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On E-Bochner curvature tensor of contact metric generalized (κ,μ) space forms

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Abstract. Here we derive the necessary and sufficient condition for the Sasakian structure corresponding to the contact metric generalized (κ,μ)-space forms. Further, we study the contact metric generalized (κ,μ)-space forms satisfying $B^e(\xi,X)\cdot\varphi=0$, $B^e(\xi,X)\cdot h=0$, and $B^e(\xi,X)\cdot S=0$, where B^e is a E-Bochner curvature tensor, $h:=\frac{1}{2}\pounds_\xi\varphi$ and S is the Ricci tensor.

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1. Introduction

In differential geometry the Riemannian curvature tensor plays a prominent role in the study of Riemannian manifolds. It is well known that sectional curvatures determine curvature tensor completely, and it is defined that Riemannian manifold with constant sectional curvature c is a real space form, its curvature tensor is given by

$$R(X,Y)Z=c\{g(Y,Z)X-g(X,Z)Y\}, \tag{1.1}$$

In 2004 [1], P. Alegra, A. Carriazo, and D. E. Blair introduced the notion of generalized Sasakian space form as an extension of the real space forms in the background of an almost contact metric geometry. Generalized Sasakian space is an almost contact metric manifold whose curvature tensor R satisfies the following condition:

$$R=f_1R_1+f_2R_2+f_3R_3, \tag{1.2}$$

where f_1, f_2, f_3 are smooth functions on M and the curvature tensors R_1, R_2, and R_3 are given by

$$R_1(X,Y)Z=g(Y,Z)X-g(X,Z)Y,$$
$$R_2(X,Y)Z=g(X,\varphi Z)\varphi Y-g(Y,\varphi Z)\varphi X+2g(X,\varphi Y)\varphi Z,$$
$$R_3(X,Y)Z=\eta(X)\eta(Z)Y-\eta(Y)\eta(Z)X+g(X,Z)\eta(Y)\xi-g(Y,Z)\eta(X)\xi.$$

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In the expression R of generalized Sasakian space forms, if $f_2 = f_3 = 0$ then it reduces to real space forms. Later, in 2012 [10] the notion of generalized (κ, μ)-space forms was defined as an almost contact metric manifold M whose curvature tensor satisfies the following condition.

$$R = f_1R_1 + f_2R_2 + f_3R_3 + f_4R_4 + f_5R_5 + f_6R_6, \tag{1.3}$$

where f_1, f_2, f_3, f_4, f_5, f_6 are smooth functions on M and R_1, R_2, R_3, R_4, R_5, R_6 are the curvature tensors defined as

$$R_1(X,Y)Z = g(Y,Z)X - g(X,Z)Y,$$
$$R_2(X,Y)Z = g(X,\varphi Z)\varphi Y - g(Y,\varphi Z)\varphi X + 2g(X,\varphi Y)\varphi Z,$$
$$R_3(X,Y)Z = \eta(X)\eta(Z)Y - \eta(Y)\eta(Z)X + g(X,Z)\eta(Y)\xi - g(Y,Z)\eta(X)\xi,$$
$$R_4(X,Y)Z = g(hY,Z)hX - g(hX,Z)hY + g(\varphi hX,Z)\varphi hY - g(\varphi hY,Z)\varphi hX,$$
$$R_5(X,Y)Z = g(Y,Z)hX - g(X,Z)hY + g(hY,Z)X - g(hX,Z)Y,$$
$$R_6(X,Y)Z = \eta(X)\eta(Z)hY - \eta(Y)\eta(Z)hX + g(hX,Z)\eta(Y)\xi - g(hY,Z)\eta(X)\xi.$$

For $f_4 = f_5 = f_6 = 0$ it reduces to generalized Sasakian space forms.

Generally Sasakian implies K-contact, but the converse is true only in case of 3-dimensional manifolds. But in [2], P. Alegre proved that generalized Sasakian space form with K-contact structure is Sasakian. In [3], A. Carriazo et.al., studied the generalized (κ, μ) space forms in the frame of contact metric geometry. They conclude that being K-contact and being Sasakian are equivalent concepts and obtained some beautiful results. With the support of this result in [10] A. Carriazo et. al., proved that being Sasakian and K-contact are equivalent concepts in generalized (κ, μ) space forms.

