apparent rotation velocity introduced by the Earth's revolution, $\boldsymbol{\omega}_2$, lies in the direction toward the south ecliptic pole π_S. This velocity can be represented by its two components: $\boldsymbol{\omega}_3 = \boldsymbol{\omega}_2 \cos\beta$ colinear with $\boldsymbol{\omega}_1$ (β being the inclination of the solar equator to the ecliptic), and $\boldsymbol{\omega}_4 = \boldsymbol{\omega}_2 \sin\beta$ perpendicular to the solar rotation axis. It is now obvious that $\boldsymbol{\omega}_3$ can be readily subtracted from $\boldsymbol{\omega}_1$ in (1), resulting in

$$V_s = \Delta\,(\omega_1 - \omega_2 \cos\beta) \cos\varphi_m \sin\lambda_m \cos B_0. \tag{2}$$

Taking into account the numerical value of ω_2, one finds the mean angular velocity $\overline{\omega_3} = 1.975072 \times 10^{-7}$ rad s^{-1}, or as a peripheral velocity at the solar equator $V_3 = 137.5$ m s^{-1}. This value is only about 1 m s^{-1} smaller than the one usuelly applied, namely 138.6 m s^{-1}.

The factor $\cos\beta$ at the right-hand side of (2) is in contradiction with the usual relation between the sidereal and synodic solar rotation:

$$\text{sidereal} - \text{synodic rotation} = \text{Earth orbital motion}, \tag{3}$$

given in Allen (1964). In terms of angular velocities relation (3) perhaps may be understood as

$$\omega_{syn} = \omega_1 - \omega_2, \tag{3a}$$

where ω_{syn} is the angular velocity of synodic solar rotation. The relations (3) or (3a) do not show any dependance of ω_{syn} on the inclination of the solar equator.

This problem we are trying to solve scrutinizing the classical definition of synodic solar rotation.

TRIGONOMETRIC DESCRIPTION OF SYNODIC SOLAR ROTATION

According to the generally accepted definition, the synodic period of rotation of the solar globe is the interval between two successive passages of a given heliographic meridian through the centre of the solar disk. All necessary geometrical parameters are shown in Figure 2. Heliocentric longitudes of the Earth L_1 and L_2 are measured from the ascending node Ω. Then ΔL is the change of heliocentric longitude of the Earth during one solar synodic rotation period and $\Delta\lambda$ is the corresponding heliographic longitude difference between one synodic and sidereal solar rotation turn.

During one sidereal rotational turn of the solar globe, T_{sid}, the given central meridian, $P_N C_1 P_S$, completes 360° returning at the same starting position. Meanwhile the Earth moves along the ecliptic so that the centre of the solar disk moves from C_1 toward C_2 and the selected central meridian has to turn for an additional interval in heliographic longitude, $\Delta\lambda$, to reach C_2 and to become the central meridian again. We can take the Earth's longitude change ΔL as constant (neglecting the elipticity of the Earth's orbit) and approximately equal to 27°. In spite of ΔL

being constant along the ecliptic, $\Delta\lambda$ changes during the revolution of the Earth and depends on the inclination of the solar equator, β.

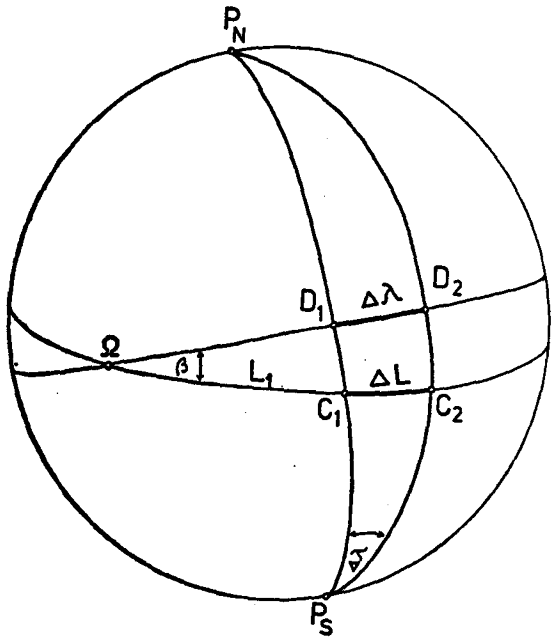

Fig. 2. Trigonometrical definiton of synodic solar rotation. P_N and P_S are the north and south solar rotation poles, $\Omega\, D_1\, D_2$ is the solar equator, and $\Omega\, C_1\, C_2$ is the ecliptic. C_1 and C_2 are centres of solar disk at the moments of two successive passages of the same heliographic meridian through the centre, with ΔL and $2\pi+\Delta\lambda$ being the Earth's heliocentric and heliographic longitude increments corresponding to one synodic solar turn. $C_1\, D_1 = B_{01}$ and $C_2\, C_2 = B_{02}$ are the heliographic latitudes of the centres of the solar disk C_1 and C_2 respectively.

