Publications of the Department of Astronomy — Beograd, № 10, 1980

UDC 523.24; 521.1/3 OSP

THE EARTH UPPER ATMOSPHERE AND THE MOTION OF THE ARTIFICIAL SATELLITES

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Received January 30, 1980

Summary. The theory of the motion of an artificial satellite in the terrestrial upper atmosphere is described. The atmospherical density definition includes the diurnal effect as well as thr flattening of the atmosphere. An example of computing the analytical formulae using computee algebra is given. The theory of the atmospherical lift is shortly discussed and the effect of the rotation of the atmosphere is shown on several examples of determination of the upper atmcsphere rotational velocity.

Ladislav Sehnal, VISOKA ZEMLJINA ATMOSFERA I KRETANJE VEŠTAČKIH SATELITA — Opisana je teorija kretanja veštačkog satelita u visokoj Zemljinoj atmosferi. Definiicija atmosferske gustine podrazumeva i dnevni efekt kao i spljoštenost atmosfere. Dat je primer izračunavanja analitičkih formula koristeći računarsku algebru. Ukratko je diskutovana teorija atmosferskog lifta, a uticaj rotacije atmosfere je pokazan na nekoliko primera određivanja brzine rotacije visoke atmosfere.

The influence of the Earth upper atmosphere is the most important disturbing effect of non-gravitational origin in the motion of the artificial satellite of the Earth. It is so at least for the so called „close satellites" — this concept can be readily defined on the basis of perceptible atmospheric perturbations.

To study quantitatively the non-gravitational forces, we shall make use of the equations for the variations of constants (with the usual orthogonal components of the disturbing forces — Gauss's equations), a general form of which can be expressed as

$$\frac{\partial\sigma}{dt}=\sum_{k} K^{(k)} F^{(k)}(t), \qquad k=\mathrm{S, T, W}.$$

This is the change of an arbitrary element σ, caused by the components S, T and W of the disturbing accelerations (S — radial, T — perpendicular to the radius-vector in the orbital plane, W — normal to the orbital plane).

The computation of the changes of the elements consists then from two parts, basically:

a) the mathematical description of the components of the disturbing forces S, T and W

b) the solution of the equations for variations of constants with known S, T and W.

The problem ad a) cannot be always solved in time as the independent variable, so that we use some suitable anomaly (true, excentric). The problem ad b) can be solved in most cases only by using the series developments in a suitable small parameter (excentricity e or R/a ratio, etc.). This depends principally on the form of the expressions for S, T and W. Sometimes we can find closed formulae for those disturbing forces but the integration of the equations of variations of constants can not be still made; then again the series expressions are necessary.

In the following, we shall deal with the disturbing effects of the upper atmosphere in terms of drag, lift and the influence of the rotation of the atmosphere.

The drag force F_D is described by the expression

$$F_D = -\frac{1}{2} C_D \frac{A}{m} \rho V^2,$$

where C_D is the drag coefficient, A/m is the satellite effective area/mass ratio, ρ is the density of the atmosphere and V is the relative velocity of the satellite to the ambient air. Thus, the quantitative determination of the magnitudes of the drag effects requires also a knowledge ot the factors A/m and C_D (1).

The determination of the A/m ratio may cause some difficulties in case of uncontrolled tumbling of the satellite, and especially a body of spheroidal shape may produce rotation with precessing motion of the axis of rotation. Nevertheless, the A/m ratio value is usually the best known quantity in the equation for F_D and once determined, it may be considered as constant for drag studies.

The drag coefficient C_D is considered usually to be constant and to be equal to 2.2. But the detailed analysis shows that it changes quite distinctly with height, since it depends on the properties of the satellite's surface as this interacts with the surrounding medium. The coefficient C_D is usually determined theoretically, since laboratory measurements encounter great difficulties when imitating the conditions of melecular motions at great heights. The widely used value of 2.2 seems now to be rather low even in lower regions of the atmosphere. The theoretical analyses show that up to heights of 400 km this coefficient may be taken as constant, but then it increases rapidly with increasing height. Some practically used values took the coefficient C_D as 2.8 in 1400 km and equal to 3.6 in 3300 km height. Thus, neglecting the variation of C_D with height might lead to substantial errors, almost of a factor of 2.

