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PERTURBATIONS IN THE MOTION OF THE QUASICOMPLANAR MINOR PLANETS FOR THE CASE PROXIMITIES ARE UNDER 10000 KM

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Summary. Mutual gravitational action during proximities of 12 quasicomplanar minor planets pairs have been investigated, whose minimum distances were under 10000 km. In five of the pairs perturbations of several orbital elements have been stated, whose amounts are detectable by observations from the Earth.

J. Lazović, M. Kuzmanoski, POREMEĆAJI ELEMENATA KRETANJA KVAZIKOMPLANARNIH MALIH PLANETA U PROKSIMITETIMA NJIHOVIH PUTANJA SA DALJINAMA MANJIM OD 10000 KM — Ispitali smo međusobna gravitaciona dejstva pri proksimitetima 12 parova kvazikomplanarnih malih planeta sa minimalnim daljinama ispod 10000 km. Kod pet parova nađeni su poremećaji u više putanjskih elemenata, čiji bi se iznosi mogli ustanoviti posmatranjima sa Zemlje.

Earlier we have found out 13 quasicomplanar minor planets pairs (the angle I between their orbital planes less than $0\overset{\circ}{.}500$), whose minimum mutual distances (ρ_{min}) were less than 10000 km (Lazović, Kuzmanoski, 1978). The shortest proximity distance of only 600 km among these pairs has been stated with minor planet pair (215) Oenone and 1851 ≡ 1950 *VA*. The corresponding perturbation effects in this pair have been calculated (Lazović, Kuzmanoski, 1979a). This motivated us to investigate the mutual perturbing actions in the 12 remaining minor planets pairs for the case they found themselves within the proximities of their corresponding orbits. We used thereby the proximity distances already determined by way of the orbital elements given in Ephemeris of Minor Planets for 1977.

In this we applied the classical calculus of the special perturbations of the first order, we earlier already made use of, whose formulae are given in Lazović (1971) and Kuzmanoski (1973). The values of minor planet masses (m) are used from Lazović (1979). In the pair (j, $\boldsymbol{k}$) of the numbered minor planets, the first one is the perturbing minor planet, which, being the larger, gives rise to the perturbations in the motion of the second, the latter thus being the perturbed minor planet. This second minor planet is demarked in bold-face type in order to stress the fact that it was its orbital elements where the perturbations have been looked for. One single among these pairs only had the same values of its components masses. The change in the argument of perihelion $\Delta\omega$ has been derived from changes in the longitude of perihelion and the longitude of the ascending node calculated by the formulae $\Delta\omega = \Delta\tilde{\omega} - \Delta\Omega$. The instant of proximity has been taken as the zero instant $t_p = 0$, and with respect to it the instant preceeding proximity is designated by t_- and the one succeeding it by t_+. That is why these instants have the signs $-$ or $+$ in the Tables enclosed. In all of cases considered here it proved that the so called second term of the change of the mean anomaly does not affect the perturbation increment of the mean anomaly attributable to the double integral of derivation of the mean motion. Thus, in as much as there occurs a perturbing effect in the mean anomaly, it has its origin in the first term of the mean anomaly's increment, being due to the variation of the mean anomaly. The time interval in which the perturbing effect of one minor planet upon the other during proximity becomes sensible is designated by Δt. In the enclosed Tables we note three such intervals, which, as a rule, are different for different orbital elements, being asymmetrical with reference to the instant of proximity. Under each of the time intervals we give the corresponding perturbing change in the orbital element considered. The programme of the numerical integration, accomplished by means of our computer, of the relevant differential equations of the elliptic elements has been conceived in such a way as to derive the cumulative changes of elements for equidistant intervals of the integration step w, expressed in mean ephemeris days. It was, thus, possible to pursue these changes from the instant antecedant to the proximity, when they just begin to be noticeable through their measurable amounts, to state their amounts at the moment of the proximity and, finally, the concluding phase, after proximity until the moment when these changes cease to be perceivable.