A convenient way to find $\Delta\lambda$ as a function of ΔL and β is from the spherical triangle $C_1 P_S C_2$, namely

$$\cos \Delta L = \sin B_{01} \sin B_{02} + \cos B_{01} \cos B_{02} \cos \Delta\lambda. \tag{4}$$

The heliographic latitude of the centre of the solar disk, B_0, depends on the Earth's longitude in the following way·

$$\sin B_{0i} = \sin \beta \sin L_i, \tag{5}$$

where $i = 1$ or 2. From (4) and (5) one finds

$$\Delta\lambda = \arccos \frac{\cos (L_2 - L_1) - \sin^2 \beta \sin L_1 \sin L_2}{\sqrt{(1 - \sin^2 \beta \sin^2 L_1)(1 - \sin^2 \beta \sin^2 L_2)}} \tag{6}$$

A similar relation has been publish by Graff (1974).

The influence of parameter β on $\Delta\lambda$ up to the fictive value $\beta = 90°$ is shown in Figure 3. The function $\Delta\lambda$ changes smoothly with β from a constant 27° — value to as close as 90° when obtained the discrete form $\Delta\lambda = 0°$, for $0° < L_1 < 90° - \Delta L$ and $90° < L_1 < 180°$, but $\Delta\lambda = 180°$ for $90° - \Delta L < L_1 < 90°$. The last value appears if the solar rotatin pole, when $\beta \to 90°$, falls into the ΔL — intervel at the ecliptic, and the former one appears if pole falls out of that interval.

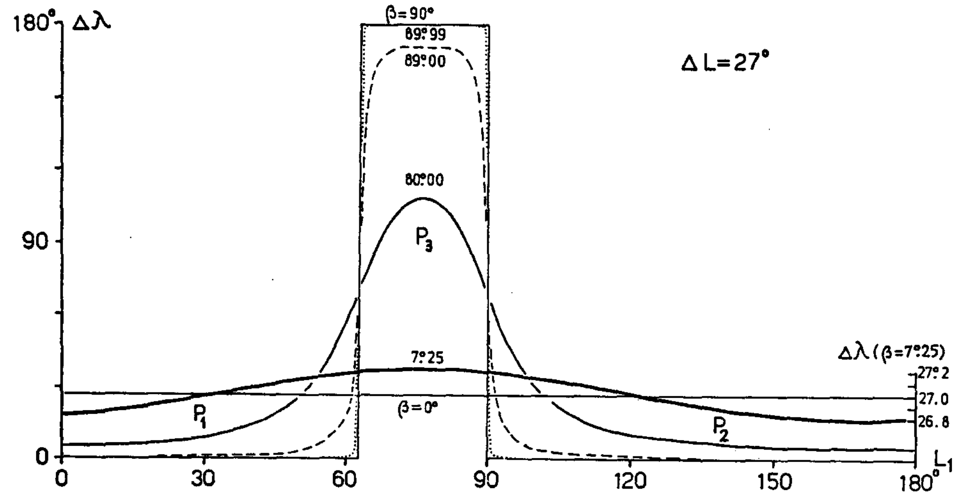

Fig. 3. Heliographic longitude difference, $\Delta\lambda$, between one synodic and sidereal solar rotation turn is given as a function of the Earth's longitude L_1 and parameter β for $\Delta L = 27°$. Six $\Delta\lambda$ — curves for $\beta = 0°$ to $\beta = 90°$ are shown. The ordinate scale for the curve $\beta = 7°25$ has been enlarged 50 times and centered at $\Delta\lambda = 27°$ (at the right). Notice the constancy of the mean value of $\Delta\lambda$ — function within the shown six-month interval and throughout the whole interval of β — values.

One can also see that for $\beta = 0°$ at any instant $\Delta\lambda = \Delta L = 27°$. For other values of β the mean integral value of $\Delta\lambda$ within the observed 180° — interval, $\overline{\Delta\lambda}$, equals to ΔL or 27°, including the case $\beta = 90°$ when the three rectangular areas on both sides of the ordinate 27° satisfy the relation $P_1 + P_2 = P_3$. As each value of $\Delta\lambda$ defines a synodic turn of the solar globe, such a behavior of this quantity means that the mean value of synodic period T_{syn} within a six-month oı an annual time interval is constant and independent of the inclination of the solar equator. Hence, when we deal with such mean synodic periods, or with $\beta = 0°$, we can take the relation (3) as a correct one.