The velocity V is the velocity of the satellite relative to the ambient rotating atmosphere; we suppose that the atmosphere rotates with the same velocity as the solid Earth.

Then, determining the disturbing force, we have something to do with the density of the atmosphere. This quantity is probably the least known value. On the other hand, the analysis of the drag effects is used very much exactly for the backwards determination of the density. The main difficulty lies in the fact, that the density depends strongly on the height above the Earth surface and change in place as well as in time.

The rapid variability of the density forces us then to compute the drag perturbations for a short time interval only, since even if drag makes secular effects (diminishing of the excentricity, semi-major axis), the secular decrease is not linear over longer time. Therefore, the best way is to compute the changes after one revolution and to make a simple summation of the effect. This is again very important, since the influence of drag is concentrated around the perigee (owing to very rapid diminishing of the density with height) so that we can take for such a limited region the coefficient C_D to be constant.

Thus, in the following, we shall suppose a constant value for the coefficient C_D as well as for A/m and describe the density of the atmosphere as

$$\rho(z) = \rho_0 \left((1 + \alpha \cos^{n'} \frac{\Psi}{2} \right),$$

where

$$\log \rho_0 = \bar{a} + bz + c \exp(-0.01\, z).$$

The first expression describes the change of the density with respect to the position of the Sun, since ψ is the angle between the radius-vector of the satellite and the Sun. The second equation expresses the dependence of the density on the height; allowing for the oblateness of the Earth we have

$$z = a - ae \cos E - R_E (1 - f \sin^2 \varphi).$$

The constants α, n'; $\bar{a}$, b, c are to be experimentally found. They can be computed from the measured value of the solar flux at 10.7 cm wavelenght (2).

Now, we can transform the right sides of the equations for variations of orbital elements explicitely into the time-dependent quantities. We shall use the excentric anomaly as the independent variable, so that

$$\rho \sim k_0 \exp(k_{11} \cos E) \exp[k_2 \exp(k_3 \cos E)] \cos^{n'} \frac{\Psi}{2} \cdot$$

This expression will be then transformed into a series in powers of excentricity with arguments as functions of the excentric anomaly so that we shall be able to make the integration. The change $\Delta\sigma$ of a general element σ will be then expressed as

$$\Delta\sigma = K_\sigma k_0 Z_0 nt + K_\sigma k_0 \sum_i \frac{Z'_i}{i} \sin iE + K_\sigma k_0 \sum_i \frac{W_i}{i} (1 - \cos iE).$$

Here, K and k_0 are constants, for Z'_i we have

$$Z'_i = eZ_0 + Z_1 \quad (i = 1), \qquad Z'_i = Z_i \quad (i = 1)$$

and the coefficients Z_i and W_i are given as

$$Z_i = \sum_q e^q \sum_r d_i^{(r)} \sum_s \sum_p g_{q-p}^{(r-s)} \sum_t k_5^{(s-t)} \sum_l \sum_j k_j^{(l)} x_{t-j}^{l,\, p-j}$$

$$W_i = \sum_q e^q \sum_r c_i^{(r)} \sum_s \sum_p g_{q-p}^{(r-s)} \sum_t k_5^{(s-t)} \sum_l \sum_j k_j^{(l)} y_{t-j}^{l,\, p-j}$$

From those expressions, the individual terms should be computed. Unfortunately, this would be only a hopeless work since the summation limits can be high and the number of terms will be then extremely great. Therefore, we shall try to find general analytical formulae for the individual coefficients, in form of functions of the indeces or in the form of reccurent relations, e. g.:

a) $d_i^{(r)} = \frac{1}{2^{r-1}} \left(\begin{matrix} r \\ \frac{r-i}{2} \end{matrix} \right)$

b) $x_i^{l+1,0} = \frac{n_{l+1}}{n_l} (A x_{i-1}^{l,0} + B y_i^{l,0} - B y_{i-2}^{l,0})$

