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MEAN PLANE OF ASTEROIDAL ORBITS

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Summary. Making use of area vectors of numerated minor planets from „EMP for 1979" the mean plane of asteroid orbits is determined, as its position with respect to orbital planes of Mars and Jupiter and the Laplace invariable plane. The inclination and longitude of node distributions are given as the bimodal frequency for the same orbital elements.

M. Kuzmanoski, SREDNJA RAVAN PLANETOIDSKIH PUTANJA — Polazeći od vektora površina numerisanih malih planeta iz „EMP for 1979" određena je srednja ravan planetoidskih putanja i njen položaj u odnosu na putanjske ravni Marsa i Jupitera i Laplasovu invarijabilnu ravan. Data je raspodela po nagibu i longitudi čvora, kao i dvojna frekvencija po istim elementima.

According to the idea presented in a previons paper (B. Popović [1]) it is possible to determine the mean plane of asteroidal orbits, making use of area vectors $\boldsymbol{C}_k$. Starting from orbital elements of minor planets, the components of area vectors are derived:

$$C_{xk} = C_k \sin \Omega_k \sin i_k$$

$$C_{yk} = -C_k \cos \Omega_k \sin i_k$$

$$C_{zk} = C_k \cos i_k$$

The mean plane is defined by the mean area vector $\boldsymbol{C}_s$, which components are obtained as the arithmetical mean values of the components of vectors $\boldsymbol{C}_k$:

$$C_{xs} = \frac{1}{n} \sum_{k=1}^{n} C_{xk}$$

$$C_{ys} = \frac{1}{n} \sum_{k=1}^{n} C_{yk}$$

$$C_{zs} = \frac{1}{n} \sum_{k=1}^{n} C_{zk}$$

The longitude of the node Ω_s of the mean plane and the inclination i_s, relative to the ecliptic plane, is obtained from

$$\operatorname{tg} \Omega_s = -\frac{C_{xs}}{C_{ys}}$$

$$\cos i_s = \frac{C_{zs}}{C_s}$$

Taking the orbital elements of minor planets from „EMP for 1979", we obtained following numerical values for the components of the mean area vector:

$$\begin{aligned} C_{xs} &= \quad 0.00027971 \\ C_{ys} &= -0.00005873 \\ C_{zs} &= \quad 0.02758096 \end{aligned}$$

and there from the values of the longitude of the node Ω_s and the inclination i_s:

$$\Omega_s = 78^\circ.141$$

$$i_s = 0^\circ.594$$

In order to examine whether the mean plane, defined in such a way varies when increasing the number of minor planets, the values of Ω_s and i_s for different samples are determined: for the first hundred of asteroids, then for the first two hundred, etc. The results are presented in Table 1.

TABLE 1

n	Ω_s	i_s	n	Ω_s	i_s
100	96°.360	0°.703	1100	93°.901	0°838
203	91.267	1.005	1200	91.674	0.692
300	116.062	0.997	1300	88.102	0.589
400	108.722	0.926	1400	89.293	0.607
500	112.166	0.951	1500	78.305	0.539
600	105.854	1.130	1600	77.757	0.561
700	105.277	0.877	1700	78.818	0.563
800	100.564	0.864	1800	86.211	0.538
900	109.869	0.907	1900	85.721	0.547
1000	95.360	0.732	2000	82.931	0.573

Due to the chosen sample of asteroids, the inclination of the mean plane varies from 0°.538 to 1°.130 and the longitude of the node from 77°.757 to 116°.062.

Although somewhat greater variations of the longitude of the node appear, we can establish that the mean plane holds its orientation in space, and one should not expect for it more serions changes when increasing the number of asteroids.

The position of the mean plane is in a good accordance with orbital planes of Mars and Jupiter and the Laplace invariable plane, which could be one of the indicators in the investigation of minor planet origins. The values of the longitude of the node Ω and the inclination i for these planes are given in Table 2, as well as the inclination i_0 the mean plane makes with them.

TABLE 2

	Ω	i	i_0
Mars	49°.318	1°.854	1°.364
Jupiter	99.865	1.307	0.787
Laplace inv. pl.	108.200	1.642	1.168

Let us examine now the distribution of inclinations i_k and longitudes of node Ω_k of minor planets with respect to the mean plane. In the new coordinate system, x-axis is in the intersection of the ecliptic plane and the mean plane (defined by the angle $\Omega_s = 78°.141$), y-axis is normal to it and z-axis is in the direction of the mean vector $\boldsymbol{C}_s$.

Forming the groups of inclinations in a class-intervals of 1°, we have obtained the frequencies of inclination distributions of orbital planes with respect to the mean plane (Fig. 1.), while the frequency of node distribution are carried out in class-intervals of 10° (Fig. 2.).