In the second column of the enclosed Tables we find the perturbations in the motion elements in the time interval to the moment of proximity, in the third the perturbations in the interval following the proximity and in the fourth column the perturbations in the interval comprehending both intervals. Accordingly, the forth column gives the total changes in the orbital elements of the second minor planet caused by the action of the first around the positions of the proximity of their orbits.

In Table I are presented the perturbations of the orbital elements of the minor planet (1393) **Sofala** due to the action of the minor planet (110) Lydia. We gather from its title that the proximity distance of minor planets under consideration is $\rho_{min} = 9400$ km, and that the mass of the perturbing minor planet is $m_{110} = 3.3 \times 10^{-12}$ of the solar mass. We find in the paper of Lazović and Kuzmanoski (1978) that the mutual inclination of their orbits is $I = 0\overset{\circ}{.}141$. Here the integration step was $w = 0\overset{d}{.}01$. By proceeding in this way we were able to state that the change in the mean anomaly in consequence of this interaction attains its maximum positive value $+2\overset{''}{.}25$ in the interval of 2.12 mean ephemeris days preceeding the proximity, while its amount after proximity is $-2\overset{''}{.}44$ within 2.28 days. Its total change, therefore, for the whole time interval around proximity of $\Delta t = 4.40$ days amounts to $-0\overset{''}{.}19$. This interval, during which occurs the perturbing effect of the first minor planet on the relevant motion element of the second minor planet, we termed „the dynamical duration of proximity". The dynamical duration

TABLE I. (110) Lydia, **(1393) Sofala,** $\rho_{min} = 9400$ km, $m_{110} = 3.3 \times 10^{-12}$

Δt ΔM	$-2^{d}.12$ to 0 $+2''.25$	0 to $+2^{d}.28$ $-2''.44$	$-2^{d}.12$ to $+2^{d}.28$ $-0''.19$
Δt $\Delta\omega$	$-2^{d}.09$ to 0 $-1''.48$	0 to $+2^{d}.28$ $+2''.26$	$-2^{d}.09$ to $+2^{d}.28$ $+0''.78$
Δt $\Delta\Omega$	$-0^{d}.20$ to 0 $-0''.32$	0 to $+0^{d}.23$ $-0''.31$	$-0^{d}.20$ to $+0^{d}.23$ $-0''.63$
Δt $\Delta\tilde{\omega}$	$-2^{d}.09$ to 0 $-1''.80$	0 to $+2^{d}.28$ $+1''.95$	$-2^{d}.09$ to $+2^{d}.28$ $+0''.15$
Δt Δi	$-0^{d}.20$ to 0 $-0''.31$	0 to $+0^{d}.25$ $-0''.29$	$-0^{d}.20$ to $+0^{d}.25$ $-0''.60$
Δt $\Delta\varphi$	$-0^{d}.84$ to 0 $+0''.32$	0 to $+0^{d}.86$ $-0''.32$	$-0^{d}.84$ to $+0^{d}.86$ $0''.00$
Δt Δn	$-2^{d}.09$ to 0 $+0''.00188$	0 to $+2^{d}.28$ $-0''.00188$	$-2^{d}.09$ to $+2^{d}.28$ $0''.00000$

of proximity in this pair is about six times as long as its kinematic duration of proximity found earlier (Lazović and Kuzmanoski, 1979b). Let us remember „the kinematic duration of proximity" is time interval around the instant of proximity, during which mutual distance ρ between two minor planets $\rho_{min} \leqslant \rho \leqslant \rho_f$, ρ_f being some maximum distance chosen beforehand, $\rho_f = 0.0004$ AU, before and after passing the proximity positions. The amount of the dynamical duration of proximity just indicated is at the same time the greatest of all derived for all the elements investigated in connection with this pair. The largest mutual distaces of these minor planets ρ_- and ρ_+, before and after their proximity, corresponding to the instants t_- and t_+, when the perturbation changes in the particular orbital elements begin and stop being perceptible, are $\rho_- = 0.002249$ AU and $\rho_+ = 0.002418$ AU, thus about six times larger than the upper limit value, accepted ealier at defining the kinematic proximity duration. The angular widths Δv of the parts of orbits around proximity, within which the perturbing interaction of these minor planets is sensible, assume the largest values $\Delta v_{110} = 0^{\circ}.98537$ and $\Delta v_{1393} = 0^{\circ}.92757$, which is also true of the change in the mean anomaly. In the last column of Table I can be seen that the maximum total change in the argument of perihelion during proximity is $\Delta\omega = +0''.78$, within the time interval of $\Delta t = 4.37$ days.