On the other hand, in [8], S. Bochner introduced a Kähler analogue of the Weyl conformal curvature tensor which is cited as the Bochner curvature tensor. In [4] D. E. Blair interpreted geometrical meaning of the Bochner curvature tensor. In [17] Matsumoto and G. Chuman constructed the C-Bochner curvature tensor by using Boothby-Wang's fibration [9]. As an extension to C-Bochner curvature tensor H. Endo in [13] defined E-Bochner curvature tensor which is called as an extended C-Bochner curvature tensor. In [15], J. S. Kim et.al., proved (κ, μ)-contact metric manifold with vanishing E-Bochner curvature tensor is Sasakian. Further, Bochner curvature has been extensively studied in [14], [12], [11], [18].

Our paper is organized as follows: after preliminaries, in Section 3, we study the E-Bochner curvature tensor and obtain a necessary and sufficient condition for Sasakian structure corresponding to contact metric generalized (κ, μ)-space forms. In Section 4, we study the contact metric generalized (κ, μ)-space forms satisfying $B^e(\xi, X)\cdot\varphi = 0$, and we draw several corollaries. In Section 5 and in 6 we study contact metric generalized (κ, μ)-space forms satisfying $B^e(\xi, X)\cdot h = 0$ and $B^e(\xi, X) \cdot S = 0$ in a non-Sasakian case.

By assuming κ, μ as smooth functions in [16], Koufogiorgos and Tschlias defined the notion of generalized (κ, μ) contact metric manifolds and proved their existence for the 3-dimensional case and non-existence for greater than

3-dimension. Motivated by this result in this paper and with the help of [7], we pay attention to the 3-dimensional case.

2. Preliminaries

In this section, we briefly recall some basic definitions and properties of contact metric manifolds. Further we infer some results which are useful for our paper.

If an odd dimensional smooth manifold M together with $(1,1)$ tensor field φ, a vector field ξ, and a 1-form η satisfies following conditions,

$$\varphi^2 = -I + \eta \otimes \xi, \quad \eta(\xi) = 1, \tag{2.1}$$

for all $X, Y \in TM$, then M is said to be an *almost contact manifold.* In an almost contact manifold, relation (2.1) implies,

$$\varphi\xi = 0, \quad \eta \circ \varphi = 0, \tag{2.2}$$

and the rank of φ is $2n$. It is well known that the first relation from (2.1) together with any relation from (2.2) also defines an almost contact manifold.

In an almost contact manifold there is always exits a positive definite metric g such that

$$g(\varphi X, \varphi Y) = g(X, Y) - \eta(X)\eta(Y), \tag{2.3}$$

or equivalently,

$$g(\varphi X, Y) = -g(X, \varphi Y) \quad \textit{and} \quad g(X, \xi) = \eta(X) \tag{2.4}$$

for all $X, Y \in TM$, which is called a *compatible Riemannian metric* and corresponding almost contact manifold is an *almost contact metric manifold.*

A $(2n+1)$-dimensional manifold is said to be a *contact manifold* if it admits a global 1-form η such that $\eta \wedge (d\eta)^n \neq 0$ everywhere on M^{2n+1}.

If an almost contact metric manifold M^{2n+1} with almost contact metric structure (φ, ξ, η, g) holds the property $d\eta(X, Y) = g(X, \varphi Y)$, then it said to be a *contact metric manifold.*

Let M be a $(2n+1)$-dimensional contact metric manifold with contact metric structure (φ, ξ, η, g). If it has the vanishing torsion tensor $[\varphi, \varphi] + 2d\eta \otimes \xi$, where $[\varphi, \varphi]$ is a Nijenhuis tensor of φ then it is said to be a *Sasakian manifold.*

In a contact metric manifold, the tensor $h = \frac{1}{2}\pounds_\xi\varphi$ of type $(1,1)$ is a symmetric operator and satisfies the following relations:

$$h\xi = 0, \quad h\varphi = -\varphi h, \quad trace(h) = trace(\varphi h) = 0, \quad \eta \circ h = 0. \tag{2.5}$$

For more details we refer to [5].

