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CONTRIBUTION TO THE PROXIMITY DETERMINATION OF NON-QUASICOMPLANAR ELLIPTICAL ORBITS OF CELESTIAL BODIES

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Summary. Simple formulae are derived for determination of approximate true anomalies of proximity positions of two celestial bodies moving in non-quasicomplanar elliptical orbits. These anomalies are necessary for the finding of the minimum mutual distance of these bodies. The method seems appropriate to serial investigation and can be applied to the discussion of the quasicomplanar orbits as well. The tangents of the true anomalies as functions of the known vector constants **P, Q, R** are used at deriving the formulae.

J. Lazović, PRILOG ODREĐIVANJU PROKSIMITETA NEKVAZIKOMPLANARNIH ELIPTIČNIH PUTANJA NEBESKIH TELA — Izvedeni su jednostavni obrasci za nalaženje približnih vrednosti pravih anomalija položaja proksimiteta dvaju nebeskih tela koja se kreću po nekvazikomplanarnim eliptičnim putanjama. Poznavanje ovih anomalija je neophodno za određivanje najmanje daljine između ovih tela. Postupak je efikasan i za serijsko ispitivanje, a može da bude koristan i kod razmatranja kvazikomplanarnih putanja. Formule su date preko tangensa ovih anomalija, a u funkciji poznatih vektorskih konstanata **P, Q** i **R.**

Earlier I have derived the exact equations $f(v_j, v_k) = 0$, $g(v_j, v_k) = 0$ valid for the general solution of the problem of proximity (minimum distance) of two elliptical orbits (Lazović, 1967). Thus, they apply both to the quasicomplanar and non-quasicomplanar elliptical orbits of two celestial bodies, i. e. to those with the very low mutual inclination and at the same time to those with high mutual inclination I. These equations being transcendent, their solution can be performed by successive approximations, provided their approximate solutions are known. I have up to now exposed several ways of solving these equations (Lazović, 1967, 1974, 1976, 1978) a graphic method, a graphic-numerical one and a purely numerical solutions. With these values, regarded as initial ones, one is able to proceed to the calculus of the corresponding corrections to the true anomalies. This calculus is pushed on, until final solutions of the above equations with the necessary accuracy are obtained. However, these solutions were all related to the quasicomplanar orbits of celestial bodies. We made use of them in our earlier investigations of the quasicomplanar minor planets, moving in nearly the same plane. The upper limit of inclination between their respective orbital planes has been taken to be $0\overset{\circ}{.}500$. Now, however, we are going to consider the non-quasicomplanar orbits, with mutual inclinations considerably exceeding the above stated value.

The requisite approximate values of true anomalies v_j and v_k of the proximity positions of two celestial bodies j and k, moving in non-quasicomplanar elliptical orbits, can be determined by assuming them to correspond to the relative nodes positions or to the positions of „crossings" of their orbits. By the relative nodes of the elliptical orbits of the pair (j' k) of celestial bodies are termed the points where one of the orbits cuts the orbital plane of the second celestial body. It is obvious that the proximity will be all closer to the relative node as the angle I between the orbital planes is higher and the orbital eccentricities lower. The heliocentric directions of the straightline, joining the relative nodes of two orbits, are, as a matter of fact, perpendicular to the perpendiculars to the orbital planes of two celestial bodies under consideration. This perpendicularity will be exploited in our deducing of the formulae serving for the calculus of the anomalies needed.

The mutual inclination I of the elliptical orbits of the pair (j, k) of the celestial bodies can be determined by means of the formulae presented in the paper Lazović, Kuzmanoski (1974). Let $\mathbf{r}_j$ and $\mathbf{r}_k$ be heliocentric position vectors of the bodies j and k in the direction of one of the relative nodes of their orbits. Thus, these vectors are co-linear. If $\mathbf{R}_j$ and $\mathbf{R}_k$ are the unit vectors in the directions of the perpendiculars to the orbital planes of these bodies then, on account of the above indicated perpendicularity, we can write these two equations for the scalar products of the corresponding vectors

$$(\mathbf{r}_j \, \mathbf{R}_k) = 0, \qquad (\mathbf{r}_k \, \mathbf{R}_j) = 0. \tag{1}$$