$y_i^{l+1,0} = \frac{n_{l+1}}{n_l} A y_{i-1}^{l,0} + B x_i^{l,0})$

Having all the necessary expressions, we can make a program for a computing machine using all these relations and we will get a numerical result for specific initial values. However, this has a disadvantage since we do not know the form of separate coeficients and we compute everything even if maybe majority of the terms has zero value. But on the other side this is a burden only for the computer.

The only possibility to obtain the explicite analytical forms of the individual coefficients is to use the algebra manipulation languagues — we used for solving the above quoted expressions the languague „REDUCE", especially modified for computing individual terms of multiples of series (Fig. 1). The computation was done on the EC 1040 computer (3).

```
                              3           3
 K2*K1*AG00 + 32*K2*K1 + 2*K1 *AG00 + 4*K1  + 16*K

     2    2   2            2   2                     2   2
 (48*A *K2 *K3 *N21 + 48*A *K2 *K3*K1*N21 + 12*A *K2

     2      2          2                      2
 24*A *K2*K3 *N21 + 48*A *K2*K3*K1*N21 + 24*A *K2*K1

 2   2          2               2  3                  2
 *K1 *N21 + 64*A *N21 + 32*A*K2 *K3 *N10 + 48*A*K2

   2              2              2  3
 K1 *N10 + 64*A*K2 *K3*N10 + 4*A*K2 *K1 *N10 + 32*A*(

                  2                        2
 + 24*A*K2*K3 *K1*N10 + 24*A*K2*K3*K1 *N10 + 64*A*K1

                                   3
 + 64*A*K2*K1*N10 + 8*A*K1 *N10 + 64*A*K1*N10 + 16*1
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Fig. 1. An example of a computer output of the algebraic formula computation.

The results of the computation gives us the shortperiodic as well as secular perturbations; secular after one revolution. The effect of drag concetrates very much around perigee. According to our definition of the atmospheric density, we can have separate results for the effect of the atmospherical bulge (Figs. 2, 3).

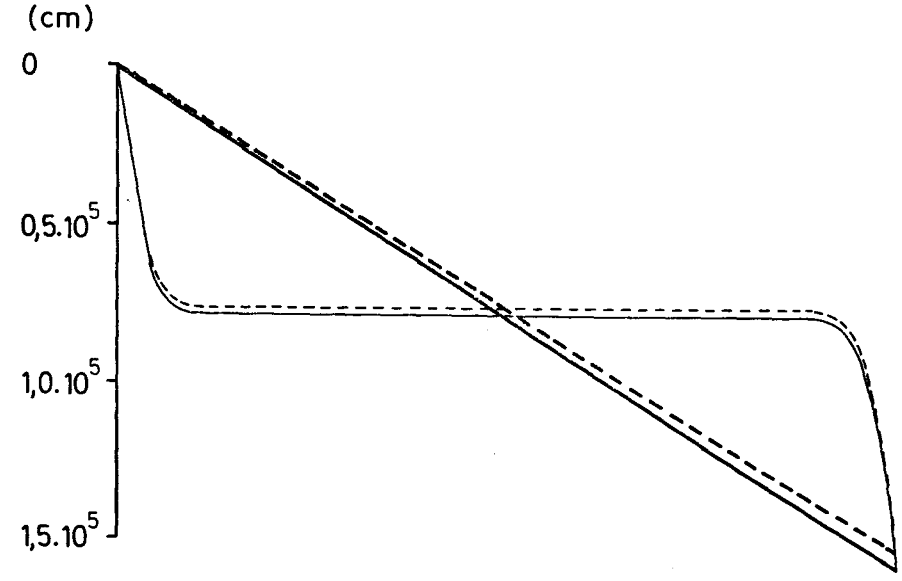

Fig. 2. The secular and shortperiodic changes of the semimajor axis caused by the drag force. The dashed lines indicate the changes without considering the atmospherical bulge effect.

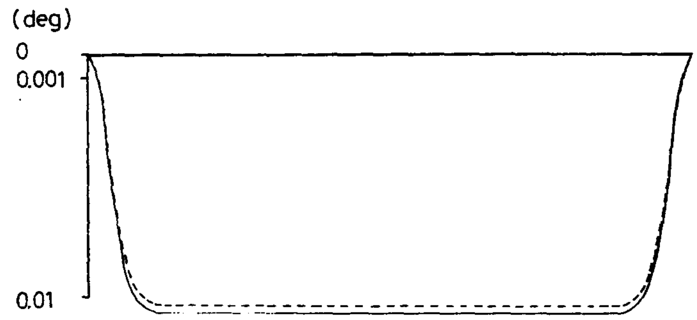

