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THE RADIATION PRESSURE AND THE MOTION OF THE ARTIFICIAL SATELLITES

L. Sehnal

Astronomical Institute of the Czechoslovak Academy of Sciences, Observatory Ondřejov

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Summary. The paper deals with the radiative effects on the motion of the Earth artificial satellites. Separately, the solar and terrestrial radiation sources are taken into account and the Poynting-Robertson effect is treated, too. The magnitude of the disturbing forces is compared to that of the atmospheric drag and relevant effective heights are computed. The components of the disturbing forces are determined and the changes of the elements are obtained from the planetary Lagrange-Gauss equations.

Ladislav Sehnal, PRITISAK ZRAČENJA I KRETANJE VEŠTAČKIH SATELITA — Ovaj rad uzima u obzir uticaj zračenja na kretanje Zemljinih veštačkih satelita. Uzeti su u razmatranje ponaosob sunčano i zemaljsko zračenje, a takođe je razmatran i Poynting-Robertsonov efekt. Veličina poremećajnih sila upoređena je sa veličinom atmosferskog otpora i izračunate su odgovarajuće efektivne visine. Određene su komponente poremećajnih sila i dobijene promene elemenata iz planetskih Lagranž-Gausovih jednačina.

For most satellites, at least for those which come close to the Earth, the atmospherical drag is the most important non-graitational disturbing effect. Next to this is the radiation pressure which can outgrow the drag in greater distances from the Earth.

Let us just quote briefly the different kinds of radiation effects:

1) Direct solar radiation pressure
2) Earth albedo radiation pressure
3) specularly reflected radiation pressure
4) infrared radiation pressure
5) Poynting-Robertson effect.

All those effects, like drag, depend on the satellite effective area/mass ratio. Close to the Earth, the influence of the atmosphere prevails; with growing height the density of the atmosphere rapidly diminishes and the drag effect declines, too. Making a ratio of all the above quoted effects to drag, the satellite area/mass ratio disappears so that for all satellites, we can find a certain height, for which the radiation pressure effects become greater than drag.

We can write for the already mentioned disturbing accelerations the approximate formulas under some simplifying conditions:

a) Drag:

$$F_D = -\frac{1}{2} C_D \frac{A}{m} \rho V^2$$

b) Direct solar radiation pressure:

$$F_S = K \frac{A}{m} Q$$

c) Earth albedo radiation pressure:

$$F_A = \frac{1}{4} K \frac{A}{m} Q \alpha \left(\frac{R}{r}\right)^2$$

d) Infrared radiation pressure:

$$F_{IR} = K \frac{A}{m} A_0 \left(\frac{R}{r}\right)^2 + \frac{1}{4} K \frac{A}{m} A_2 \left(\frac{R}{r}\right)^2$$

e) Poynting-Robertson effect:

$$F_{PR} = \frac{A}{m} Q \frac{V}{c}$$

We omit in our analysis the pressure of the specularly reflected radiation (from the Earth surface) as a very random effect (depending strongly on ocean-land difference, cloud cover etc.).

The infrared radiation pressure has two components — the first one comes from the homogeneous spherically symetrical component which actually does not contribute to any perturbations of the elements. As we shall see later, it acts only like diminishing the effective gravity. The disturbing component originates from the second term with A_2. The constants A_0, A_2, etc. are the constants in the spherical harmonic representation of the terrestrial infrared radiative field σ.

Making the ratio of the radiative disturbing forces to drag equal to unity, we have the conditions for determination the radius-vector or height above which the radiative forces prevail:

a) Direct solar radiation pressure:

$$\rho / r = \frac{1}{2} \frac{Q}{GM}$$

b) Albedo radiation pressure:

$$r \rho = \frac{1}{4} \frac{Q \alpha R^2}{GM}$$

c) Infrared radiation pressure:

$$r \rho = \frac{1}{2} \frac{A_0 R^2}{GM}$$

d) Disturbing infrared radiation pressure:

$$r\,\rho = \frac{1}{8}\,\frac{A_2 R^2}{GM}$$

e) Poynting-Robertson effect:

$$\rho/r = \frac{1}{2}\,\frac{Q}{GM}\,\frac{V}{c}\,.$$

On the left side, we have the terms depending on the radius-vector (or height) and if we make choice of a specific model of the atmosphere we can solve those expressions for r. However, since the density depends strongly on the solar activity, we shall have accordingly different values of r. Fig. 1 shows such dependence; the upper regions indicate the heights above which the radiative effects prevail.