A (κ, μ)-*contact metric manifold* is the class of $(2n+1)$-dimensional contact metric manifold M in which its curvature tensor R satisfies following relation

$$R(X, Y)\xi = (\kappa I + \mu h)\{\eta(Y)X - \eta(X)Y\}, \tag{2.6}$$

where $(\kappa,\mu)\in\mathbb{R}^2$, for all $X,Y\in TM$, with $h^2=(\kappa-1)\varphi^2$, and $\kappa\leq 1$. For more information we recommend the reference [6]. We recall some results here, due to A. Carriazo et. al., [10].

Theorem 2.1. *Let $M(f_1,f_2,f_3,f_4,f_5,f_6)$ be a generalized (κ,μ)-space form. If M is a contact metric manifold with $f_1-f_3=1$, then it is Sasakian.*

Theorem 2.2. *Let $M(f_1,f_2,f_3,f_4,f_5,f_6)$ be a generalized (κ,μ)-space form. If M is a Sasakian manifold, then $f_1-f_3=1$.*

Theorem 2.3. *If $M(f_1,f_2,f_3,f_4,f_5,f_6)$ is a non-Sasakian contact metric generalized (κ,μ)-space form of dimension greater than or equal to 5, then M is a $(-f_6,1-f_6)$-space with constant φ-sectional curvature $c=2f_6-1$, where $f_6=constant>-1$ and $f_4=1$.*

Theorem 2.4. *Let M be a non-Sasakian contact metric generalized (κ,μ)-space form of dimension greater than or equal to 5, then M is a (κ,μ)-contact metric manifold, that is, the functions $\kappa=f_1-f_3$ and $\mu=f_4-f_6$ become constants.*

In [19], due to K. Mirji et. al., we recall

Theorem 2.5. *If $M^3(f_1,f_2,f_3,f_4,f_5,f_6)$ is a contact metric generalized (κ,μ)-space form, then the following conditions are equivalent to one another:*

- *M^3 is η-Einstein,*
- *$Q\varphi=\varphi Q$, where Q denotes the Ricci operator,*
- *$f_4-f_6=0$,*
- *M^3 is pseudo symmetric,*
- *M^3 is ξ-projectively flat.*

3. ξ - E - Bochner flat contact metric generalized (κ,μ) space forms

In [17], Matsumoto and G. Chuman constructed the *C-Bochner curvature tensor* in an almost contact metric manifold as follows:

$$\begin{aligned}B(X,Y)Z=&R(X,Y)Z-\frac{m-4}{2n+4}R_0(Y,X)Z+\frac{1}{2n+4}\{R_0(QY,X)Z\\&-R_0(QX,Y)Z+R_0(Q\varphi Y,\varphi X)Z-R_0(Q\varphi X,\varphi Y)Z\\&+2g(Q\varphi X,Y)\varphi Z+2g(\varphi X,Y)Q\varphi Z+\eta(Y)R_0(QX,\xi)Z\\&+\eta(X)R_0(\xi,QY)Z\}-\frac{m+2n}{2n+4}\{R_0(\varphi Y,\varphi X)Z+2g(\varphi X,Y)\varphi Z\}\\&+\frac{m}{2n+4}\{\eta(Y)R_0(\xi,X)Z+\eta(X)R_0(Y,\xi)Z\},\end{aligned}\tag{3.1}$$

where $m = \frac{2n+r}{2n+2}$, r is a scalar curvature, Q is the Ricci operator. Generally the term $R_0(X,Y)Z$ is defined as

$$R_0(X,Y)Z = g(Y,Z)X - g(X,Z)Y.$$

In [13], H. Endo constructed *E-Bochner curvature tensor* by using C-Bochner curvature tensor like this

$$\begin{aligned} B^e(X,Y)Z =& B(X,Y)Z - \eta(X)B(\xi,Y)Z \\ & - \eta(Y)B(X,\xi)Z - \eta(Z)B(X,Y)\xi. \end{aligned} \tag{3.2}$$