Fig. 3. The shortperiodic change of the argument of perigee caused by the drag force. The dashed line indicates the change without considering the atmospheric bulge effect.

Let us now turn our attention to the lift force, which might be of importance in case of an oriented Earth satellite. Especially if the satellite has some larger flat surface planes (e. g. like solar paddles) (Fig. 4), this effect could come into consideration (4).

The lift disturbing acceleration can be described by the same equation as that for drag:

$$F = \frac{1}{2} C_L \frac{A}{m} \rho V^2,$$

where the lift coefficient is now dependent on the angle of the incident molecules with the plane of the satellite surface (η). Besides, we have to distinguish between

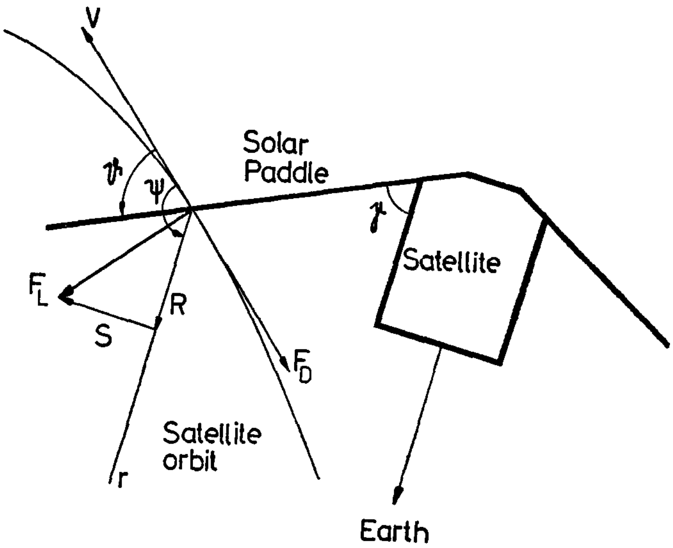

Fig. 4. The geometry of a motion of an oriented satellite with solar paddles through the atmosphere. F_D is the drag force, F_L is the lift.

the diffuse and specular reflection of incident molecules. We thenhave according to Moe (5):

$$C_L\,(\text{diffuse}) = 2/3\,(1 - \alpha)^{1/2} \sin 2\,\eta$$

$$C_L\,(\text{specular}) = 2 \sin \eta \sin 2\,\eta,$$

where α is the accomodation coefficient which defines the adjustment of the energy of the reemitted molecules to the temperature of the surface.

We shall make a simplifying assumption about the Earth atmosphere, considering its density to be given by a simple formula

$$\rho = \rho_0 \exp\,[(r_0 - r)/H],$$

where H is the density scale height.

From the character of the disturbing acceleration, we see immediately that the secular change of semi-major axis vanishes (the disturbing acceleration acts perpendicularly to the velocity vector). Moreover, since we suppose the lift force acting in the orbital plane, the changes of the inclination and of the ascending node vanish, too. Therefore we shall pay our attention mainly to the radius-vector. After introducing the Fourier expansion we can integrate and will get a series with the Bessel functions of the first kind of imaginary argument. The result for radius vector r will be then

$$r = -1/3a^2 (1-e^2)^{1/2} A/m\, \rho_p \exp(-z) \cdot$$

$$\cdot \sum_{j=0}^{2} E_{(j)} I_{(j)} nt + \sum_{i=1}^{P} \frac{1}{i} \sum_{j=0}^{2} \frac{1}{z} F_{(j)} I_{(i,j)} \cos iE +$$

