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PROXIMITIES OF ASTEROIDS (1) CERES, (2) PALLAS, (3) JUNO AND (4) VESTA

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Summary. Determination is carried out of some parameters, related to the first four minor planets, such as: mutual inclinations of their respective orbital planes, true anomalies of their orbits' relative nodes, the minimal (proximity) distances of their orbits and true anomalies of the proximity positions. The lowest mutual inclination of orbital planes is 4°92761, for asteroids (1) Ceres and (4) Vesta. The highest inclination 36° 65323 is between the orbital planes of asteroids (1) Ceres and (2) Pallas. The shortest proximity distance is 0.0626963 AU, occuring with the asteroid pair (1) Ceres and (2) Pallas. The minimum angular distances of the proximity positions from the relative node by this same asteroid pair were found to be somewhere about 0°5 and 0°6. The least difference of the true longitudes of the proximity positions is 0°00853, being associated with the pair (1) Ceres and (3) Juno.

Dynamical consequences of these proximities are investigated. The greatest change obtained is the one, $\Delta M_4 = -0''04$, which would be produced by (1) Ceres in the orbit of the minor planet (4) Vesta during the interval $\Delta t = 32.2$ days and with the angular widths $\Delta v_1 = 7°22$, $\Delta v_4 = 7°34$ of their being in the region of their sensible mutual perturbing action. With this pair we have $\Delta\tilde{\omega}_4 = +0''03$ for 25ᵈ2. With the pair (1) Ceres and (2) Pallas is $\Delta M_2 = -0''03$ for $\Delta t = 28^d0$ and $\Delta v_1 = 5°30$, $\Delta v_2 = 5°42$. And for (2) Pallas and (1) Ceres is $\Delta M_1 = +0''03$ for $\Delta t = 12^d6$ and $\Delta\tilde{\omega}_1 = -0''03$ for 16.2. The results obtained show that it is possible to make use of the proximities of these minor planets for the checking of estimates of their masses.

J. Lazović, M. Kuzmanoski, PROKSIMITETI IZMEĐU ASTEROIDA (1) CERES, (2) PALLAS, (3) JUNO I (4) VESTA — Za četiri prve numerisane male planete određeni su: nagibi između njihovih putanjskih ravni, prave anomalije relativnih čvorova njihovih putanja, najmanje (proksimitetske) daljine između njihovih putanja i prave anomalije položaja proksimiteta. Najmanji međusobni nagib je 4°92761 između putanja asteroida (1) Ceres i (4) Vesta, a najveći je 36°65323 između ravni putanja asteroida (1) Ceres i (2) Pallas. Dobijena najmanja vrednost proksimitetske daljine iznosi 0.0626963 AJ, koju ima asteroidski par (1) Ceres i (2) Pallas; za ovaj par su dobijena i najmanja ugaona rastojanja položaja proksimiteta od njihovog relativnog čvora, oko 0°5 i 0°6. Najmanja razlika pravih longituda položaja proksimiteta je 0°00853 kod para (1) Ceres i (3) Juno.

Ispitali smo dinamičke posledice ovih proksimiteta. Najveća promena zbog poremećaja je ona koju bi (1) Ceres izazvao u putanji (4) Vesta: $\Delta M_4 = -0''04$ za vreme $\Delta t = 32.2$ dana i ugaonim širinama delova putanja oko proksimiteta, u kojima se oseća međusobno poremećajno dejstvo, $\Delta v_1 = 7°22$, $\Delta v_4 = 7°34$. Za ovaj par je $\Delta\tilde{\omega}_4 = +0''03$ za 25ᵈ2. Za par (1) Ceres i (2) Pallas je $\Delta M_2 = -0''03$ za $\Delta t = 28.0$; $\Delta v_1 = 5°30$, $\Delta v_2 = 5°42$. A za (2) Pallas i (1) Ceres je

$\Delta M_1 = +0\farcs03$ za $\Delta t = 12.6$ i $\Delta\tilde{\omega}_1 = -0\farcs03$ za 16ª2. Dobijeni rezultati ukazuju na mogućnost iskorišćavanja proksimiteta ovih malih planeta za proveru ocena njihovih masa.

Up to now we have dealt with the proximities, that is, with the minimum distance positions of the quasicomplanar asteroids, the orbital planes of which were mutually inclined at small angles, not exceeding $0\fdg500$ (Lazović, 1964, 1967; Lazović, Kuzmanoski, 1978). Then, by means of these positions, we determined the corresponding minimum, i.e. proximity distances between the orbits of the asteroid pair concerned. The four first numbered minor planets were absent in these earliest investigations of ours, as the inclination between their orbits are considerably higher. But now we desired just to determine the proximities between of the asteroids (1) Ceres, (2) Pallas, (3) Juno and (4) Vesta. Their orbital elements are borrowed from Ephemeris of Minor Planets for 1980.