For a contact metric generalized (κ, μ)-space form M^{2n+1}, we have the following relations:

$$\begin{aligned} S(X,Y) =& \{2nf_1 + 3f_2 - f_3\}g(X,Y) + \{(2n-1)f_4 - f_6\}g(hX,Y) \\ & - \{3f_2 + (2n-1)f_3\}\eta(X)\eta(Y), \end{aligned} \tag{3.3}$$

$$R(X,Y)\xi = (f_1 - f_3)\{\eta(Y)X - \eta(X)Y\} + (f_4 - f_6)\{\eta(Y)hX - \eta(X)hY\}, \tag{3.4}$$

$$\begin{aligned} R(\xi,Y)Z =& (f_1 - f_3)\{g(Y,Z)\xi - \eta(Z)Y\} + (f_4 - f_6)\{g(hY,Z)\xi - \eta(Z)hY\} \\ =& - R(Y,\xi)Z. \end{aligned} \tag{3.5}$$

From (3.1), (3.4) and (3.5) we have the following relations

$$\begin{aligned} B(X,Y)\xi =& \frac{2(f_1 - f_3 - 1)}{(n+2)}\{\eta(Y)X - \eta(X)Y\} \\ & + (f_4 - f_6)\{\eta(Y)hX - \eta(X)hY\}, \end{aligned} \tag{3.6}$$

$$\begin{aligned} B(\xi,Y)Z =& \frac{2(f_1 - f_3 - 1)}{(n+2)}\{g(Y,Z)\xi - \eta(Z)Y\} \\ & + (f_4 - f_6)\{g(hY,Z)\xi - \eta(Z)hY\}, \end{aligned} \tag{3.7}$$

$$\begin{aligned} B(\xi,Y)\xi =& \frac{2(f_1 - f_3 - 1)}{(n+2)}\{\eta(Y)\xi - Y\} \\ & + (f_4 - f_6)\{-hY\}. \end{aligned} \tag{3.8}$$

By using (3.2) and (3.8), we get,

$$\begin{aligned} B^e(X,Y)\xi =& \frac{2(f_1 - f_3 - 1)}{(n+2)}\{\eta(X)Y - \eta(Y)X\} \\ & + (f_4 - f_6)\{\eta(X)hY - \eta(Y)hX\}, \textit{and} \end{aligned} \tag{3.9}$$

$$B^e(\xi, Y)Z = \frac{2(f_1 - f_3 - 1)}{(n+2)}\{\eta(Z)Y - \eta(Y)\eta(Z)\xi\} + (f_4 - f_6)\{\eta(Z)hY\}. \tag{3.10}$$

Clearly from (3.9) and (3.10) we have

$$\eta(B^e(X, Y)\xi) = 0, \qquad \eta(B^e(\xi, Y)Z) = 0 \quad \textit{and} \tag{3.11}$$

$$B^e(\xi, Y)\xi = \frac{2(f_1 - f_3 - 1)}{(n+2)}\{Y - \eta(Y)\xi\} + (f_4 - f_6)hY. \tag{3.12}$$

Now we are going to prove the following theorem;

Theorem 3.1. *Let M^{2n+1} $(n \geq 1)$ be a contact metric generalized (κ, μ)- space form then it is Sasakian if and only if it is ξ-E-Bochner flat.*

Proof. Let M^{2n+1} be a contact metric generalized (κ, μ)-space form which is ξ-E-Bochner flat, then from relation (3.9) we have

$$\frac{2(f_1 - f_3 - 1)}{(n+2)}\{\eta(X)Y - \eta(Y)X\} + (f_4 - f_6)\{\eta(X)hY - \eta(Y)hX\} = 0.$$

On contracting the above equation over X with respect to an orthonormal basis we get,

$$\frac{2(f_1 - f_3 - 1)}{(n+2)}2n\eta(Y) = 0,$$

this implies

$$f_1 - f_3 = 1, \quad for \quad n \geq 1.$$

Then by using Theorem 2.1 we can conclude that, for $n \geq 1$, ξ-E-Bochner flat contact metric generalized (κ, μ)-space form M^{2n+1} is Sasakian.