In Table II are shown the perturbations in the pair (205) Martha, **(992) Swasey.** Here we have $I = 0^{\circ}.163$, $w = 0^{d}.01$. The greatest total change, taking all the elements into account, is $\Delta\Omega = +0''.24$ within 0.20 days about proximity. The greatest values connected with this pair are: $\rho_- = 0.000192$ AU, $\rho_+ = 0.000261$ AU; $\Delta v_{205} = 0^{\circ}.09068$, $\Delta v_{992} = 0^{\circ}.09443$ for the dynamical proximity duration of $\Delta t = 0.45$ days, with the mean diurnal motion n of the minor planet (992) Swasey. However, it is just with this element that changes before and after proximity have the same value but with opposite signs, resulting in the total change being equal to zero.

TABLE II. (205) Martha, **(992) Swasey,** $\rho_{min} = 5200$ km, $m_{205} = 2.4 \times 10^{-13}$

Δt	$-0^{d}.14$ to 0	0 to $+0^{d}.24$	$-0^{d}.14$ to $+0^{d}.24$
ΔM	$-0''.10$	$+0''.02$	$-0''.02$
Δt	$-0^{d}.07$ to 0	0 to $+0^{d}.16$	$-0^{d}.07$ to $+0^{d}.16$
$\Delta \omega$	$-0''.08$	$-0''.13$	$-0''.21$
Δt	$-0^{d}.07$ to 0	0 to $+0^{d}.13$	$-0^{d}.07$ to $+0^{d}.13$
$\Delta \Omega$	$+0''.12$	$+0''.12$	$+0''.24$
Δt	$-0^{d}.06$ to 0	0 to $+0^{d}.16$	$-0^{d}.06$ to $+0^{d}.16$
$\Delta \tilde{\omega}$	$+0''.04$	$-0''.01$	$+0''.03$
Δt	$-0^{d}.03$ to 0	0 to $+0^{d}.08$	$-0^{d}.03$ to $+0^{d}.08$
Δi	$+0''.03$	$+0''.04$	$+0''.07$
Δt	$-0^{d}.09$ to 0	0 to $+0^{d}.08$	$-0^{d}.09$ to $+0^{d}.08$
$\Delta \varphi$	$+0''.05$	$-0''.05$	$0''.00$
Δt	$-0^{d}.19$ to 0	0 to $+0^{d}.26$	$-0^{d}.19$ to $+0^{d}.26$
Δn	$-0''.00019$	$+0''.00019$	$0''.00000$

TABLE III. (227) Philosophia, **(1737) Severny,** $\rho_{min} = 1000$ km, $m_{227} = 3.6 \times 10^{-13}$

Δt	$-0^{d}.204$ to 0	0 to $+0^{d}.236$	$-0^{d}.204$ to $+0^{d}.236$
ΔM	$+2''.35$	$-2''.09$	$+0''.26$
Δt	$-0^{d}.212$ to 0	0 to $+0^{d}.294$	$-0^{d}.212$ to $+0^{d}.294$
$\Delta \omega$	$-2''.65$	$+2''.47$	$-0''.18$
Δt	$-0^{d}.004$ to 0	0 to $+0^{d}.008$	$-0^{d}.004$ to $+0^{d}.008$
$\Delta \Omega$	$-0''.04$	$-0''.05$	$-0''.09$
Δt	$-0^{d}.212$ to 0	0 to $+0^{d}.294$	$-0^{d}.212$ to $+0^{d}.294$
$\Delta \tilde{\omega}$	$-2''.69$	$+2''.42$	$-0''.27$
Δt	$-0^{d}.010$ to 0	0 to $+0^{d}.024$	$-0^{d}.010$ to $+0^{d}.024$
Δi	$-0''.16$	$-0''.17$	$-0''.33$
Δt	$-0^{d}.028$ to 0	0 to $+0^{d}.024$	$-0^{d}.028$ to $+0^{d}.024$
$\Delta \varphi$	$-0''.10$	$+0''.10$	$0''.00$
Δt	$-0^{d}.008$ to 0	0 to $+0^{d}.060$	$-0^{d}.008$ to $+0^{d}.060$
Δn	$+0''.00001$	$-0''.00007$	$-0''.00006$