However, this conclusion does not seem to be consistent with the vectorial relation (2) except for the case $\beta = 0°$. To point out the difference between the trigonometric and vectorial description of synodic solar rotation, let us look at the Figure 4a where the vicinity of the centre of the solar disk, C, is represented and the uniform apparent motion of the north solar rotation pole, P_1 to P_{17}, along the ecliptic, E — W, is shown for the case $\beta = 90°$. An arbitrary heliographic meri-

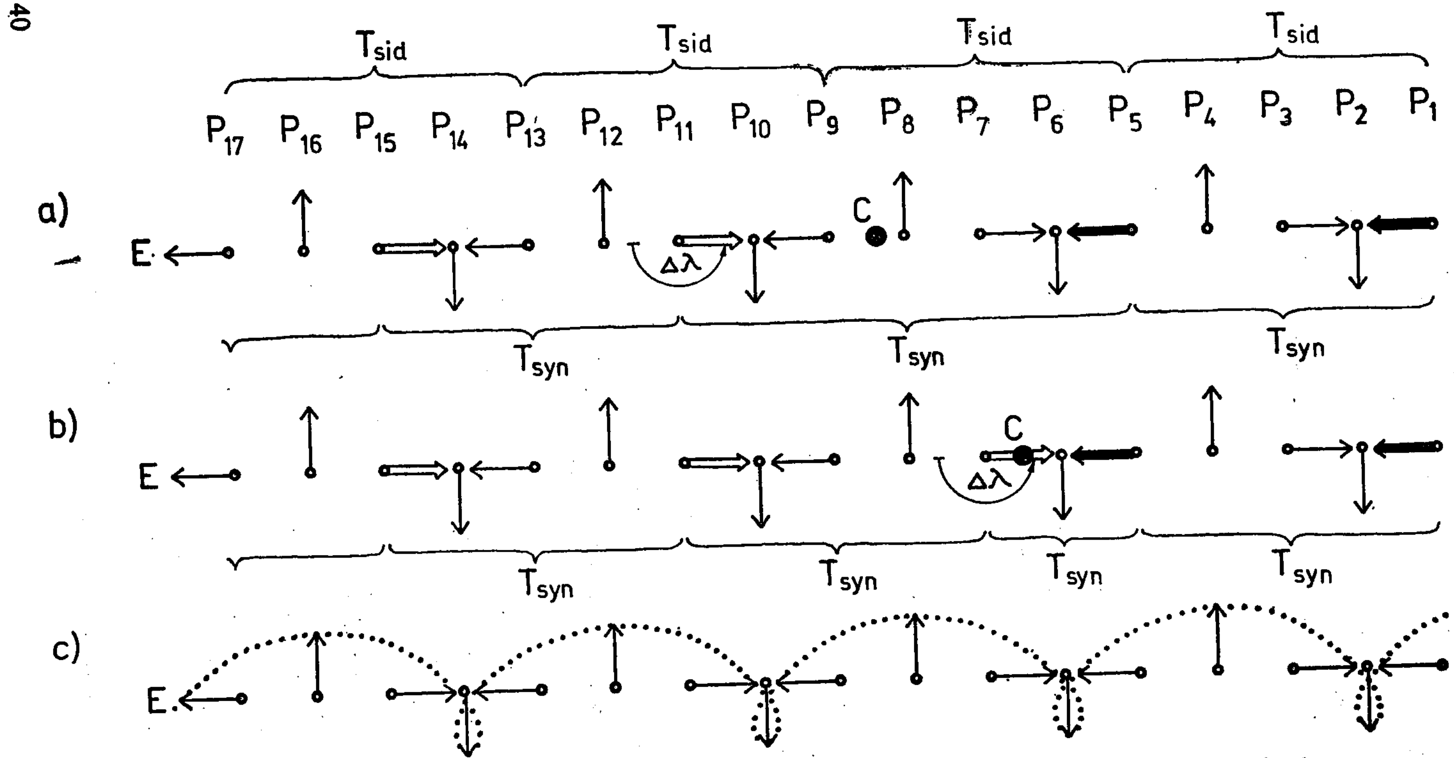

Fig. 4. As follows from the text (for $\beta = 90°$) T syn $= T$ sid except if the centre C and the pole P simultaneously fall in the same ΔL — interval when T syn $\gtrless T$ sid. Due to direction-dependent geometrical definition of central meridian, two different positions of C are possible: between P_7 and P_9 in a) and between P_5 and P_7 in b). In both cases, a) and b), $\Delta\lambda = 180°$ for the central rotation period, and $\Delta\lambda = 0°$ for the others. Notice that the synodic angular velocity (e.g. the dotted curve at 4c) as well as the sidereal and apparent annual velocities of the solar globe have no discontinuities.