We at first determined the inclination I between their orbits, that is, between the respective orbital planes of the two selected asteroids, using the formulae from Lazović, Kuzmanoski (1974). The values obtained for each considered pair of numbered asteroids j and k are presented in Table I. In this way was found that the lowest mutual inclination of the asteroid orbits has the value $4\fdg92761$. The orbits concerned are those of (1) Ceres and (4) Vesta. All the other values of this angle exceed 12°, the largest $36\fdg65323$ being stated with the pair (1) Ceres and (2) Pallas. We were thus faced by the need of investigating 4 non-quasicomplanar asteroids, in contrast to our earlier investigations, bearing upon the quasicomplanar asteroids.

The problem of determining of the approximate true anomalies of the proximity positions in the case of non-quasicomplanar elliptical orbits, such as we are confronted now with, was resolved by means of relative nodes. These approximate anomalies are necessary for the solution of our rigorous transcendent equations $f = 0$, $g = 0$, valid for and proximity, both for the non-quasicomplanar asteroid pairs and the quasicomplanar ones (Lazović, 1964, 1967). By the term „relative nodes" the projections are understood on the celestial sphere of the cuting points of the orbit of one asteroid with orbital plane of the second asteroid, as seen from the Sun's centre. There are two relative nodes with every pair of asteroid orbits, true anomalies of which differing by 180° in one and the same orbit. True anomalies of the relative nodes of orbits of the numbered asteroids j and k are denoted by v_{j0} and v_{k0}. Their determination was accomplished by the formulae in Lazović (1980). Let it be observed that these true anomalies can equally be obtained by the formulae, appearing in Lazović and Kuzmanoski (1974), as the argument of latitude is $u = \omega + v$.

In Table I the values are first listed of the true anomalies relating to that relative node, to which the shorter proximity distance of the asteroid pair concerned is nearer, while underneath are values associated with the second relative node, which is nearer the second, larger, proximity distance. The proximity is nearer that relative node for which the absolute difference of the true anomalies of the proximity positions and the relative node is inferior to 90°.

By making use of the true anomalies of the relative nodes from Table I we were to compute the corresponding corrections and to finally derive true anomalies v_j and v_k of the positions of proximity required. We next determined the shortest (proximity) distance ρ_{min} in astronomical units (AU) and kilometers (rounded off to 100 km). This calculus was performed according to the formulae in Lazović and Kuzmanoski (1978). Table II summarizes the values of the proximities obtained for all the combinations of pairs of the four first numbered minor planets. Our formulae for proximity $f = 0$ and $g = 0$ were satisfied in all instances.

TABLE I. — *Mutual Inclination and True Anomalies of Relative Nodes of Asteroid Orbits*

Asteroid pair j, k	I	v_{j0}	v_{k0}
(1) Ceres, (2) Pallas	36°65323	213°32170 33.32170	247°96828 67.96828
(1) Ceres, (3) Juno	16.69387	54.52716 234.52716	152.78792 332.78792
(1) Ceres, (4) Vesta	4.92761	251.04627 71.04627	151.55001 331.55001
(2) Pallas, (3) Juno	21.82501	51.73880 231.73880	117.27988 297.27988
(2) Pallas, (4) Vesta	32.87497	62.41189 242.41189	289.25719 109.25719
(3) Juno, (4) Vesta	12.04906	326.06895 146.06895	128.51639 308.51639

TABLE II. — *Minimum Distances Between Asteroids and True Anomalies of Proximities*

Asteroid pair j, k	ρ_{min} AU	ρ_{min} km	v_j	v_k
(1) Ceres, (2) Pallas	0.0626963 0.1643610	9379200 24588100	212°85695 34.47908	247°32283 69.60308
(1) Ceres, (3) Juno	0.5094437 0.8400314	76211700 125666900	29.83102 228.70166	126.88881 325.22982
(1) Ceres, (4) Vesta	0.1767204	26437000	283.94331	184.33076
(2) Pallas, (3) Juno	0.4826982 0.7775778	72210600 116324000	39.50826 252.50782	102.93462 314.53897
(2) Pallas, (4) Vesta	0.0842314 0.4497816	12600800 67286400	60.78821 251.78751	287.74000 117.78025
(3) Juno, (4) Vesta	0.2728517 0.4345448	40818000 65007000	297.96865 92.60748	100.56873 255.47269