Table III summarizes the perturbations of the pair (227) Philosophia, **(1737) Severny**; $I = 0^{\circ}.246$, $w = 0^{d}.002$. The largest total change connected with this

TABLE IV. (389) Industria, **(972) Cohnia,** $\rho_{min} = 1300$ km, $m_{389} = 1.2 \times 10^{-12}$

Δt	$-0^d.080$ to 0	0 to $+0^d.096$	$-0^d.080$ to $+0^d.096$
ΔM	$+0''.50$	$-0''.49$	$+0''.01$
Δt	$-0^d.016$ to 0	0 to $+0^d.018$	$-0^d.016$ to $+0^d.018$
$\Delta\omega$	$-0''.71$	$+0''.66$	$-1''.36$
Δt	$-0^d.016$ to 0	0 to $+0^d.018$	$-0^d.016$ to $+0^d.018$
$\Delta\Omega$	$+0''.69$	$+0''.66$	$+1''.35$
Δt	$-0^d.004$ to 0	0 to $+0^d.006$	$-0^d.004$ to $+0^d.006$
$\Delta\tilde{\omega}$	$-0''.02$	$+0''.01$	$-0''.01$
Δt	$-0^d.010$ to 0	0 to $+0^d.034$	$-0^d.010$ to $+0^d.034$
Δi	$+0''.24$	$+0''.24$	$+0''.48$
Δt	$-0^d.054$ to 0	0 to $+0^d.158$	$-0^d.054$ to $+0^d.158$
$\Delta\varphi$	$+0''.30$	$-0''.31$	$-0''.01$
Δt	$-0^d.140$ to 0	0 to $+0^d.202$	$-0^d.140$ to $+0^d.202$
Δn	$-0''.00119$	$+0''.00120$	$+0''.00001$

pair is that of the inclination, $\Delta i = -0''.33$ for $\Delta t = -0^d.034$. However, the greatest change is stated in longitude of perihelion, $\Delta\tilde{\omega} = -2''.69$, but it takes place in a time interval preceeding proximity. Other parameters characterizing this pair have, also for this orbital element, the greatest values: $\rho_- = 0.000374$ AU, $\rho_+ = 0.000518$ AU; $\Delta v_{227} = 0^\circ.10327$, $\Delta v_{1737} = 0^\circ.10345$ for $\Delta t = 0^d.506$.

In Table IV we find the perturbations in the pair (389) Industria, **(972) Cohnia**; $I = 0^\circ.251$, $w = 0^d.002$. Here the largest total change is $\Delta\omega = -1''.36$ for $\Delta t = 0^d.034$. Here the largest values are: $\rho_- = 0.000402$ AU, $\rho_+ = 0.000581$ AU; $\Delta v_{389} = 0^\circ.07584$, $\Delta v_{972} = 0^\circ.08010$ for $\Delta t = 0^d.342$ and for n_{972}. This mean daily motion changes by $-0''.00119$ up to the moment of proximity. Its change in the interval after proximity is $+0''.00120$. Accordingly its total change is merely $+0''.00001$.