Conversely, suppose M^{2n+1} is a Sasakian manifold then by Theorem 2.2 we have the condition

$$f_1 - f_3 = 1.$$

Substituting this in (3.9), directly we get

$$B^e(X, Y)\xi = 0.$$

This completes the proof. □

Definition 3.2. A contact metric manifold M is said to be η-Einstein if its Ricci tensor satisfies the following condition

$$S(X, Y) = ag(X, Y) + b\eta(X)\eta(Y),$$

where a and b are smooth functions on M.

Corollary 3.3. *If M^{2n+1} $(n \geq 1)$ is a ξ-E-Bochner flat contact metric generalized (κ, μ)-space form then it is always η-Einstein.*

Since M^{2n+1} is a ξ-E-Bochner flat contact metric generalized (κ, μ)-space form, from Theorem 3.1 it is Sasakian, this implies $h = 0$, so by relation (3.3) we have

$$S(X, Y) = (2nf_1 + 3f_2 - f_3)g(X, Y) - (2nf_1 + 3f_2 - f_3)\eta(X)\eta(Y),$$

where $(2nf_1 + 3f_2 - f_3)$ and $(2nf_1 + 3f_2 - f_3)$ are smooth functions on M^{2n+1}. Then from Definition (3.2) we can say that M^{2n+1} is η-Einstein.

Corollary 3.4. *Let M^{2n+1} $(n > 1)$ be a contact metric generalized (κ, μ)-space form. If it is not ξ-E-Bochner flat then M^{2n+1} reduces to $(-f_6, 1 - f_6)$-space, ($f_6 =$ constant > -1) with constant φ-sectional curvature $c = 2f_6 - 1 > -3$.*

Let M^{2n+1} $(n > 1)$ be a contact metric generalized (κ, μ)-space form. If it is not ξ-E-Bochner flat, then by Theorem 3.1 it is non-Sasakian. Then the proof of the corollary follows from Theorem 2.3.

Corollary 3.5. *Let $M^3(f_1, f_2, f_3, f_4, f_5, f_6)$ be a ξ-E-Bochner flat contact metric generalized (κ, μ)-space form. Then it satisfies the following conditions;*

- $f_4 - f_6 = 0$,
- *M^3 is pseudo symmetric,*
- *M^3 is ξ-projectively flat.*

Let $M^3(f_1, f_2, f_3, f_4, f_5, f_6)$ be a ξ-E-Bochner flat contact metric generalized (κ, μ)-space form, then the rest of the proof of the corollary follows from Corollary 3.3 and Theorem 2.5.

Remark 3.6. Theorem 3.1, Corollary 3.3, Corollary 3.4 and Corollary 3.5 are valid for ξ -C-Bochner flat contact metric generalized (κ, μ) space forms, also.

4. Contact metric generalized (κ, μ)-space forms satisfying $B^e(\xi, X) \cdot \varphi = 0$

Let M^{2n+1} be a contact metric generalized (κ, μ) space form satisfying the condition

$$B^e(\xi, X) \cdot \varphi = 0. \tag{4.1}$$

The condition (4.1) gives

$$B^e(\xi, X)\varphi Y - \varphi B^e(\xi, X)Y = 0 \tag{4.2}$$

for all X, Y $\in$ TM. Using (3.10), we get

$$\frac{2(f_1 - f_3 - 1)}{(n + 2)}\{\eta(Y)\varphi X\} + (f_4 - f_6)\{\eta(Y)\varphi hX\} = 0. \tag{4.3}$$

Putting $X = \varphi X$ and then contracting over X with respect to an orthonormal basis we obtain

$$2n\eta(Y)\frac{2(f_1 - f_3 - 1)}{(n+2)} = 0, \tag{4.4}$$

which gives $f_1 - f_3 = 1$.