dian is represented by the short arrows starting at the pole positions and pointing along the intantaneous directions of the selected meridian. Its starting direction at P_1 has been arbitrarily chosen eastward along the ecliptic and the counter-clockwise sidereal solar rotation around the pole is shown in 90°-steps. The corresponding sidereal periods of rotation (from one eastward position of the observed meridian till its next eastward position) T_{sid}, have been marked above. The first shown synodic period, T_{syn}, begins also at P_1 as the observed meridian, pointing eastward and passing through C, represents the central meridian as well (heavy arrow). It lasts till P_5 where the next eastward direction of the observed meridian again has the role of the central meridian. Here the synodic period of rotation is equal to the corresponding sidereal one. In such cases $\Delta\lambda = 0°$. But if during a sidereal period the rotation pole passes the centre of the solar disk, the observed meridian has to reach the opposite direction to become the new central meridian (white arrow at P_{11}). Between the last eastward position of the meridian, at P_9, and its new position at P_{11} a conversion of direction of the central meridian, followed by the heliographic longitude increment $\Delta\lambda = 180°$, takes place. In the remaining part of a six-month interval all synodic periods are again equal to the corresponding sidereal ones.

An interesting case seems to appear in Figure 4b. Namely, if the centre of the solar disk, C, happens to fall in the first half of an arbitrarily started sidereal period (e.g. between P_5 and P_7 for the shown distribution of sidereal periods), the conversion of direction of the central meridian takes place at P_7 (the white arrow) with the consequence $T_{syn} < T_{sid}$. This case indicates a whole series of formal solutions for $\Delta\lambda$ and T_{syn} (even with T_{syn} close to zero for $\beta<90°$) that do not take into account the necessary completion of one full sidereal turn within the observed synodic one. This condition is so important that it should be included os understood in the classical trigonometric definition of the synodic rotation of the Sun. Provided the one-sidereal-turn condition is satisfied, the case b) in Figure 4 becomes a) and the observed synodic turn, lasting for (3/2) T_{sid}, completes at P_{11}.

Excluding the critical ΔL interval (P_5 to P_9) with a kind of an „artificial" discontinuity of centrel meridian direction (amounting to $\Delta\lambda$) we can take $T_{syn} = T_{sid}$ or $\omega_{syn}=\omega_1$ what also follows from the earlier vectorial result for $\beta = 90°$. Besides, it is worth noticing in Figure 4 that the mentioned discontinuity — causing a prolongation of T_{syn} compared to T_{sid} — does not influence either of the two component solar motions involved (the sidereal rotation and the apparent effect of the Earth's revolution) as well as the resulting synodic rotation. The last motion is shown in Figure 4c as an uninterrupted and smooth cycloidal motion of an arbitrary photospheric point, namely the end of the arrow indicating the selectead heliographic meridian (the dotted curve). Unlike the synodic solar rotation defined through $\Delta\lambda$ and T_{syn}, the smooth cycloidal synodic motion at 4c) can be described by corresponding vectorial angular velocity of synodic solar rotation.

CONCLUSION

Although fictious, the extreme case $\beta = 90°$ nicely shows the substantial discrepancy between the classical trigonometric definition of synodic solar rotation and the alternative vectorial notion — the angular velocity of synodic solar rotation.

Therefore:

1) In the trigonometric definition of synodic solar turn, besides the fundamental quantity $\Delta\lambda$, we have to imply the completion of one full sidereal turn as well. We also may apply the relation (3) in connection with notion as „synodic rotation period" only when we deal with long-period (six-moth or annual) mean values. Otherwise, the relations (3) or (3a) can be taken as an approximation only.

2) The physical (vectorial) notion of synodic rotation angular velocity depends on β, seems free of any discontinuities and can be regarded as suitable for evaluation of instantaneous velocities of individual points at the solar globe.

3) It seems promissing to pay some more attention to the vectorial approach of the synodic rotation and to develop it to a greater extent.

*

This work is a part of the research project of the Basic Organization of Associated Labour for Mathematics, Mechanics and Astronomy of the Belgrade Facnlty of Sciences, funded by thə Republic Community of Sciences of Serbia.

REFERENCES

Allen, C.W.: 1964, Astrophys. Quantities, Univ. of London, The Athlone Press, p. 179.

Graff, W.: 1974, Solar Phys., 37, 257—260.

Kubičela, A. and Karabin, M.: 1982, Proc. VI European Regional Meeting in Astronomy, Dubrovnik, 19—23. X 1982, eds. Fricke, W. and Teleki, G., Reidel Publ. Comp., Dordrecht, 1982, in press.