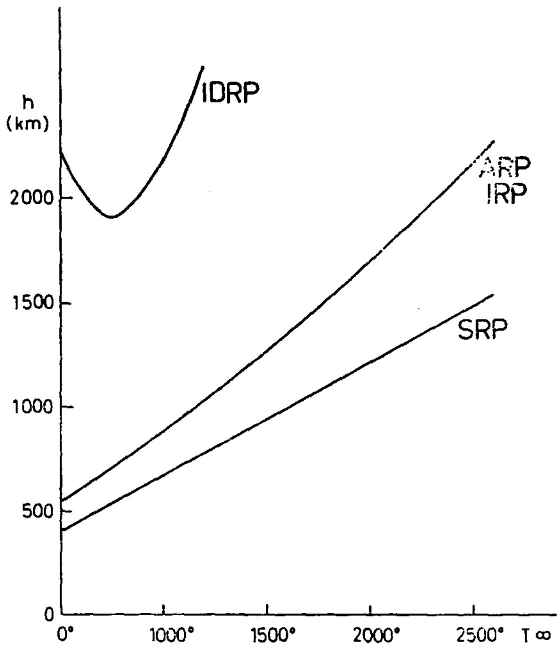

Fig. 1. The heights at which the different radiative effects exceed the atmospherical drag, in terms of growing solar activity represented by exospheric temperature T_∞.

In the following, we shall discuss in some more detail the individual radiative effects.

To compute the perturbations caused by the direct solar radiation pressure we can use the equation for F_S, where the constant Q means the pressure exerted on a unit surface in 1 astr. unite distance from the Sun, and it is $Q = 4.63 \times 10^{-5}$ in cgs system.

The computation is not complicated in case the satellite does not enter the Earth shadow. We suppose that the direction to the Sun does not change and the disturbing function can be then written as

$$F_S = -Q\frac{A}{m}(x\cos L_S + y\cos\varepsilon\cos L_S + z\sin\varepsilon\sin L_S),$$

where x, y, z are the geocentric coordinates of the satellite, ε and L_S is the ecliptical obliquity and longitude of the Sun (1).

Immediately after introducing the expression for F_S into the equations for variations of constants, we see that there are no secular perturbations of the semi--major axis. However, this does not hold in case the satellite enters the Earth shadow during its revolution around the Earth. In this case we can, of course, integrate the equations within limits corresponding to the entry and exit of the satellite into or from the shadow.

Another method which could enable to integrate over whole revolution and so it would be independent on the conditions of the shadow entry, lies in introducing the so called „shadow function". This function should gain a value of unity on the illuminated portion of the orbit and drop to zero during the shadow crossing. Multiplying the disturbi*k* function by this „shadow" function, we have actually the exact description of the conditions of a real satellite orbit.

Basic problem in construction of the shadow function lies in the mathematical approximation of a discontinuous function. The geometrical situation can be shown on following Fig. 2:

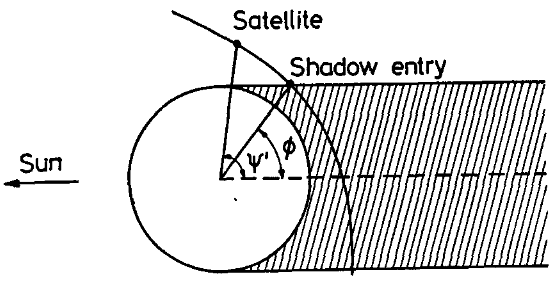

Fig. 2. The shadow function ϰ.

The angular distance λ of the satellite from the center of the shadow can be expressed as

$$\cos\lambda = A_1\cos v + A_2\sin v,$$

where the coefficients A_1 and A_2 depend on the longitude of the Sun L_S, obliquity of the ecliptic and on the orbital elements. Similarly, the angle Φ can be written as

$$\sin\phi = (R/p)(1 + e\cos v).$$

If we write the difference $\lambda - \phi = x$, then the function

$$\varkappa = \frac{1}{2}(1 + x/|x|) = \frac{1}{2}[1 + \sin x/(1-\cos^2 x)^{1/2}]$$

has the properties we are looking for (2).

This function $\varkappa$ can be developed into a power series where the argument is the true anomaly v. For usage of this function in the Lagrange equations we transform the true anomaly into the mean one using the Hansen coefficients and then we have for the change of an arbitrary element σ the expression

$$\triangle\sigma = F_D[SM + \sum_j P_j \sin jM + \sum_j Q_j(1 - \cos jM)]$$

This is exactly the same form as we got for the change of arbitrary element σ caused by drag; since the coefficients S_j, P_j and Q_j are given again by multiplication of several series, we shall use the semi-analytical method, consisting of expressing the coefficients as the functions of indeces or as reccurent formulas.

The shadow function can be introduced through several other mathematical means, e. g. the Fourier series (3) or Legendre polynomials (4). It seems, however, that the power series give the fastest convergence.