Then by the Theorem 2.1 it is Sasakian, and so we can state the following theorem;

Theorem 4.1. *A contact metric generalized (κ, μ)-space form M^{2n+1} $(n \geq 1)$, satisfying $B^e(\xi, X) \cdot \varphi = 0$ is Sasakian.*

Corollary 4.2. *A contact metric generalized (κ, μ)-space form M^{2n+1} $(n \geq 1)$ satisfying $B^e(\xi, X) \cdot \varphi = 0$ is η-Einstein.*

Corollary 4.3. *Let $M^3(f_1, f_2, f_3, f_4, f_5, f_6)$ be a contact metric generalized (κ, μ)-space form satisfying $B^e(\xi, X) \cdot \varphi = 0$, then it satisfies following conditions:*

- $f_4 - f_6 = 0$,
- *M^3 is pseudo symmetric,*
- *M^3 is ξ-projectively flat.*

Remark 4.4. Theorem 4.1, Corollary 4.2, and Corollary 4.3, are valid for contact metric generalized (κ, μ)-space forms satisfying $B(\xi, X) \cdot \varphi = 0$, also.

5. Contact metric generalized (κ, μ)-space forms satisfying $B^e(\xi, Y) \cdot h = 0$

Let M^{2n+1} be a contact metric generalized (κ, μ)-space form satisfying the condition

$$(B^e(\xi, X) \cdot h)Y = 0,$$

for all $X, Y \in M$, then we have

$$B^e(\xi, X)hY - hB^e(\xi, X)Y = 0. \tag{5.1}$$

Using (3.10), in the above equation we get

$$\frac{2(f_1 - f_3 - 1)}{(n+2)}\{\eta(Z)hY\} + (f_4 - f_6)\{\eta(Z)h^2Y\} = 0.$$

On contracting over Y we obtain

$$2n(f_1 - f_3 - 1)(f_4 - f_6) = 0. \tag{5.2}$$

By Theorem (2.4), we can state the following theorem

Theorem 5.1. *Let M^{2n+1} $(n > 1)$ be a non-Sasakian contact metric generalized (κ, μ)-space form satisfying $B^e(\xi, X) \cdot h = 0$, then it is a $N(\kappa)$-contact metric manifold.*

Corollary 5.2. *Let M^{2n+1} $(n > 1)$ be a non-Sasakian contact metric generalized (κ, μ)-space form satisfying $B^e(\xi, X) \cdot h = 0$, then it is a $(-1, 0)$-contact metric manifold with constant φ-sectional curvature equal to 1.*

Let M^{2n+1} $(n > 1)$ be a non-Sasakian contact metric generalized (κ, μ)-space form, then $\kappa = f_1 - f_3 \neq 1$ is a constant. If it satisfies the condition $B^e(\xi, X) \cdot h = 0$, then by the relation (5.2) we obtain,

$$f_4 - f_6 = 0.$$

Then the rest of the proof follows from Theorem 2.3.

6. Contact metric generalized (κ, μ)-space forms satisfying $B^e(\xi, X) \cdot S = 0$

Suppose

$$(B^e(\xi, X) \cdot S)(U, V) = 0. \tag{6.1}$$

Put $V = \xi$ then we get,

$$S(B^e(\xi, X)U, \xi) + S(U, B^e(\xi, X)\xi) = 0. \tag{6.2}$$

By using relations, $S(X, \xi) = 2n(f_1 - f_3)\eta(X)$, (3.11) and (3.12), in (6.2) becomes

$$\frac{2(f_1 - f_3 - 1)}{(n + 2)}\{S(U, X) - \eta(X)S(U, \xi)\} + (f_4 - f_6)S(U, hX) = 0. \tag{6.3}$$

Put $U = X = e_i$ and taking the summation from $i = 1$ to $2n + 1$, we obtain

(6.4)
$$\frac{2(f_1 - f_3 - 1)}{(n + 2)}\{r - 2n(f_1 - f_3) - n(n + 2)(f_4 - f_6)\{(2n - 1)f_4 - f_6\}\} = 0,$$

where r is the scalar curvature. By using Theorem 2.3 and Theorem 2.4 we can state the following theorem

Theorem 6.1. *Let M^{2n+1} $(n > 1)$ be a non-Sasakian contact metric generalized (κ, μ) space form satisfying $B^e(\xi, X) \cdot S = 0$, then it has a constant scalar curvature $r = 2n(f_1 - f_3) + n(n + 2)(1 - f_6)\{(2n - 1) - f_6\}$.*

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