The heliocentric position vectors of these bodies as a functions of their true anomalies are

$$\mathbf{r}_i = r_i \cos v_i \, \mathbf{P}_i + r_i \sin v_i \, \mathbf{Q}_i,$$
$$r_i = \frac{a_i (1 - e_i^2)}{1 + e_i \cos v_i}, \quad i = j,\ k, \tag{2}$$

whereby $\mathbf{P}_i$ and $\mathbf{Q}_i$ are unit vectors in the orbital plane, the first one in the direction of the perihelion and the second one perpendicular to the first. The heliocentric ecliptical rectangular coordinates of the vector constants $\mathbf{P}_i$, $\mathbf{Q}_i$ and $\mathbf{R}_i$ can be derived from the elliptical orbital elements of the bodies under consideration by the known expressions

$$\left.\begin{aligned}
\mathbf{P}_i &= \begin{cases} \cos \omega_i \cos \Omega_i - \sin \omega_i \sin \Omega_i \cos i_i, \\ \cos \omega_i \sin \Omega_i + \sin \omega_i \cos \Omega_i \cos i_i, \\ \sin \omega_i \sin i_i, \end{cases} \\
\mathbf{Q}_i &= \begin{cases} -\sin \omega_i \cos \Omega_i - \cos \omega_i \sin \Omega_i \cos i_i, \\ -\sin \omega_i \sin \Omega_i + \cos \omega_i \cos \Omega_i \cos i_i, \\ \cos \omega_i \sin i_i, \end{cases} \\
\mathbf{R}_i &= \begin{cases} \sin \Omega_i \sin i_i, \\ -\cos \Omega_i \sin i_i, \qquad i = j,\ k. \\ \cos i_i, \end{cases}
\end{aligned}\right\} \tag{3}$$

From (1) and (2) we obtain

$$r_j \cos v_j (\mathbf{P}_j \mathbf{R}_k) + r_j \sin v_j (\mathbf{Q}_j \mathbf{R}_k) = 0,$$

$$r_k \cos v_k (\mathbf{P}_k \mathbf{R}_j) + r_k \sin v_k (\mathbf{Q}_k \mathbf{R}_j) = 0,$$

whence, by performing the division of the first expression by r_j and the second by r_k,

$$\sin v_j (\mathbf{Q}_j \mathbf{R}_k) + \cos v_j (\mathbf{P}_j \mathbf{R}_k) = 0,$$

$$\sin v_k (\mathbf{Q}_k \mathbf{R}_j) + \cos v_k \mathbf{P}_k \mathbf{R}_j) = 0,$$

is found. Hence the simple formulae

$$\operatorname{tg} v_j = -\frac{(\mathbf{P}_j \mathbf{R}_k)}{(\mathbf{Q}_j \mathbf{R}_k)}, \qquad \operatorname{tg} v_k = -\frac{(\mathbf{P}_k \mathbf{R}_j)}{(\mathbf{Q}_k \mathbf{R}_j)} \tag{4}$$

can be derived, by means of which the searched approximate values of the true anomalies v_j and v_k of the proximity of two non-quasicomplanar elliptical orbits of the celestial bodies j and k are obtained. On the right-hand sides of the expressions (4) are the scalar products of the vector constants $\mathbf{P}_i$, $\mathbf{Q}_i$, $\mathbf{R}_i$, $i = j, k$, which, with respect to (3), can be regarded as known.

In the practice, each one of the expressions (4) yields two solutions: v_{j1}, v_{j2} and v_{k1}, v_{k2}, whose values differ, for the same body, by 180°. As we are unable to decide in advance which one of them corresponds to the proximity of the two considered orbits, the question of the correct choice can be solved in the following way. With the four values of the true anomalies we determine four radius vectors, using thereby the second equation in (2), and four true longitudes according the formula $L_i = \Omega_i + \omega_i + v_i$. Next we form the differences $\Delta r_{jk} = r_j - r_k$ of the radius vectors combinations and the differences $\Delta L_{jk} = L_j - L_k$ of the true longitudes combinations for the two bodies. The required values v_j and v_k are those resulting from the smallest absolut values of these differences. It is known that the proximity positions are nearer to the one of the two existing relative nodes of the pair of orbits considered, and that the heliocentric position vectors in the direction of one of the relative nodes are oriented in the same sense. So, for instance, if it turned out that the smallest absolute values have the differences $r_{j2} - r_{k1}$ and $L_{j2} - L_{k1}$, then the solutions v_{j2} and v_{k1} would, in fact, represent the required approximate values. By using these values we would then get down to the calculus of their corrections. In so doing we would find out the final, i. e. exact values for the proximity according to the formulae given in the already mentioned papers Lazović (1967, 1974) and Lazović, Kuzmanoski (1978).