The analytical description of the albedo effect is very difficult even when we take into account the simplest model of the albedo — a constant value, homogeneously distributed over the Earth's surface. In this simple case it is possible to find an analytical expression which agrees with the numerical model in its limiting values (vanishes when satellite enters the Earth shadow). Such a function can be expressed, e. g. as

$$F_{ARP} = F_{ARPC}\cos^2\eta\,(\pi/2 + \cos^{-1}R/r)\,\pi/2$$

where η is the angle between the satellite radius-vector and the Earth-Sun line. The quantity F_C is the maximum value of the albedo radiation pressure which can be computed analyticaly in a close form (5).

In last years, there were attempts to find the albedo effects using some special methods. In this sense we have to mention the french satellite D-5-B having on board the microaccelerometer with which it was possible to measure even small accelerations effecting the motion of the satellite. When the satellite was illuminated ba the Sun, the direct solar radiation pressure, infrared radiation pressure and the albedo radiation was active. Since the direct radiation has a constant character, as well as the infrared, the changes of the acceleration could have been assigned to the albedo radiation. This was in those part of the orbit, when the drag effects were small owing to great distance from the Earth.

The measurents made by the microaccelerometer showed clearly the distinction between land and oceans and also the latitudinal dependence. Thus, the albedo can be represented by two formulas (6):

$$\alpha = 0.1 + 0.3\sin|\varphi| \qquad \text{for ocean,}$$
$$\alpha = 0.2 + 0.3\sin|\varphi| \qquad \text{for land,}$$

where φ is the latitude.

This is a description of the albedo distribution by discrete values; however, by Fourier analysis one can get a continuous model. Since the D-5-B satellite's 30° inclination, the higher latitudes were not covered by the microaccelerometer measurements, so that the snow covered arctic areas got smaller albedo values. We tried to make a model of the Earth albedo using spherical harmonic analysis and taking into account the surface effects. The comparison of the longitudinal averages with the values made by the satellite Tiros 7 show good agreement (7), see Figs. 3, 4.

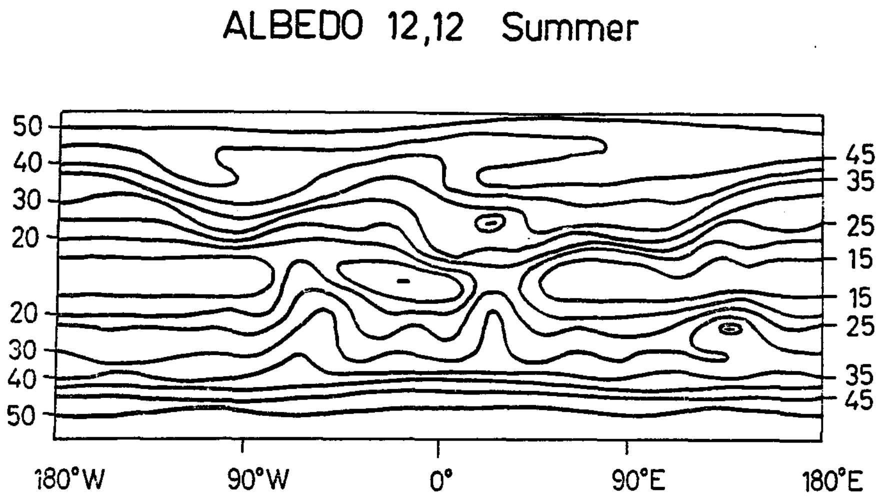

Fig. 3. The summer Earth albedo model 12, 12. The values of the albedo isolines are in percents.

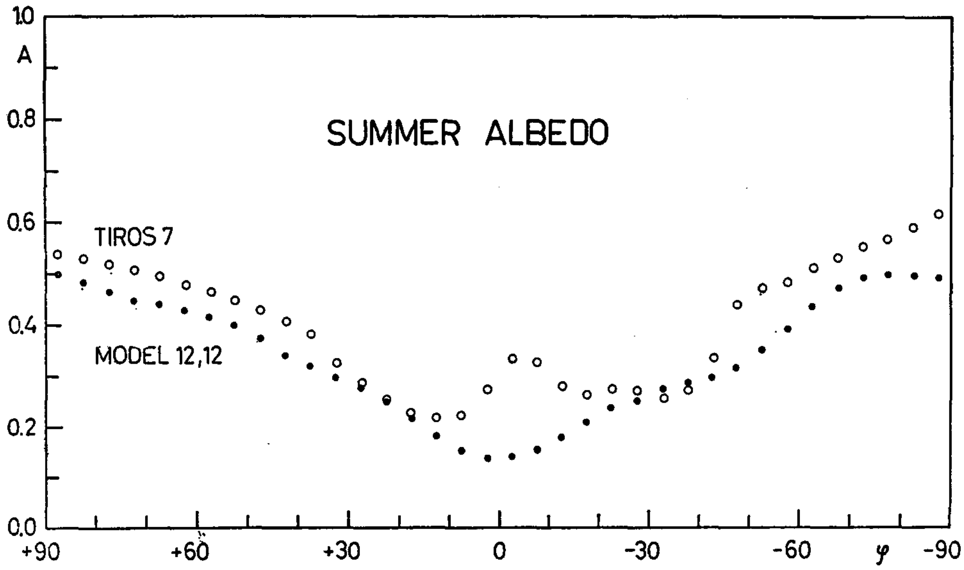

Fig. 4. The longitudinal averages of the albedo in 5deg latitudinal (φ) zones as measured by Tiros 7 and as given by our summer albedo model.