$$+ \sum_{i=1}^{P} \frac{1}{i} \sum_{j=0}^{2} G_{(j)} I_{(i,j)} \sin iE$$

where the coefficients $E_{(j)}$, $F_{(j)}$ and $G_{(j)}$ are the functions of e and angle γ, which is made by the plane of the solar paddles and the radius-vector. If the satellite is Earth-oriented, the angle γ is constant.

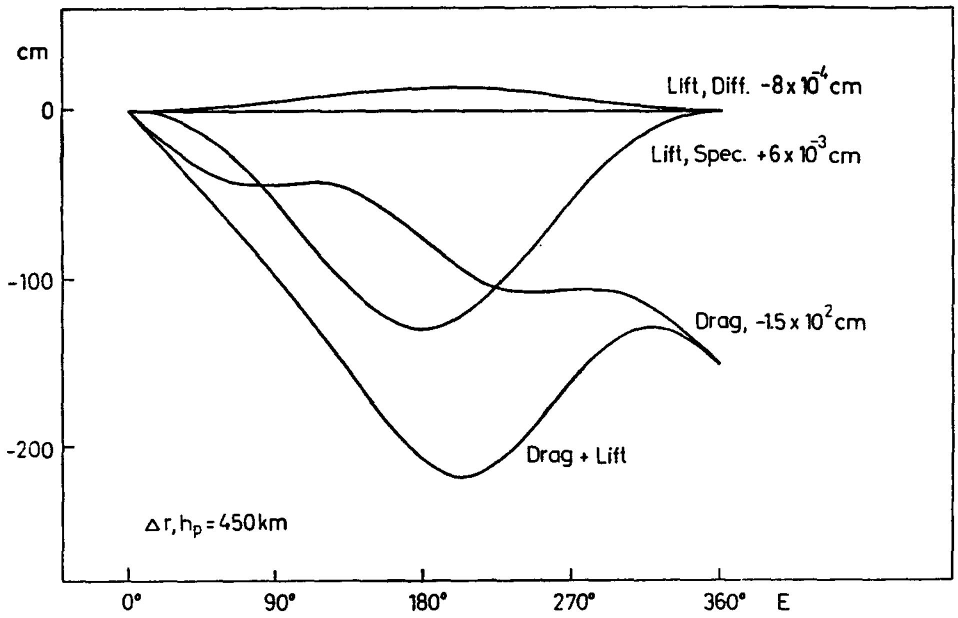

Fig. 5. The effect of the lift force on the changes of the radius-vcctor during one revolution.

The results on Fig. 5 show us that the secular changes are of several orders less than that caused by drag, whereas the shorperiodic ones are of the same order. This is of importance especially for the computation of orbits if the individual observed satellite postitions are compared to the computed ones.

We shall now deal with the effect of the rotation of the atmosphere. This perturbing effect is pronounced mainly in the component perpendicular to the orbital plane. Looking at the Lagrange equations we see that it means the changes of Ω, ω and i. The changes of Ω and ω are very much outbalanced by the effectl of the irregularities of the Earth gravitational field. On the other side, the orbital inclination i suffers only very slight perturbations of mainly periodical character from the odd zonal harmonics and lunisolar terms. Therefore, the rotation of the atmosphere will be the principal disturbing force causing secular change of the inclination.

The change of the inclination can be expressed as

$$\Delta i = -\frac{1}{2}\sqrt{\frac{a}{GM}}\,\Lambda\pi\, a^2\, \omega_E\, C_D A/m \exp(-a/H) \sin i \cdot$$
$$\cdot \left[I_0 + (1+c)\, I_2 \cos 2\omega - 2e\, I_1 \cos^2 \omega + \frac{1}{2}\, c\, (I_0 + I_4 \cos 4\omega) \right]$$

the terms of the order of c^2 and e^2 being omitted.

From this we see that the rotation of the atmosphere steadily diminishes the inclination and this diminishing is proportional to the rotational speed of the atmosphere, since Λ is the ratio of the atmospherical angular speed of rotation to that of the solid Earth.

In the above equation, H is the density scale height, ω_E is the rotational speed of the Earth and I_i are the Bessel functions of first kind of imaginary argument.