The formulae derived have been applied to the pair of two first numbered minor planets (1) Ceres and (2) Pallas, whose orbital elements are taken from Ephemeris of Minor Planets for 1980. As the inclination between their orbital planes is $I = 36\overset{\circ}{.}65323$, this pair of the minor planets must be regarded as a highly non-quasicomplanar one. The formulae (4) furnish $\operatorname{tg} v_1 = 0.6574196$ and $\operatorname{tg} v_2 = 2.471147$, whence it follows: $v_{11} = 33\overset{\circ}{.}32170$ and $v_{12} = 213\overset{\circ}{.}32170$ for (1) Ceres, and $v_{21} = 67\overset{\circ}{.}96828$ and $v_{22} = 247\overset{\circ}{.}96828$ for (2) Pallas. Upon the scrutiny of these values we arrived at the conclusion that the approximate values of the true anomalies of the proximity positions in this particular instance are $v_1 = 213^\circ\!.32170$ and $v_2 = 247^\circ\!.96828$, which in fact correspond to a relative node of these minor planets. By inserting these anomalies in our equations of conditions for proximity we get $f = -0.002393596$, $g = 0.012147263$. Upon calculating the first corrections new approximate values $v_1 = 212^\circ\!.85919$, $v_2 = 247^\circ\!.32579$ are determined, yielding $f = -0.000006956$, $g = 0.000052247$. Finally, after second corrections to the true anomalies are derived, we obtain $v_1 = 212^\circ\!.85695$, $v_2 = 247^\circ\!.32283$ whereby the equations for the proximity are satisfied the values they assume are $f = 0.000000000$, $g = 0.000000001$. For this reason the last values of the true anomalies of the two minor planets (1) Ceres and (2) Pallas correspond to the positions of the minimum (proximity) distance between their orbits, the amount of which being $\rho_{min} = 0.0626963$ AU. Accordingly, we first calculated approximate values of the true anomalies of proximity using the formulae (4). Thereupon, after finding only two of the corrections, we obtained the required exact true anomalies of the proximity positions. Thus, the method used proved very effective. From the above values one can see that the angular distances of the proximity positions from the nearer relative node are $\Delta v_1 = 0^\circ\!.46475$, $\Delta v_2 = 0^\circ\!.64545$ and that the proximity preceds this relative node.

The same procedure has been applied to the hitherto most pronounced quasicomplanar minor planets pair (215) Oenone and (1851) $\equiv$ 1950 *VA*, the proximity of which has been determined in one of the previous papers (Lazović, Kuzmanoski, 1978). The previously used values of their orbital elements have here again been exploited. We had $I = 0^\circ\!.007$. The formulae (4) furnish these approximate values of the true anomalies of the proximity positions $v_{215} = 82^\circ\!.44317$, $v_{1851} = 61^\circ\!.77989$, which at the same time applie to one of the relative nodes of the orbits of this minor planets pair. However, from a previous paper (Lazović, Kuzmanoski, 1978), we know that the exact values of the proximity positions are $v_{215} = 83^\circ\!.09412$, $v_{1851} = 62^\circ\!.43319$, by means of which we found the proximity distance to be $\rho_{min} = 0.000004$ AU. We see that this proximity occupies a position which succeeds the given relative node at angular distances of $\Delta v_{215} = 0^\circ\!.65095$, $\Delta v_{1851} = 0^\circ\!.65330$.

Simovljevitch (1977) has deduced the formulae by which the approximate values of the eccentric anomalies of the proximity can be determined. Our formulae (4), however, seem to be more straightforward and simpler.

The procedure presented here proved to be a useful contribution to the question of the determination of proximity of non-quasicomplanar orbits of celestial bodies. But it can be very efficacious in the serial investigations. At the same time it can successfully be used in treating the quasicomplanar orbits.

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