The terrestrial infrared radiation pressure is very difficult to register, since its main part, which comes from the homogeneously radiating spherical surface does not make any changes of the elements. To treat the infrared radiation more precisely, let us represent it by a spherical harmonic expansion (**8**), where we shal take into account the zonal terms of zero and second degree (**9**):

$$\sigma = A_0 + A_2 P_2(\sin\varphi),$$

where P_2 is the Legendre polynomial of second degree. The geometry of the effect is on Fig. 5; we shall suppose the Lambert law is valid for the outgoing radiation, so that the acceleraton dF_{IR} exerted by the infrared radiation flux from an area dP on a satellite of a unit area/mass ratio and of a unit surface reflectivity will be given as

$$dF_{IR} = 1/\pi \ \sigma/c \ 1/\rho^2 \cos z \ dP.$$

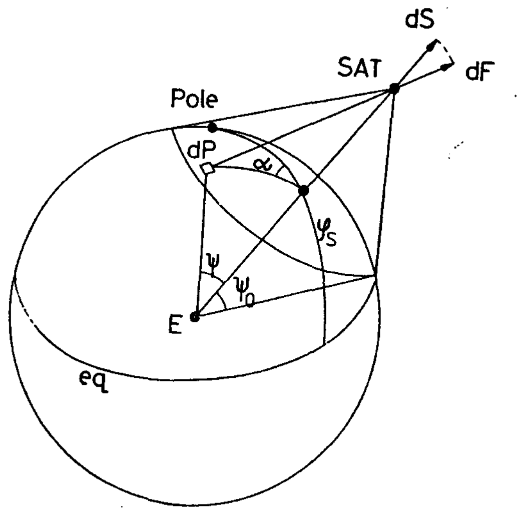

Fig. 5. The geometry of the terrestrial infrared radiation pressure dF (and its radial component dS) from the infinitessimal area dP.

The integration of dF over the whole cap visible to the satellite can be performed in a closed form, however, after introducing those expressions into the equations for variations of constants we would not be able to integrate them. Therefore we shall use the development using the Gegenbauer (ultraspheric) polynomials and shall have the result e. g. for S:

$$S = A_0 r^{-2} + A_2 P_2 (\sin \varphi_s) \, r^{-2} \left[\frac{1}{4} + \frac{4}{5} r^{-1} + \frac{1}{4} r^{-2} - \frac{18}{35} r^{-3} - \frac{4}{105} r^{-5} \right]$$

The expressions for the other terms have similar form, exept the terms with A_0 vanish, as we could expect.

To integrate the equations for the variations of constants, we shall introduce the mean anomaly and get for the changes after one revolution the integrals of the form:

$$\left(\frac{r}{a} \right)^n \cos mv \, dM,$$

where v is the true and M the mean anomalies. We can use e. g. Tisserand's treatise (10) to solve the integrals and finally we shall come to the changes for the elements in form of series in powers of $(R/p)^n$, where R is the equatorial radius of the Earth and p is the orbital parameter.

To check the results numerically, we could use only the measurements made by the microaccelerometer on board of D-5-B. All measured data were assigned to the effect of the radial component S. The numerical results computed from our theory are in complete agreement ($S = 3.537 \times 10^{-9}$ m/sec^2). The statistics of the observed data showed a slight growth of the infrared radiation effect from equatorial to tropical regions. This would contradict to the results of our theory but let us remind that the measurements give rather statistical estimate of the effect and on the other side, the theory includes just one disturbing term (second zonal harmonics).

The changes of the elements were not yet observed, but let us give at least predicted values (per day):

	D-5-B	Lageos
Δa	3.7 cm	0.0005 cm
Δr_p	$-$ 12.5 cm	$-$ 0.05 cm
Δe	2.2×10^{-8}	3.8×10^{-11}
Δi	7.6×10^{-8} deg	-9.6×10^{-13} deg
$\Delta\Omega$	-5.0×10^{-5} deg	-2.5×10^{-7} deg
$\Delta\omega$	-1.4×10^{-5} deg	-6.9×10^{-8} deg

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