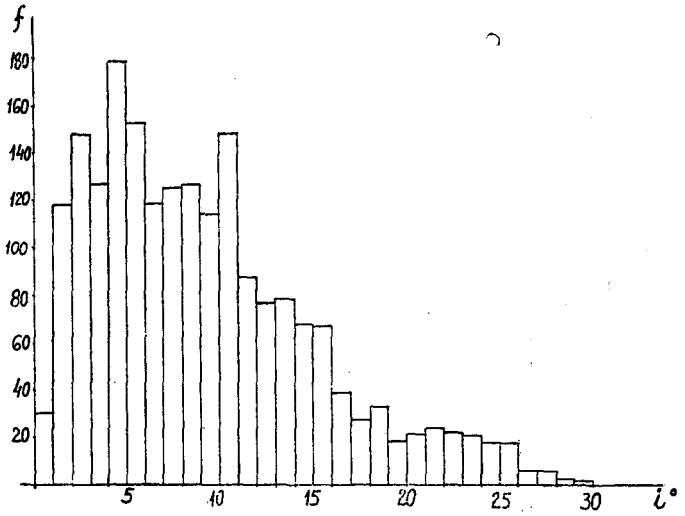

Fig. 1. The histogram of inclination distributions

The comparison of above distibtions with corresponding distributions relative to the ecliptic plane (that we made with the same elements) has not shown excessive changes. For the inclination distributions only, the maxima of both plane and spherical subsystems, are more significant, whereas the longitude node distributions show greater regularity.

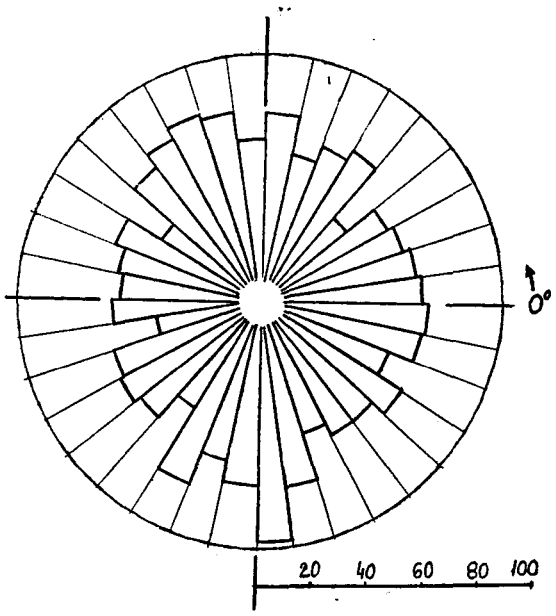

Fig. 2. The histogram of longitude of node distributions.

Observing the bimodal frequencies in longitude of node class-intervals of 20° and in inclination class-intervals of 2° (Table 3), the space disposition of minor planet orbits can be established.

TABLE 3

Ω° \ i°	2	4	6	8	10	12	14	16	18	20	22	24	26	>26
20	8	17	18	10	16	19	10	8	1	1	1	4	3	0
40	8	23	11	12	12	14	7	7	3	3	2	3	2	0
60	4	15	14	13	12	12	10	8	5	1	2	3	1	1
80	7	16	19	9	15	14	6	4	2	5	1	2	5	6
100	7	16	25	9	10	26	5	8	3	3	5	1	1	4
120	11	20	17	18	12	21	14	11	4	2	1	3	2	0
140	7	20	19	16	11	16	7	7	5	1	6	6	2	2
160	9	12	14	12	13	5	11	7	3	3	2	1	2	2
180	5	14	15	11	15	9	10	12	3	2	2	1	2	1
200	8	15	16	15	10	8	3	7	0	4	2	1	1	1
220	8	11	16	16	22	13	8	5	5	1	2	2	1	2
240	9	8	17	8	17	8	6	9	5	3	4	2	1	2
260	6	11	20	13	17	12	17	7	9	3	2	4	3	2
280	13	18	24	25	14	16	8	11	3	9	4	1	3	1
300	10	13	20	19	14	4	7	11	3	1	3	5	3	1
320	13	15	20	19	8	10	9	2	2	3	1	2	1	1
340	3	16	20	12	8	17	12	5	7	2	4	0	1	1
360	12	15	25	8	16	12	6	8	4	4	1	2	2	4

As can be seen from Table 3, the distribution is nearly uniform. The highest frequency obtained for given classes is 26. But, if taking into consideration the orientations and dimensions of orbits, one draws the conclusion that the existence of larger groups of minor planets with similar orbital elements is not evident in the present structure of the asteroidal belt.

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The computation has been performed on IBM 360/44 in the Computing Centre of the Institute for Mathematics in Beograd.

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[1] Popović B., Elazar S., Neke osobenosti statističkog rasporeda elemenata malih planeta, *Matematički vesnik*, **10** (**25**), 1973, 249—254.