Table V comprehends the changes of the orbital elements of the minor planet **(891) Gunhild** due to the gravitational action of the minor planet (412) Elisabetha. Here $I = 0^\circ.312$, $w = 0^d.01$. The greatest total change occurs in the longitude of the ascending node, $\Delta\Omega = +0''.24$ in the interval of $\Delta t = 0.42$ days. We see that the argument of perihelion would have changed by $\Delta\omega = -0''.49$ till the moment of proximity, in the interval of 1.16 days. The mean anomaly would have changed, only on account of the perturbations, by $\Delta M = -0''.53$ during the interval of 1.94 days following the moment of proximity. Here are the maximum values: $\rho_- = 0.000837$ AU, $\rho_+ = 0.001307$ AU; $\Delta v_{412} = 0^\circ.64622$, $\Delta v_{891} = 0^\circ.65800$ for $\Delta t = 3^d.18$, appearing at determining the perturbing effect in the mean anomaly of the minor planet (891) Gunhild.

For all of these pairs, save the pair (205) Martha and (992) Swasey, we found their maximum dynamical proximity durations to be larger than their kinematic proximity durations cited in Lazović, Kuzmanoski (1979b). It can also be stated that the maximum values for ρ_- and ρ_+, of their greatest mutual distances at which the action of one minor planet upon the other begins, resp. stops to be noticeable, are higher than the upper, earlier accepted value 0.000400 AU. In all of these cases it was $\rho_{+max} > \rho_{-max}$.

TABLE V. (412) Elisabetha, **(891) Gunhild,** $\rho_{min} = 9700$ km, $m_{412} = 3.1 \times 10^{-13}$

Δt ΔM	$-1^d\!.24$ to 0 $+0''\!.47$	0 to $+1^d\!.94$ $-0''\!.53$	$-1^d\!.24$ to $+1^d\!.94$ $-0''\!.06$
Δt $\Delta\omega$	$-1^d\!.16$ to 0 $-0''\!.49$	0 to $+1^d\!.82$ $+0''\!.33$	$-1^d\!.16$ to $+1^d\!.82$ $-0''\!.16$
Δt $\Delta\Omega$	$-0^d\!.15$ to 0 $+0''\!.11$	0 to $+0^d\!.27$ $+0''\!.13$	$-0^d\!.15$ to $+0^d\!.27$ $+0''\!.24$
Δt $\Delta\tilde{\omega}$	$-1^d\!.16$ t0 0 $-0''\!.38$	0 to $+1^d\!.82$ $+0''\!.46$	$-1^d\!.16$ t0 $+1^d\!.82$ $+0''\!.08$
Δt Δi	$-0^d\!.05$ to 0 $+0''\!.03$	0 to $+0^d\!.14$ $+0''\!.03$	$-0^d\!.05$ to $+0^d\!.14$ $+0''\!.06$
Δt $\Delta\varphi$	$-0^d\!.24$ to 0 $+0''\!.05$	0 to $+0^d\!.42$ $-0''\!.04$	$-0^d\!.24$ to $+0^d\!.42$ $+0''\!.01$
Δt Δn	$-0^d\!.60$ to 0 $-0''\!.00013$	0 to $+0^d\!.60$ $+0''\!.00013$	$-0^d\!.60$ to $+0^d\!.60$ $0''\!.00000$

Concerning the minor planet pair (311) Claudia and **(1397) Umtata** we had: $I = 0^\circ\!.360$, $\rho_{min} = 9900$ km, $m_{311} = 9.0 \times 10^{-14}$, $w = 0^d\!.01$. The following perturbations were found: the mean anomaly of the second minor planet changes at first by $\Delta M = -0''\!.01$ at the instant $t_- = -0^d\!.01$, but its change at the instant $t_+ = +0^d\!.03$ amounts to $\Delta M = +0''\!.01$, accordingly its total change, only on account of the perturbations, during proximity equals to zero. Ω_{1397} changes from the instant $t_- = -0^d\!.03$ up to the instant $t_+ = +0^d\!.03$, the complete change during that time $\Delta t = t_+ - t_- = 0^d\!.06$ being $\Delta\Omega = +0''\!.06$. The orbit's inclination i_{1397} changes by mere $\Delta i = -0''\!.01$ at the instant $t_+ = +0^d\!.03$.