The above written equation for Δi can be numerically integrated to get the course of the inclination during a longer time interval. Then, comparing the theoretical and observed values, we can find the initial constant Λ which determines the rate of decline of i. We made such determinations of Λ analysing the inclination changes of the Interkosmos satellites. Those satellites were taken into our consideration mainly since we had at our disposal some original unpublished elements and since some of the orbits were analysed also from other points of interest. The results obtained up till now are summarized in Table 1:

Satellite Interkosmos	Λ	σ	h_{ef} (km)	*life* (d)	i (deg)
1	1.02	0.3	260	78	0.027
3	1.20	0.05	222	121	0.055
4	1.05	0.3	280	95	0.017
5	1.25	0.05	213	127	0.048
7	1.15	0.15	271	97	0.016
9	1.10	0.05	211	180	0.053
10 a)	1.30	1.90	286	44	0.0028
b)	1.20	0.16	286	280	0.0244
c)	1.20	0.23	275	119	0.0138
d)	1.10	0.26	247	39	0.0219

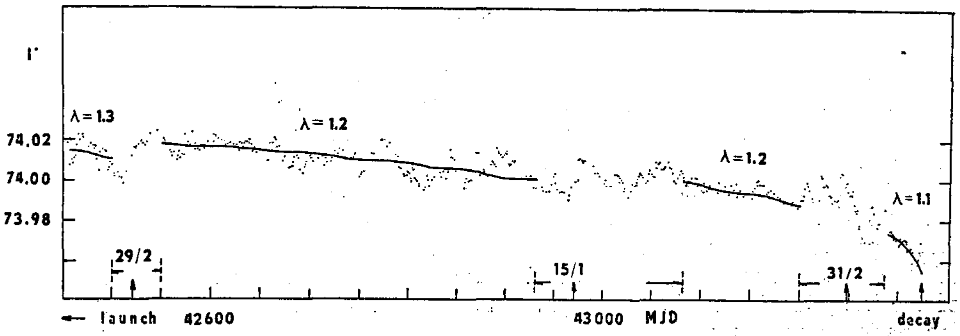

Fig. 6. The change of the inclination of the orbit of the INTERKOSMOS 10. The rotation velocity Λ of the atmosphere was determined between the intervals of resonances.

The orbit of the satellite Interkosmos 10 (Fig. 6) had to be divided cnto four parts since at certain intervals the satellite came into the resonance conditions (resonance of the satellite mean daily motion with the angular rotation rate of the Earth) which distorted substantially the otherwise continuous decrease of the inclination (6). The rotation of the atmosphere must be, of course, understood in terms of winds filowing with prevailing west-east directions.

The higher rotational speed of the atmosphere is not yet completely physically explained; we tried to find some other possible source of that effect and we looked at the Lorentz force, which appears as a result of motion of an electrically charged satellite in the geomagnetical field. We shall not go into details of this; to quote just the result, we computed the voltage of the satellite necessary to change the initial inclination in agreement with the observations. We came to a magnitude of 10^3 volts, which was neither observed nor it can be supposed theoretically (*7*).

Thus, we believe that the „superrotation" of the atmosphere in the respective heights (200 — 350 km) is real.

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