Two proximities were obtained with each asteroid pair, exept for the pair (1) Ceres and (4) Vesta. One single proximity was stated with this pair. In Table II are presented first the values corresponding to the shorter distance, followed below by the values associated with the larger proximity distance. The first proximity of the asteroid pairs, given in Table II, is nearer its first relative node, found in Table I. In the determination of these proximities, the distances between the selected asteroids in the direction of the same relative node of their orbits, proved helpful. These distances were obtained as the differences of the corresponding heliocentric distances of the asteroids concidered. The distance of asteroids in the direction of one their relative node was larger than their proximity distance, which is nearer that relative node. In this investigation an exception was identified by the pair (1) Ceres and (4) Vesta. Table I contains, for this pair too, the values of its two relative nodes. It appeared, however, that the mutual distance of these asteroids in the direction of their second relative node, was shorter than their mutual distance obtained within the procedure of determining their alleged proximity distance nearer that second relative node. Note that the proximity can occur in the direction of the relative node as well, yet it is usually near this node, in analogy to what hitherto has been found. Nevertheless this signaled us to examine more closely this pair. We therefore computed the mutual distances of these asteroids along their entire orbits at intervals of 0°5 of their true anomalies. From the values thus derived we were able to establish that the asteroids (1) Ceres and (4) Vesta had but one single proximity, found in Table II. It is nearer the first relative node of their orbits. The distance of these orbits attains a maximum, amounting to 0.7613946 AU or 113903000 km, occupying the positions of true anomalies $v_1 = 148°.57837$ and $v_4 = 49°.94953$. It follows from the comparison of these anomalies with the true anomalies of these asteroids' relative nodes that the maximum stated was nearer their second relative node. The true anomalies of positions and the maximum mutual distance of the orbits of asteroids (1) Ceres and (4) Vesta satisfied our equations $f = 0$ and $g = 0$. Remember in this connection that these equations apply both to the maximum and the minimum distance of two elliptical orbits, as they have been derived under the condition set for the extreme of the orbits' distance.

The lowest angular distances of the proximity positions from the relative node amount to 0°46475 and 0°64545, being associated with the first proximity to the first asteroid pair (1) Ceres and (2) Pallas, relative to their first relative node. The largest angular distances 53°46147 and 53°04370 are found with the second proximity of the pair (3) Juno and (4) Vesta, relative to their second relative node. Proximity positions' true longitudes ($L = \Omega + \omega + v$) have easily been obtained using true anomalies from Table II. By following this procedure it is found that the minimum difference of the proximity positions' true longitudes have the value 0°00853 or 30″71, appearing with the first proximity of the asteroid pair (1) Ceres and (3) Juno. The largest difference 3°80913 is stated with the second proximity of the pair (1) Ceres and (2) Pallas.

The least mutual distance among any two of the four quoted asteroids is that between (1) Ceres and (2) Pallas, its value being 0.0626963 AU or 9379200 km. Proximity distances found with these asteroids are considerably larger than those found with the quasicomplanar asteroids, Lazović (1964, 1967), Lazović and Kuzmanoski (1978). The minimum distance referred to above is almost 160 times as large as the maximum value and 15700 times as large as the minimum distance found earlier.

Note that by means of the true anomalies v_j and v_k presented in Table II one can easily obtain the corresponding eccentric anomalies of the positions of the minimal distances between the orbits of asteroids treated. These eccentric anomalies, as well as the corresponding proximity distances, are in disaccord with those found by Herget in 1971 and cited in Gehrels et al. (1971). But these disagreements are wholly understandable taking into account that different values of the orbital elements of these asteroids, obtained at two different osculating epochs, have been used in these calculations.

Although the approachings of these minor planets are not very close, we examined their dynamical consequences in all the possible combinations of pairs of these four minor panets. The following values of their masses in the solar mass units are adopted from Transact. IAU XVI B: (1) Ceres 5.9×10^{-10}, (2) Pallas 1.1×10^{-10}, (4) Vesta 1.2×10^{-10}. By the first of the formulae (13) in Lazović (1979) we obtained (3) Juno 1.0×10^{-10}. The mutual perturbations of these minor planets, in the event they found themselves in the vicinity of the proximities (the first ones in Table II), turned out to be small, and even this applies to only some of the combinations of these bodies. However, the amounts of the dynamical proximity durations Δt and the angular widths Δv of the parts of orbits around proximity, inside which a sensible perturbing effect occurs, have been found to be considerably greater than found in Lazović, Kuzmanoski (1979).