No measurable changes whatever were stated in the pair (1130) Skuld and (703) **Noëmi.** Otherwise, with them we had: $I = 0^\circ\!.310$, $\rho_{min} = 2100$ km, $m_{1130} = 3.8 \times 10^{-15} = m_{703}$.

Nor hade we obtained any sensible perturbations in the pair (763) Cupido and **(985) Rosina.** We had with them: $I = 0^\circ\!.048$, $\rho_{min} = 2100$ km, $m_{763} = 2.2 \times 10^{-15}$.

In the pair (938) Chlosinde and **(1815)** $\equiv$ **1932** ***CE*₁** we had: $I = 0^\circ\!.362$, $\rho_{min} = 6400$ km. However, the masses in this pair have the same values $m_{938} = m_{1815} = 1.7 \times 10^{-14}$, and $w = 0^d\!.002$. The only change occurs with Ω_{1815}, its amount attaining $\Delta\Omega = +0''\!.03$ in the interval from $-0^d\!.010$ to $+0^d\!.016$. When the two minor planets reverse their roles the only change that takes place is that in Ω_{938}, amounting to $\Delta\Omega = -0''\!.03$ in the interval from $-0^d\!.010$ to $+0^d\!.014$.

In the pair (954) Li and **(1898) Cowell** we had: $I = 0^\circ.105$, $\rho_{min} = 7600$ km, $m_{954} = 3.9 \times 10^{-14}$, $w = 0^d002$. The only change found is that in Ω_{1898}, its amount being $\Delta\Omega = +0''.03$ in the interval from $t_- = -0^d.018$ to $t_+ = +0^d016$.

Concerning the pair (960) Birgit and **(1818) ≡ 1939 *PE*** we had: $I = 0^\circ041$, $\rho_{min} = 1500$ km, $m_{960} = 1.4 \times 10^{-15}$, $w = 0^d01$. The mean anomaly of the second minor planet, only on account of the perturbations, is first changed by $\Delta M = +0''.02$ in the interval from $-0^d.04$ to -0^d03, whereupon its change is $\Delta M = -0''.02$ in the interval from -0^d03 to $+0^d.12$. Thus, its total change is $0''.00$. $\tilde{\omega}_{1818}$ changes first by $\Delta\tilde{\omega} = -0''.02$ in the interval from $-0^d.05$ to $-0^d.04$, while its change in the interval from -0^d04 to $+0^d.12$ is $\Delta\tilde{\omega} = +0''.02$. Once again the total change is zero. i_{1818} undergoes a change of only $\Delta i = -0''.01$ at the moment $t_+ = +0^d.05$. The mean daily motion changes by $\Delta n = +0''.00003$ from the moment $-0^d.07$ to the moment $-0^d.03$, while its change from $-0^d.03$ to $+0^d.15$ attains the value $\Delta n = -0''.00003$, accordingly the total change is $0''.00000$.

In the last, twelveth, pair investigated (1736) Floriac and **(1759) ≡ 1942 *RF*** we had: $I = 0^\circ.041$, $\rho_{min} = 3300$ km, $m_{1736} = 5.0 \times 10^{-15}$, $w = 0^d01$. The change appears only in Ω_{1759} at the very moment of proximity $t_p = 0$, its amount being $\Delta\Omega = -0''.01$.

The magnitudes of perturbations in the region of the proximity that we obtained indicate that they are the greatest in the elements characterizing the positions of the orbital plane and the perihelion, followed by those in the mean anomaly and the angle of eccentricity. The perturbations in the mean daily motion assume a more significant value in the time interval shortly before and after the transit of the proximity proper, but if proximity limits are assumed to have a wider separation, then these perturbations fail occuring or are at the very limit of perceptivity. But in five of the minor planets pairs perturbing effects of such an amount are found that could be stated by observations from the Earth.

The results obtained are promising as regards estimates of the still unknown minor planet masses. These masses could be found out from the mutual perturbation actions of the minor planets during their proximities.

The calculations were carried out on the IBM 360/44 of the Computing Centre of the Institute for Mathematics in Beograd.

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