It has been found that the minor planet (1) Ceres would produce only these changes in the orbital elements of the minor planet (2) Pallas during proximity: $\Delta M = -0''.03$ by $\Delta t = 28.0$ days, $\Delta v_1 = 5^\circ.30$, $\Delta v_2 = 5^\circ.42$, the longitude of perihelion $\tilde{\omega} = \Omega + \omega$ would change by $\Delta\tilde{\omega} = -0''.02$ within 12.0 days before proximity, thereafter would change by $\Delta\tilde{\omega} = +0''.02$ within 17.2 days following the proximity. The total change, accordingly, amounts to $0''.00$. The angle of eccentricity would change by $\Delta\varphi = -0''.02$ in $\Delta t = 17.8$ days, $\Delta v_1 = 3^\circ.37$, $\Delta v_2 = 3^\circ.46$. The mean daily motion n would be affected by a change of $\Delta n = -0''.00002$ in the interval before proximity, while the change in the interval after proximity is $\Delta n = +0''.00002$, the total change, therefore, being $\Delta n = 0''.00000$. The integration step was $w = 0.2$ days.

The minor planet (2) Pallas would produce the following changes in the orbit of the minor planet (1) Ceres: $\Delta M = +0''03$ by $\Delta t = 12^d.6$, $\Delta v_2 = 2^\circ.42$, $\Delta v_1 = 2^\circ.38$; $\Delta\tilde{\omega} = -0''.03$ by $16^d.2$, $\Delta v_2 = 3^\circ.12$, $\Delta v_1 = 3^\circ.06$. Ω would change by no more than $-0''.01$ at the moment 4.8 days before the instant of proximity of these two minor planets. $w = 0\ 2$.

The effects of the minor planet (1) Ceres upon the orbit of the minor planet (4) Vesta are as follows: $\Delta M = -0''.04$ for $\Delta t = 32^d.2$, $\Delta v_1 = 7^\circ.22$, $\Delta v_4 = 7^\circ.34$ ($-0''.02$ for $15^d.2$ before and $-0''.02$ for $17^d.0$ after proximity); $\Delta\tilde{\omega} = +0''.03$ for $25^d.2$, $\Delta v_1 = 5^\circ.65$, $\Delta v_4 = 5^\circ.74$; Ω would change by only $-0''.01$ at the instant 2 0 before proximity. $w = 0^d.2$.

(2) Pallas would be responsible for the following changes of the orbital elements of (4) Vesta: M would be changed, only on account of the perturbations, by $+0''.01$ at the instant $0^d.4$ after proximity; $\Delta\Omega = -0''.01$ at the instant $2^d.8$ before proximity, $\Delta\tilde{\omega} = -0''.01$ at the instant $2^d.2$ before proximity.

(4) Vesta would produce one single change $\Delta\tilde{\omega} = +0''.01$ in the orbit of (2) Pallas at the instant 6.0 days after proximity.

The results obtained are illustrative as to the possibility of exploiting the proximities of these minor planets for the checking of estimates of their masses and offer supplementary information concerning their motion.

The computation has been performed on IBM 360/44 in the Computing Centre of the Institute for Mathematics in Beograd.

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REFERENCES

Gehrels, T., Gill, J. R., Haughey, J. W. 1971, Introduction, in *Physical Studies of Minor Planets*, Ed. T. Gehrels, NASA SP-267, Washington, p. XIV.

IAU. 1977, Proceedings of the Sixteenth General Assembly, *Transact. IAU* XVI B, 60.

Inst. Theor. Astr. Acad. Sci. USSR. 1979, *Ephemeris of Minor Planets for 1980*, Leningrad.

Lazović, J. 1964, On some of the more important features of the motion of the quasicomplanar asteroids, *Doctoral dissertation*, Belgrade Faculty of Sciences; in Serbian.

Лазович, Й. П. 1967, Определение кратчайшего расстояния между орбитами астероидов с малым взаимным наклоном, *Бюллетень Института теоретической астрономии*, Академия наук СССР, Ленинград, XI, **1**, 57—62.

Lazović, J., Kuzmanoski, M. 1974, Pairs of quasicomplanar asteroids considered with respect to mutual inclination of their orbits, *Publ. Dept. Astr., Univ. Beograd*, No. **5**, 19—49.

Lazović, J., Kuzmanoski, M. 1978, Minimum distances of the quasicomplanar asteroid orbits, *ibid.*, **8**, 47—54.

Lazović, J., Kuzmanoski, M. 1979, Perturbing effects of the asteroid 215 Oenone on the asteroid 1851 ≡ 1950 *VA* during their proximity, *ibid.*, **9**, 63—69.

Lazović, J. 1979, Masses of some numbered minor planets, *ibid.*, **9**, 55—61.

Lazović, J. 1980, Contribution to the proximity determination of non-quasicomplanar elliptical orbits of celestial bodies, *ibid.*, **10**, 43—47.